# Solving some System of Linear Fuzzy Fractional Differential Equations by Adomian Decomposition <br> <br> Method 

 <br> <br> Method}

Asim Kumar Das ${ }^{1}$ and Tapan Kumar Roy ${ }^{2}$<br>Department of Applied Mathematics<br>Indian Institute of Engineering Science and Technology<br>Shibpur, Howrah, West Bengal, India-711103<br>${ }^{1}$ Corresponding author. Email id: asd.math @gmail.com

Received 10 June 2017; accepted 17 July 2017


#### Abstract

In this paper, we present the numerical method relatively new for solving fuzzy fractional differential equations using Adomian decomposition method (ADM) where the fractional derivative is assumed in terms Riemann-Livoullie sense. The solutions here are expressed in terms of Mittag-Leffler functions. ADM method is an interesting method to solve some fractional order differential equation numerically. Our aim in this paper is to find solution of fractional differential equation numerically with fuzzy valued initial condition by applying ADM.


Keywords: Riemann-Liouville fractional differentiation, fuzzy fractional differential equation, adomian decomposition method
AMS Mathematics Subject Classification (2010): 34A07, 34A08

## 1. Introduction

A long mathematical history has been studied so far in [11,20,22]. Fractional calculus generalizes the differentiation and integration to an arbitrary order. Fractional differential equations are of great importance in real life problems, since it generalizes our concept more precisely for better description of material properties. Recently fractional calculus has been utilized in development of models area like rehonology, viscoelasticity, electrochemistry, diffusion process etc in terms of fractional differential and fractional integrals[12],[22]. Some theoretical aspects of existence and uniqueness results for fractional differential equation have been considered recently by many authors [13].

A differential and integral calculus for fuzzy valued function was developed in some papers Hukuhara [26], Dubois and Prade [14-16]; Puri and Ralescu [23-24]. The significant results method of fuzzy differential equation and their application has been discussed in the papers [2-3,8-9]. The concept of fuzzy fractional differential equation was introduced by Agarwal, Lakshikantham and Nieto [4]. Fuzzy fractional differential equation (FFDE) has already been solved analytically using integral transform method like Laplace transform method [25]. But since analytical method has some limitation to

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solve a problem, numerical method is very useful to solve the problem. Regarding this ADM is very acceptable method to solve FFDE. The aim of this paper is to technique of findings the numerical solution of some liner fractional differential equation with initial fuzzy value using ADM. Here the obtained solution are expressed in form of MittagLeffler function, which has been discussed in [22].

The rest of the paper is organized as follows: In section 2 , we introduce some basic concepts of fuzzy mathematics and the definition and notation of Riemann-Livoullie fractional derivatives. The Adomian decomposition method (ADM) is discussed in section 3. In section 4, the solution procedure of FFDEs are determined using Adomian decomposition method (ADM). Finally, conclusion and future research are drawn in section 5.

## 2. Basic concepts

Let E be the set of all upper semi-continuous normal convex fuzzy numbers with bounded r-level intervals. We define the r-level set, if $\tilde{u} \in E \tilde{u}(r)=\{\mathrm{x} \in \mathrm{R}: \mathrm{u}(\mathrm{x}) \geq \mathrm{r}\}$, $0 \leq \mathrm{r} \leq 1$ which is a closed interval and we denoted by $[\mathrm{u}(\mathrm{r})]=[\underline{u}(r), \bar{u}(r)]$ and there exist a $x_{0} \in \mathrm{R}$ such that $\mathrm{u}\left(x_{0}\right)=1$. Two fuzzy numbers $\tilde{u}$ and $\tilde{v}$ are called equal $\tilde{u}=\tilde{v}$ iff $\mathrm{u}(\mathrm{x})=\mathrm{v}(\mathrm{x})$ for all $\mathrm{x} \in \mathrm{R}$. it follows that $\tilde{u}=\tilde{v}$ iff $[\mathrm{u}(\mathrm{r})]=[\mathrm{v}(\mathrm{r})]$ for all $\mathrm{r} \in(0,1)$.
It is clear that following statements are true

1. $\underline{u}(r)$ is a bounded left continuous non decreasing on $[0,1]$
2. $\bar{u}(r)$ is a bounded right continuous non decreasing on $[0,1]$
3. $\underline{u}(r) \leq \bar{u}(r)$ for all $\mathrm{r} \in[0,1]$

The following arithmetic operation on fuzzy numbers are well known and frequently used bellow.
If $u, v \in E$, then $[\tilde{u}(r)+\tilde{v}(r)]=[\underline{u}(r)+\underline{v}(r), \bar{u}(r)+\bar{v}(r)]$

$$
[\tilde{u}(r)-\tilde{v}(r)]=[\underline{u}(r)-\bar{v}(r), \bar{u}(r)-\underline{v}(r)]
$$

And $[\lambda \tilde{u}(r)]=\lambda[\tilde{u}(r)]=\left\{\begin{array}{l}{[\lambda \underline{u}(r), \lambda \bar{u}(r)] i f \lambda \geq 0} \\ {[\lambda \bar{u}(r), \lambda \underline{u}(r)] i f \lambda<0}\end{array} \quad \lambda \in R\right.$
For a real interval $\mathrm{I}=[0, \mathrm{a}]$, a mapping $\tilde{q}: \mathrm{I} \rightarrow \mathrm{E}$ is called a fuzzy function. We denote the r-cut representation of fuzzy valued function as $[\tilde{q}(t ; r)]$ and defined by $[\tilde{q}(t ; r)]=$ $[\underline{q}(t ; r), \bar{q}(t ; r)]$,for $\mathrm{t} \in \mathrm{I}$ and $\mathrm{r} \in(0,1]$. The derivative of a fuzzy function $\tilde{q}(t ; r)$ is given by $\left[\tilde{q}^{\prime}(t ; r)\right]=\left[\underline{q^{\prime}}(t ; r), \bar{q}^{\prime}(t ; r)\right], \quad \mathrm{r} \in(0,1]$. Provided that $q^{\prime}(t) \in \mathrm{E}$. the fuzzy integral $\int_{a}^{b} \tilde{q}(t ; r) d t$ is defined by $\left[\int_{a}^{b} \tilde{q}(t ; r) d t\right]=\left[\int_{a}^{b} \underline{q}(t ; r) d t, \int_{a}^{b} \bar{q}(t ; r) d t\right]$ provided that the integral on the right side exist.

We introduce our definition of Riemann-Liouville integrals and derivatives. Let $C^{F}[a, b]$ be the set of all continuous function on $[\mathrm{a}, \mathrm{b}]$ and $L^{F}[a, b]$ denote the space of all Lebesque integrable function on the bounded interval $[\mathrm{a}, \mathrm{b}] \subset \mathrm{R}$. Let $q \in C^{F}[a, b] \cap$ $L^{F}[a, b]$. The Riemann-Liouville fractional integral of fractional order ' $\alpha$ ' of $\mathrm{q}(\mathrm{t})$ is

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 Decomposition Methoddefined as $J^{\alpha} q(t)=\frac{1}{\Gamma(\alpha)} \int_{a}^{t} \frac{q(\tau) d \tau}{(t-\tau)^{1-\alpha}}, \mathrm{t}>\mathrm{a}, 0<\alpha \leq 1$. And the Riemann-Liouville fractional derivative of fractional order ' $\alpha$ ' of $\mathrm{q}(\mathrm{t})$ is defined as $D^{\alpha} q(t)=\frac{1}{\Gamma(-\alpha)} \int_{a}^{t} \frac{q(\tau) d \tau}{(t-\tau)^{\alpha+1}}$
We now define the r-cut representation of fuzzy fractional Riemann-Liouville integral of fuzzy valued function $\tilde{q}(\mathrm{t} ; \mathrm{r})$ based on the lower and upper function as following:
$J^{\alpha} \tilde{q}(t ; r)=\left[J^{\alpha} \underline{q}(t ; r), J^{\alpha} \bar{q}(t ; r)\right]$, for $0 \leq \mathrm{r} \leq 1$
where $J^{\alpha} \underline{q}(t ; r)=\frac{1}{\Gamma(\alpha)} \int_{a}^{t} \frac{q(\tau ; r)}{(t-\tau)^{1-\alpha}} d \tau$ and $J^{\alpha} \bar{q}(t ; r)=\frac{1}{\Gamma(\alpha)} \int_{a}^{t} \frac{\bar{q}(\tau ; r)}{(t-\tau)^{1-\alpha}} d \tau$
And the r-cut representation of fuzzy fractional Riemann-Liouville derivative of fuzzy valued function $\widetilde{q}(t ; r)$ based on the lower and upper function is defined as following:

$$
D^{\alpha} \tilde{q}(t ; r)=\left[D^{\alpha} \underline{q}(t ; r), D^{\alpha} \bar{q}(t ; r)\right], \text { for } 0 \leq \mathrm{r} \leq 1
$$

where $D^{\alpha} \underline{q}(t ; r)=\frac{1}{\Gamma(-\alpha)} \int_{a}^{t} \frac{q(\tau ; r)}{(t-\tau)^{\alpha+1}} d \tau$ and $D^{\alpha} \bar{q}(t ; r)=\frac{1}{\Gamma(-\alpha)} \int_{a}^{t} \frac{\bar{q}(\tau ; r)}{(t-\tau)^{\alpha+1}} d \tau$

## 3. Adomian decomposition method (ADM)

We consider the differential equation of the form: $\mathrm{Fu}(\mathrm{t})=\mathrm{g}(\mathrm{t})$, where F is a general differential operator, $F$ can be divided into three parts $L, R$ and $N$ such that

$$
\begin{equation*}
\mathrm{Lu}+\mathrm{Ru}+\mathrm{Nu}=\mathrm{g} \tag{3.1}
\end{equation*}
$$

where $L$ is the highest order derivative which is assumed to be easily invertible, $R$ is a linear differential operator of order less than $L, N$ represents the nonlinear terms, and $g$ is the source term. We may write the equation (3.1) as

$$
\begin{equation*}
\mathrm{Lu}=\mathrm{g}-\mathrm{Ru}-\mathrm{Nu} . \tag{3.2}
\end{equation*}
$$

Applying the inverse operator $L^{-1}$ (which is the inverse operator of $L$ ) on both sides of equation (1.2) one may obtain the following equation:

$$
\begin{equation*}
L^{-1} \mathrm{Lu}=L^{-1} \mathrm{~g}-L^{-1} \mathrm{Ru}-L^{-1} \mathrm{~N} \tag{3.3}
\end{equation*}
$$

For initial value problems, we define $L^{-1}$ for $L \equiv \frac{\partial^{n}}{\partial t^{n}}$ as the $n$-fold definite integral operator from 0 to $t$. for example if $L \equiv \frac{\partial^{2}}{\partial t^{2}}$ is a second order operator then $L^{-1}$ is a twofold integration operator.
We have $L^{-1} \mathrm{Lu}=u(t)-u(0)-t u^{\prime}(t)$, and therefore (3.3) becomes

$$
\begin{equation*}
u(t)=u(0)+t u^{\prime}(t)+L^{-1} \mathrm{~g}-L^{-1} \mathrm{Ru}-L^{-1} \mathrm{Nu} . \tag{3.4}
\end{equation*}
$$

According to ADM, the solution u is assumed as infinite sum of series $u=\sum_{n=0}^{\infty} u_{n}$,
and the nonlinear term Nu is decomposed as follows: $N u=\sum_{n=0}^{\infty} A_{n}$,
where $A_{n}$ are the set of Adomian polynomials. $A_{n}$ 's can be found from formula

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$$
\begin{equation*}
A_{n}=\frac{1}{n!} \frac{\partial^{n}}{\partial t^{n}} N\left(\sum_{i=1}^{n} \lambda^{i} u_{i}\right) \tag{3.7}
\end{equation*}
$$

Substituting (3.5) and (3.6) into Eqn. (1.4)

$$
\begin{equation*}
\text { we get } \quad u=\sum_{n=0}^{\infty} u_{n}=u_{0}-L^{-1} R \sum_{n=0}^{\infty} u_{n}-L^{-1} \sum_{n=0}^{\infty} A_{n} \tag{3.8}
\end{equation*}
$$

where $u_{0}=u(0)+t u^{\prime}(0)+L^{-1} g$ and $u_{n+1}=-L^{-1} R u_{n}-L^{-1} \sum_{n=0}^{\infty} A_{n}, \mathrm{n} \geq 0$
Substituting the values of $u_{0}, u_{1}, u_{2}, \ldots$. in Eqn.(3.4), we can obtain $u$. The convergence for the aforementioned series has been found in [1], [10],[18]

## 4. Solution of linear FFDE by ADM

Let us consider linear fuzzy fractional differential equation $D^{\alpha} \tilde{q}(t ; r)=a \tilde{q}(t ; r)+\mathrm{f}(\mathrm{t})$,
where $0<\alpha \leq 1$, and $f(t)$ is either constant or function of ' $t$ '.
' $a$ ' being constant. with the fuzzy initial condition; $\left[J^{1-\alpha} \tilde{q}(t ; r)\right]_{t=0}=\tilde{Q}=\left(\gamma_{1}, \gamma_{2}, \gamma_{3}\right)$, on using the relation $J^{\alpha} D^{\alpha} q(t ; r)=q(t)-\frac{t^{\alpha-1}}{\Gamma(\alpha)}\left[J^{1-\alpha} q(t ; r)\right]_{\mathrm{t}=0}$ we have,

$$
\begin{equation*}
J^{\alpha} D^{\alpha} q(t ; r)=q(t)-\frac{t^{\alpha-1}}{\Gamma(\alpha)} \tilde{Q} \tag{4.2}
\end{equation*}
$$

$\operatorname{Now}\left[D^{\alpha} \tilde{q}(t ; r)\right]=\left[D^{\alpha} \underline{q}(t ; r), D^{\alpha} \bar{q}(t ; r)\right]$, where $[\tilde{q}(t ; r)]=[\underline{q}(t ; r), \bar{q}(t ; r)]$

With $\left[J^{1-\alpha} \tilde{q}(t ; r)\right]_{t=0}=\tilde{Q} \quad$ then $\left[J^{1-\alpha} \underline{q}(t ; r), \bar{q}(t ; r)\right]_{t=0}=\tilde{Q}=$
$\left[\gamma_{1}+r\left(\gamma_{2}-\gamma_{1}\right), \gamma_{3}-r\left(\gamma_{3}-\gamma_{2}\right)\right]$
$\therefore\left[J^{1-\alpha} \underline{q}(t ; r)\right]_{t=0}=\gamma_{1}+r\left(\gamma_{2}-\gamma_{1}\right)$, and $\left[J^{1-\alpha} \bar{q}(t ; r)\right]_{t=0}=\gamma_{3}-r\left(\gamma_{3}-\gamma_{2}\right)$
Then using (4.2) we have
$J^{\alpha} D^{\alpha}[\underline{q}(t ; r), \bar{q}(t ; r)]=[\underline{q}(t ; r), \bar{q}(t ; r)]-\frac{t^{\alpha-1}}{\Gamma(\alpha)}\left[J^{1-\alpha} \underline{q}(t ; r), J^{1-\alpha} \bar{q}(t ; r)\right] \mathrm{t}=0$
$\therefore J^{\alpha} D^{\alpha} \underline{q}(t ; r)=\underline{q}(t ; r)-\frac{t^{\alpha-1}}{\Gamma(\alpha)}\left[J^{1-\alpha} \underline{q}(t ; r)\right]_{t=0}=\underline{q}(t ; r)-\frac{t^{\alpha-1}}{\Gamma(\alpha)}\left\{\gamma_{1}+r\left(\gamma_{2}-\gamma_{1}\right)\right\}$
then $J^{\alpha} D^{\alpha} \bar{q}(t ; r)=\bar{q}(t ; r)-\frac{t^{\alpha-1}}{\Gamma(\alpha)}\left\{\gamma_{3}-r\left(\gamma_{3}-\gamma_{2}\right)\right\}$
Case i: When $\mathrm{a}>0$ in (4.1) we have $\left[D^{\alpha} \underline{q}(t ; r), D^{\alpha} \bar{q}(t ; r)\right]=a[\underline{q}(t ; r), \bar{q}(t ; r)]+f(t)$

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$$
\begin{align*}
& \therefore D^{\alpha} \underline{q}(t ; r)=a \underline{q}(t ; r)+f(t)  \tag{4.3}\\
& \text { And } D^{\alpha} \bar{q}(t ; r)=\bar{q}(t ; r)+f(t) \tag{4.4}
\end{align*}
$$

Applying $J^{\alpha}$ in both side of (4.3) \& (4.4) we get,
$J^{\alpha} D^{\alpha} \underline{q}(t ; r)=a J^{\alpha} \underline{q}(t ; r)+J^{\alpha} f(t)$
$\Rightarrow \underline{q}(k ; r)-\frac{t^{\alpha-1}}{\Gamma(\alpha)}\left\{\gamma_{1}+r\left(\gamma_{2}-\gamma_{1}\right)\right\}=a J^{\alpha} \underline{q}(t ; r)+J^{\alpha} f(t)$
$\Rightarrow \underline{q}(k ; r)=\frac{t^{\alpha-1}}{\Gamma(\alpha)}\left\{\gamma_{1}+r\left(\gamma_{2}-\gamma_{1}\right)\right\}+a J^{\alpha} \underline{q}(t ; r)+J^{\alpha} f(t)$
In light of ADM we decompose $\underline{q}(k ; r)$ into $\underline{q}(k ; r)=\sum_{k=0}^{\infty} \underline{q}_{k}(t ; r)$
With $q_{0}(t ; r)=\frac{t^{\alpha-1}}{\Gamma(\alpha)}\left\{\gamma_{1}+r\left(\gamma_{2}-\gamma_{1}\right)\right\}+J^{\alpha} f(t)$
Then we can write by (4.6) $q_{1}(t ; r)=J^{\alpha} a q_{0}(t ; r), q_{2}(t ; r)=J^{\alpha} a q_{1}(t ; r)$.
$q_{k}(t ; r)=J^{\alpha} a q_{k-1}(t ; r)$. $\qquad$ $. k \geq 1$

$$
\begin{equation*}
=\left(J^{\alpha} a\right)^{k} q_{0}(t ; r) \tag{4.8}
\end{equation*}
$$

$\therefore q_{k}(t ; r)=a^{k} J^{k \alpha} q_{0}(t ; r)$
$=a^{k} J^{k \alpha}\left[\frac{t^{\alpha-1}}{\Gamma(\alpha)}\left\{\gamma_{1}+r\left(\gamma_{2}-\gamma_{1}\right)\right\}+J^{\alpha} f(t)\right]$
$=a^{k} J^{k \alpha} \frac{t^{\alpha-1}}{\Gamma(\alpha)}\left\{\gamma_{1}+r\left(\gamma_{2}-\gamma_{1}\right)\right\}+a^{k} J^{k \alpha+\alpha} f(t)$
Let $q_{h}(t)=\sum_{k=0}^{\infty} a^{k} J^{k \alpha} \frac{t^{\alpha-1}}{\Gamma(\alpha)}\left\{\gamma_{1}+r\left(\gamma_{2}-\gamma_{1}\right)\right\}$

$$
\begin{align*}
& =\sum_{k=0}^{\infty} a^{k} \frac{t^{k \alpha+\alpha-1}}{\Gamma(k \alpha+\alpha)} \frac{\Gamma(\alpha-1+1)}{\Gamma(\alpha)}\left\{\gamma_{1}+r\left(\gamma_{2}-\gamma_{1}\right)\right\} \\
& =\sum_{k=0}^{\infty} a^{k} \frac{t^{k \alpha+\alpha-1}}{\Gamma(k \alpha+\alpha)}\left\{\gamma_{1}+r\left(\gamma_{2}-\gamma_{1}\right)\right\} \\
& =\left\{\gamma_{1}+r\left(\gamma_{2}-\gamma_{1}\right)\right\} \sum_{k=0}^{\infty} \frac{\left(a t^{\alpha}\right)^{k}}{\Gamma(k \alpha+\alpha)} \\
& =\left\{\gamma_{1}+r\left(\gamma_{2}-\gamma_{1}\right)\right\} t^{\alpha-1} E_{\alpha, \alpha}\left(a t^{\alpha}\right) \tag{4.10}
\end{align*}
$$

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And $q_{p}(t)=\sum_{k=0}^{\infty} a^{k} J^{k \alpha+\alpha} f(t)$
$=\sum_{k=0}^{\infty} a^{k} \frac{t^{k \alpha+\alpha-1}}{\Gamma(k \alpha+\alpha)} * f(t)=t^{\alpha-1} \sum_{k=0}^{\infty} \frac{\left(a t^{\alpha}\right)^{k}}{\Gamma(k \alpha+\alpha)} * f(t)=t^{\alpha-1} E_{\alpha, \alpha}\left(a t^{\alpha}\right)^{*} f(t)$
Thus we have $\quad \underline{q}(t ; r)=q_{h}(t)+q_{p}(t)$

$$
\begin{equation*}
=\left\{\gamma_{1}+r\left(\gamma_{2}-\gamma_{1}\right)\right\} t^{\alpha-1} E_{\alpha, \alpha}\left(a t^{\alpha}\right)+t^{\alpha-1} E_{\alpha, \alpha}\left(a t^{\alpha}\right) * f(t) \tag{4.12}
\end{equation*}
$$

Similarly we have

$$
\begin{equation*}
\bar{q}(t ; r)=\left\{\gamma_{3}-r\left(\gamma_{3}-\gamma_{2}\right)\right\} t^{\alpha-1} E_{\alpha, \alpha}\left(a t^{\alpha}\right)+t^{\alpha-1} E_{\alpha, \alpha}\left(a t^{\alpha}\right)^{*} f(t) \tag{4.13}
\end{equation*}
$$

Therefore when $\mathrm{a}>0$ the solution of equation (4.1) is $[\widetilde{q}(t ; r)]=[\underline{q}(t ; r), \bar{q}(t ; r)]$, where $\underline{q}(t ; r) \& \bar{q}(t ; r)$ are given by (4.12) \& (4.13).
Case ii: when $\mathrm{a}<0$,
Then $D^{\alpha} \underline{q}(t ; r)=a \bar{q}(t ; r)+f(t)$

$$
\begin{equation*}
\text { And } D^{\alpha} \bar{q}(t ; r)=\underline{q}(t ; r)+f(t) \tag{4.14}
\end{equation*}
$$

Adding (4.14) \& (4.15) we get

$$
D^{\alpha}[\underline{q}(t ; r)+\bar{q}(t ; r)]=a[\bar{q}(t ; r)+\underline{q}(t ; r)+2 f(t)
$$

$$
\begin{equation*}
\text { Let } \bar{u}(t ; r)=\underline{q}(t ; r)+\bar{q}(t ; r) \text {, then } D^{\alpha} u(t ; r)=a u(t ; r)+2 f(t) \tag{4.16}
\end{equation*}
$$

With the initial condition $\left[J^{1-\alpha} \tilde{q}(t ; r)\right]_{t=0}=\left(\gamma_{1}, \gamma_{2}, \gamma_{3}\right)$

$$
\begin{aligned}
& \therefore \quad\left[J^{1-\alpha} \mathrm{q}(t ; r)\right]_{t=0}=\left(\gamma_{1}+\mathrm{r}\left(\gamma_{2}-\gamma_{3}\right)\right. \\
& \quad\left[J^{1-\alpha} \bar{q}(t ; r)\right]_{t=0}=\gamma_{3}-r\left(\gamma_{3}-\gamma_{2}\right) \\
& \therefore\left[J^{1-\alpha} u(t ; r)\right]_{t=0}=\left[J^{1-\alpha} \underline{q}(t ; r)\right]_{t=0}+\left[J^{1-\alpha} \bar{q}(t ; r)\right]_{t=0}=\gamma_{1}+\gamma_{3}+r\left(2 \gamma_{2}-\gamma_{3}-\gamma_{1}\right) \\
& J^{\alpha} D^{\alpha} u(t ; r)=u(t ; r)-\frac{t^{\alpha-1}}{\Gamma(\alpha)}\left[J^{1-\alpha} u(t ; r)\right]_{t=0} \\
& =u(t ; r)-\frac{t^{\alpha-1}}{\Gamma(\alpha)}\left\{\gamma_{1}+\gamma_{3}+r\left(2 \gamma_{2}-\gamma_{3}-\gamma_{1}\right)\right\}
\end{aligned}
$$

Therefore from (4.16) we get

$$
\begin{aligned}
& u(t ; r)-\frac{t^{\alpha-1}}{\Gamma(\alpha)}\left\{\gamma_{1}+\gamma_{3}+r\left(2 \gamma_{2}-\gamma_{3}-\gamma_{1}\right)\right\}=a J^{\alpha} u(t ; r)+2 J^{\alpha} f(t) \\
& \quad \therefore u(t ; r)=\frac{t^{\alpha-1}}{\Gamma(\alpha)}\left\{\gamma_{1}+\gamma_{3}+r\left(2 \gamma_{2}-\gamma_{3}-\gamma_{1}\right)\right\}+a J^{\alpha} u(t ; r)+2 J^{\alpha} f(t)
\end{aligned}
$$

In light of ADM we decompose $u(t ; r)$ into $u(t ; r)=\sum_{k=0}^{\infty} u_{k}(t ; r)$

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With $u_{0}(t ; r)=\frac{t^{\alpha-1}}{\Gamma(\alpha)}\left\{\gamma_{1}+\gamma_{3}+r\left(2 \gamma_{2}-\gamma_{3}-\gamma_{1}\right)\right\}+2 J^{\alpha} f(t)$

$$
\begin{aligned}
& u_{1}(t ; r)=J^{\alpha} a u_{0}(t ; r), u_{2}(t ; r)=J^{\alpha} a u_{1}(t ; r)=J^{\alpha} a J^{\alpha} a u_{0}(t ; r)=\left(J^{\alpha} a\right)^{2} u_{0}(t ; r) \\
& \qquad u_{k}(t ; r)={ }_{\left(J^{\alpha} a\right)^{k} u_{0}(t ; r) \ldots \ldots \ldots \ldots \ldots \ldots k \geq 1}^{\therefore u_{k}(t ; r)=a^{k} J^{k \alpha}\left[\frac{t^{\alpha-1}}{\Gamma(\alpha)}\left\{\gamma_{1}+\gamma_{3}+r\left(2 \gamma_{2}-\gamma_{3}-\gamma_{1}\right)\right\}+2 J^{\alpha} f(t)\right]} \\
& \left.\quad=a^{k} J^{k \alpha} \frac{t^{\alpha-1}}{\Gamma(\alpha)}\left\{\gamma_{1}+\gamma_{3}+r\left(2 \gamma_{2}-\gamma_{3}-\gamma_{1}\right)\right\}+2 a^{k} J^{\alpha} f(t)\right]
\end{aligned}
$$

Let $u_{h}(t)=\sum_{k=0}^{\infty} a^{k} J^{k \alpha} \frac{t^{\alpha-1}}{\Gamma(\alpha)}\left\{\gamma_{1}+\gamma_{3}+r\left(2 \gamma_{2}-\gamma_{1}-\gamma_{3}\right)\right\}$

$$
\begin{aligned}
& =\sum_{k=0}^{\infty} a^{k} \frac{t^{k \alpha+\alpha-1}}{\Gamma(k \alpha+\alpha)}\left\{\gamma_{1}+\gamma_{3}+r\left(2 \gamma_{2}-\gamma_{1}-\gamma_{3}\right)\right\} \\
& =\left\{\gamma_{1}+\gamma_{3}+r\left(2 \gamma_{2}-\gamma_{1}-\gamma_{3}\right)\right\} t^{\alpha-1} E_{\alpha, \alpha}\left(a t^{\alpha}\right)
\end{aligned}
$$

And $u_{p}(t)=\sum_{k=0}^{\infty} 2 a^{k} J^{k \alpha+\alpha} f(t)$

$$
\begin{aligned}
& =\sum_{k=0}^{\infty} 2 a^{k} \frac{t^{k \alpha+\alpha-1}}{\Gamma(k \alpha+\alpha)} * f(t) \\
& =2 t^{\alpha-1} \sum_{k=0}^{\infty} \frac{\left(a t^{\alpha}\right)^{k}}{\Gamma(k \alpha+\alpha)} * f(t) \\
& =2 t^{\alpha-1} E_{\alpha, \alpha}\left(a t^{\alpha}\right)^{*} f(t)
\end{aligned}
$$

Thus we have $u(t ; r)=u_{h}(t)+u_{p}(t)$

$$
\begin{equation*}
=\left\{\gamma_{1}+\gamma_{3}+r\left(2 \gamma_{2}-\gamma_{1}-\gamma_{3}\right)\right\} t^{\alpha-1} E_{\alpha, \alpha}\left(a t^{\alpha}\right)+2 t^{\alpha-1} E_{\alpha, \alpha}\left(a t^{\alpha}\right) * f(t) \tag{4.17}
\end{equation*}
$$

Again from (4.14) \& (4.15) we get
$D^{\alpha}[\underline{q}(t ; r)-\bar{q}(t ; r)]=a[\bar{q}(t ; r)-\underline{q}(t ; r)$
Let $v(t ; r)=\underline{q}(t ; r)-\bar{q}(t ; r)$, then $D^{\alpha} v(t ; r)=-a v(t ; r)$
With the initial condition
$\left[J^{1-\alpha} \tilde{q}(t ; r)\right]_{t=0}=\left(\gamma_{1}, \gamma_{2}, \gamma_{3}\right)=\left[\gamma_{1}+r\left(\gamma_{2}-\gamma_{1}\right), \gamma_{3}-r\left(\gamma_{3}-\gamma_{2}\right)\right]$

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$$
\begin{aligned}
& \therefore\left[J^{1-\alpha} v(t ; r)\right]_{t=0}=\gamma_{1}-\gamma_{3}+r\left(\gamma_{3}-\gamma_{1}\right) \\
& J^{\alpha} D^{\alpha} v(t ; r)=v(t ; r)-\frac{t^{\alpha-1}}{\Gamma(\alpha)}\left[J^{1-\alpha} v(t ; r)\right]_{t=0} \\
&=v(t ; r)-\frac{t^{\alpha-1}}{\Gamma(\alpha)}\left\{\left(\gamma_{1}-\gamma_{3}\right)+r\left(\gamma_{3}-\gamma_{1}\right)\right\}
\end{aligned}
$$

Therefore from (4.18) we get

$$
v(t ; r)=\frac{t^{\alpha-1}}{\Gamma(\alpha)}\left\{\left(\gamma_{1}-\gamma_{3}\right)+r\left(\gamma_{3}-\gamma_{1}\right)\right\}-a J^{\alpha} v(t ; r)
$$

In light of ADM we decompose $v(t ; r)$ into $v(t ; r)=\sum_{k=0}^{\infty} v_{k}(t ; r)$
With $v_{0}(t ; r)=\frac{t^{\alpha-1}}{\Gamma(\alpha)}\left\{\left(\gamma_{1}-\gamma_{3}\right)+r\left(\gamma_{3}-\gamma_{1}\right)\right\}$
$v_{1}(t ; r)=-a J^{\alpha} v_{0}(t ; r)=J^{\alpha}(-a) v_{0}(t ; r)$,
$v_{2}(t ; r)=J^{\alpha}\left\{(-a) v_{1}(t ; r)\right\}=J^{\alpha}\left\{(-a) J^{\alpha}(-a) v_{0}(t ; r)\right\}=\left\{(-a) J^{\alpha}\right\}^{2} v_{0}(t ; r)$
$\qquad$ $v_{k}(t ; r)=\left\{(-a) J^{\alpha}\right\}^{k} v_{0}(t ; r)$. $\qquad$ $. . k \geq 1$

Now $v(t ; r)=\sum_{k=0}^{\infty} v_{k}(t ; r)$

$$
\begin{equation*}
\therefore v(t ; r)=\left\{\left(\gamma_{1}-\gamma_{3}\right)+r\left(\gamma_{3}-\gamma_{1}\right)\right\} t^{\alpha-1} E_{\alpha, \alpha}\left(-a t^{\alpha}\right) \tag{4.19}
\end{equation*}
$$

Hence $\underline{q}(t ; r)=\frac{1}{2}[u(t ; r)+v(t ; r)]$

$$
\begin{align*}
& =\frac{1}{2} t^{\alpha-1}\left[\left\{\left(\gamma_{1}+\gamma_{3}\right)+r\left(2 \gamma_{2}-\gamma_{1}-\gamma_{3}\right)\right\} E_{\alpha, \alpha}\left(a t^{\alpha}\right)\right. \\
& \left.+\left\{\left(\gamma_{1}-\gamma_{3}\right)+r\left(\gamma_{3}-\gamma_{1}\right)\right\} t^{\alpha-1} E_{\alpha, \alpha}\left(-a t^{\alpha}\right)\right]+t^{\alpha-1} E_{\alpha, \alpha}\left(a t^{\alpha}\right) * f(t) \tag{4.20}
\end{align*}
$$

And $\bar{q}(t ; r)=\frac{1}{2}[u(t ; r)-v(t ; r)]$

$$
\begin{align*}
& =\frac{1}{2} t^{\alpha-1}\left[\left\{\left(\gamma_{1}+\gamma_{3}\right)+r\left(2 \gamma_{2}-\gamma_{1}-\gamma_{3}\right)\right\} E_{\alpha, \alpha}\left(a t^{\alpha}\right)\right. \\
& \left.-\left\{\left(\gamma_{1}-\gamma_{3}\right)+r\left(\gamma_{3}-\gamma_{1}\right)\right\} t^{\alpha-1} E_{\alpha, \alpha}\left(-a t^{\alpha}\right)\right]+t^{\alpha-1} E_{\alpha, \alpha}\left(a t^{\alpha}\right) * f(t) \tag{4.21}
\end{align*}
$$

Therefore when $\mathrm{a}<0$ the solution of equation (4.1) is $[\tilde{q}(t ; r)]=[\underline{q}(t ; r), \bar{q}(t ; r)]$ where $\underline{q}(t ; r) \& \bar{q}(t ; r)]$ are given by (4.20) \& (4.21)

# Solving some System of Linear Fuzzy Fractional Differential Equations by Adomian Decomposition Method 

## 5. Conclusion

Adomian decomposition method (ADM) is a powerful tool which enables us to find solutions in case of linear as well as non-linear equations. The method has been successfully applied to a system of fractional differential equation as well as fuzzy differential equation. It is shown that the applicability of Adomian decomposition method to solve the system of fuzzy fractional differential equations of fractional order $\alpha$ ( $0<\alpha$ $\leq 1)$. The Adomian decomposition method is straightforward and applicable for broader problems. It can avoid the difficulty of finding the inverse of Laplace transform for solving the fuzzy fractional differential equation by Laplace transform method.

## REFERENCES

1. K.Abbaoui and Y.Cherruault, Convergence of Adomian's method applied to different equations, Comput Math Appl., 28 (1994) 103-109.
2. T.Allahviranloo, S.Abbasbandy, S.Salahshour and S.A.Hakimzadeh, A new method for solving fuzzy linear differential equations. Computing, 92 (2011) 181-97.
3. T.Allahviranloo and M.B.Ahmadi, Fuzzy Laplace transforms, Soft Comput. 14 (2010) 235-243.
4. R.P.Agarwal, V.Lakshmikantham and J.J.Nieto, On the concept of solution for fractional differential equations with uncertainty, Nonlinear Anal., 72 (2010) 28592862.
5. A.Arara, M.Benchohra, N.Hamidi and J.J.Nieto, Fractional order differential equations on an unbounded domain, Nonlinear Anal., 72 (2010) 580-586.
6. S.Arshad and V.Lupulescu, On the fractional differential equations with uncertainty, Nonlinear Anal., 74 (2011) 3685-3693.
7. R.L.Bagley, On the fractional order initial value problem and its engineering applications, In: Nishimoto K, editor. Fractional calculus and its applications. Tokyo: College of Engineering, Nihon University; 1990. p. 12-20.
8. B.Bede and S.G.Gal, Generalizations of the differentiability of fuzzy-number-valued functions with applications to fuzzy differential equations, Fuzzy Sets Syst., 151 (2005) 581-599.
9. B.Bede, I.J.Rudas and A.L.Bencsik, First order linear fuzzy differential equations under generalized differentiability, Inform Sci., 177 (2007) 1648-1662.
10. Y.Cherruault, Convergence of Adomian's method, Kybernetes, 18 (1989) 31-38.
11. L.Debnath, A brief historical introduction to fractional calculus, International Journal of Mathematical Education in Science and Technology, (2006), pages 487501.
12. L.Debnath, Fractional integral and fractional differential equation in fluid mechanics, Fract. Calc. Anal. 6(2) (2003) 119-155.
13. K.Diethelm and N.J.Ford, Analysis of fractional differential equations, J Math Anal Appl., 265 (2002) 229-248.
14. D.Dubois and H.Prade, Towards fuzzy differential calculus - Part 1, Fuzzy Sets and Systems, 8 (1982) 1-17.

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15. D.Dubois and H.Prade; Towards fuzzy differential calculus - Part 2, Fuzzy Sets and Systems, 8 (1982) 105-116.
16. D.Dubois and H.Prade; Towards fuzzy differential calculus - Part 3, Fuzzy Sets and Systems, 8 (1982) 225-234.
17. M.Friedman, M.Ma and A.Kandel, Numerical solution of fuzzy differential and integral equations, Fuzzy Sets Syst., 106 (1999) 35-48.
18. N.Himoun, K.Abbaoui and Y.Cherruault, New result of convergence of Adomian's method, Kybernetes, 28 (1999) 423-429.
19. A.A.Kilbas, H.M.Srivastava and J.J.Trujillo; Theory and applications of fractional differential equations, Amesterdam: Elsevier Science B.V. (2006).
20. K.S.Miller and B.Ross, An Introduction to the Fractional Calculus and Fractional Differential Equations, John Wiley \& Sons, (1993).
21. J.J.Nieto, Maximum principles for fractional differential equations derived from Mittag-Leffler functions, Appl Math Lett., 23 (2010) 1248-1251.
22. I.Podlubny, Fractional differential equation. San Diego: Academic Press; 1999.
23. M.L.Puri and D.Ralescu, Differentials for fuzzy functions, J. Math. Anal. Appl., 91 (1983) 552-558.
24. M.L.Puri and D.Ralescu, Fuzzy random variables, J Math Anal Appl., 114 (1986) 409-422.
25. S.Salahshour, T.Allahviranloo and S.Abbasbandy, Solving fuzzy fractional differential equations by fuzzy Laplace transforms, Commun Nonlinear Sci Numer Simulat, 17 (2012) 1372-1381.
26. L.Stefanini, A generalization of Hukuhara difference and division for interval and fuzzy arithmetic, Fuzzy Sets Syst., 161 (2010), 1564-1584.
27. L.Stefanini and B.Bede, Generalized Hukuhara differentiability of interval-valued functions and interval differential equations, Nonlinear Anal., 71 (2009) 1311-1328.
28. D.G.Varsha and J.Hossein, Adomian decomposition: a tool for solving a system of fractional differential equations, J. Math. Anal. Appl. 301 (2005) 508-518.
29. J.Xu, Z.Liao and Z.A.Hu, Class of linear differential dynamical systems with fuzzy initial condition, Fuzzy Sets Syst., 58 (2007) 2339-2358.
