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Further Multiplicative Operations of Intuitionistic Fuzzy Matrices

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Abstract. In this paper, we investigate the algebraic properties of intuitionistic fuzzy matrices under the new multiplicative operations X_3 and X_4 . The properties of intuitionistic fuzzy matrices are also obtained in the case where these new operations are combined with the well known operations of intuitionistic fuzzy matrices.

Keywords: Fuzzy matrix, intuitionistic fuzzy set, intuitionistic fuzzy matrix

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1. Introduction

Atanassov [1] introduced the concept of intuitionistic fuzzy set (IFS) which is the generalization of fuzzy set introduced by Zadeh [21]. Since its appearance, intuitionistic fuzzy set has been investigated by many researchers and applied to many fields, such as decision making, clustering analysis etc. Using the fuzzy set theory, Kim and Roush [9] studied fuzzy matrices as a generalization of matrices over the two element Boolean algebra. Using the theory of intuitionistic fuzzy set, Im et al. [6] defined the notion of intuitionistic fuzzy matrix (IFM) as a generalization of fuzzy matrix.

IFM is very useful in the discustion of intuitionistic fuzzy relation (IFR) [4,11]. Lee and Jeong [10] obtained a canonical form of the transtive IFM. Sriram and Murugadas [15] proved the set of all IFMs form a semiring with respect to max-min composition of IFMs. Zhong et al. [20] constructed the intuitionistic fuzzy similarity matrix and then utilize it to derive a method for clustering analysis.

Simultaneously, Pal et al. [7] defined the IFM and Pal [14] introduced the intuitionistic fuzzy determinant, studied some properties on it. Khan and Pal [8] studied some operations on IFMs. Shyamal and Pal [17] defined the two new operators on fuzzy matrices and studied their algebraic properties. Recently, Pal [23,24] defines new kind of fuzzy matrices. In these matrices rows and columns are also uncertain.

In Zhang and Zheng [22] defined two fuzzy operators and four cut sets for fuzzy matrices and studied their algebraic properties. Boobalan and Sriram [3,16] studied the algebraic sum and algebraic product of two intuitionistic fuzzy matrices and their algebraic

properties. Also they proved the set of all intuitionistic fuzzy matrices forms a commutative monoid with respect to these operations.

Muthuraji et al. [12] introduced a new composition operator for IFMs and studied the algebraic properties also obtained a decomposition of an IFM. Muthuraji and Sriram [13] defined two operators namely Lukasiwicz disjunction and conjunction for IFMs and investigated its algebraic properties also results which connects the above set operators with the other existing operators.

Emam and Fndh [5] defined some kinds of IFMs, the max-min and min-max composition of IFMs. Also they derived several importent results by these compositions and constract an idempotent intuitionistic fuzzy matrix from any given one through the min-max composition. In [2] Atanassov, five new intuitionistic fuzzy operations on intuitionistic fuzzy sets containing multiplication were introduced and their properties are studied.

Venkatesan and Sriram [19] extended Multiplicative operations of IFMs of two operators namely X_1 and X_2 and investigated its algebraic properties. In this paper, we extend amoung two of these operations to IFMs and investigate their algebraic properties.

2. Preliminaries

In this section, we give to some basic definitions of intuitionistic fuzzy matrix that are necessary for this paper.

Definition 2.1. ([14]) Consider a matrix $A = (a_{ij})_{m \times n}$ where $a_{ij} \in [0,1]$, $1 \le i \le m$ and $1 \le j \le n$. Then A is fuzzy matrix.

Definition 2.2. ([6]) An intuitionistic fuzzy matrix(IFM) is a matrix of pairs $A = (\langle a_{ij}, a'_{ij} \rangle)$ of a non negative real numbers satisfying $0 \le a_{ij} + a'_{ij} \le 1$ for all i, j.

Definition 2.3. ([14]) For any two IFMs *A* and *B* of same size, we have (*i*) The max-min composition of *A* and *B* is defined by $A \lor B = ((\max(a_{ij}, b_{ij}), \min(a'_{ij}, b'_{ij})))).$ (*ii*) The min-max composition of *A* and *B* is defined by $A \land B = ((\min(a_{ij}, b_{ij}), \max(a'_{ij}, b'_{ij})))).$

Definition 2.4. ([14]) For any two IFMs A and B of same size, $A \ge B$ iff $a_{ij} \ge b_{ij}$ and $a'_{ii} \le b'_{ij}$ for all i, j.

Definition 2.5. ([3]) The $m \times n$ zero IFM *O* is an IFM all of whose entries are (0,1). The $m \times n$ universal IFM *J* is an IFM all of whose entries are (1,0).

Definition 2.6. ([14]) The complement of an IFM A which is denoted by A^{C} and is defined by $A^{C} = (\langle a'_{ij}, a_{ij} \rangle)$.

Lemma 2.7. ([20]) Let a, b and c be real numbers. Then the following equalities hold: (*i*) max $(a, \min(a, b) = a, \min(a, \max(a, b)) = a$. (*ii*) max $(a, \max(b, c)) = \max(\max(a, b), c), \min(a, \min(b, c) = \min(\min(a, b), c)$.

Lemma 2.8. ([20]) Let $a, b \in [0,1]$, we have (*i*) max $(a, \min(b, c)) = \min(\max(a, b), \max(a, c)),$ min $(a, \max(b, c)) = \max(\min(a, b), \min(a, c)).$ (*ii*) max $(a, b) \max(a, c) \le \max(a, bc), \min(a, b) \min(a, c) \ge \min(a, bc).$

Definition 2.9. ([19]) For any two IFMs *A* and *B* of same size, then we define $(i)AX_1B = (\langle \max(a_{ij}, b_{ij}), a'_{ij}b'_{ij} \rangle).$ $(ii)AX_2B = (\langle a_{ij}b_{ij}, \max(a'_{ij}, b'_{ij}) \rangle).$ $a_{ij}b_{ij}$ and $a'_{ij}b'_{ij}$ are the ordinary multiplications.

3. Main results

In this section, we difine operations X_3 and X_4 of IFMs and investigate their algebraic properties.

Definition 3.1. For any two IFMs *A* and *B* of same size, then we define $(i)AX_3B = (\langle \min(a_{ij}, b_{ij}), a'_{ij}b'_{ij} \rangle).$ $(ii)AX_4B = (\langle a_{ij}b_{ij}, \min(a'_{ij}, b'_{ij}) \rangle).$ $a_{ij}b_{ij}$ and $a'_{ij}b'_{ij}$ are the ordinary multiplications.

Property 3.2. If *A* and *B* are any two IFMs of same size, we have $AX_4B \le AX_3B$. **Proof:** Let $A = (\langle a_{ij}, a'_{ij} \rangle)$ and $B = (\langle b_{ij}, b'_{ij} \rangle)$ be two IFMs of same size. Since $a_{ij}b_{ij} \le \min(a_{ij}, b_{ij}) \le \max(a_{ij}, b_{ij})$ and $\min(a'_{ij}, b'_{ij}) \ge a'_{ij}b'_{ij}$, for all i, j. Hence, $AX_4B \le AX_3B$.

Property 3.3. For any IFM *A*, we have (*i*) $AX_3A \neq A$. (*ii*) $AX_4A \neq A$. **Proof:** Let $A = (\langle a_{ij}, a'_{ij} \rangle)$ be an IFM. Then (*i*) $AX_3A = (\langle a_{ij}, a'_{ij} \rangle)$ $\neq (\langle a_{ij}, a'_{ij} \rangle)$. Hence, $AX_3A \neq A$. (*ii*) $AX_4A = (\langle a^2_{ij}, a'_{ij} \rangle)$ $\neq (\langle a_{ij}, a'_{ij} \rangle)$. Hence, $AX_4A \neq A$.

The following properties are obvious. The operations X_3 and X_4 are commutative as well as associative.

Property 3.4. Let *A*, *B* and *C* be any three IFMs of same size, we have (*i*) $AX_3B = BX_3A$. (*ii*) $(AX_3B)X_3C = AX_3(BX_3C)$.

 $\begin{array}{l} (iii) \ AX_4B = BX_4A. \\ (iv) \ (AX_4B)X_4C = AX_4(BX_4C). \end{array}$

Property 3.5. For any three IFMs *A*, *B* and *C* of same size, we have (*i*) Nullity: $AX_3J = J$, $AX_4O = O$. (*ii*) Identity: $AX_3O = A$, $AX_4J = A$. (*iii*) Distributivity: $AX_3(BX_4C) \neq (AX_3B)X_4(AX_3C)$ and $AX_4(BX_3C) \neq (AX_4B)X_3(AX_4C)$. (*iv*) Absorption: $AX_3(AX_4B) \neq A$ and $AX_4(AX_3B) \neq A$. **Proof:** (*i*) and (*ii*) are clear by the definition of X_3 and X_4 . (*iii*) $AX_3(BX_4C) = ((\min(a_{ij}, b_{ij}c_{ij}), a'_{ij}\min(b'_{ij}, c'_{ij})))$ ($AX_3B)X_4(AX_3C) = ((\min(a_{ij}, b_{ij})\max(a_{ij}, c_{ij}), \max(a'_{ij}b'_{ij}, a'_{ij}c'_{ij})))$ $= ((\max(a_{ij}, b_{ij})\min(a_{ij}, c_{ij}), a'_{ij}\min(b'_{ij}, c'_{ij})))$ Since $\min(a_{ij}, b_{ij})\max(a_{ij}, c_{ij}) \leq \min(a_{ij}, b_{ij}c_{ij})$, for all i, j. Hence, $AX_3(BX_4C) \neq (AX_3B)X_4(AX_3C)$. Similarly we can prove the other one. (*iv*) $AX_3(AX_4B) = ((\min(a_{ij}, a_{ij}b_{ij}), a'_{ij}\min(a'_{ij}, b'_{ij})))$ $= (\langle a_{ij}b_{ij}, a'_{ij}\min(a'_{ij}, b'_{ij}) \rangle)$

 $\neq (\langle a_{ij}, a'_{ij} \rangle).$ Hence, $AX_3(AX_4B) \neq A.$

Similarly we can prove the other one.

4. Results on complement of IFM

The operator complement obey the De Morgan's laws for the operations X_3 and X_4 . This is established in the following properties.

Property 4.1. For the IFMs *A* and *B* of same size, we have (*i*) $(AX_3B)^C = A^C X_4 B^C$. (*ii*) $(AX_4B)^C = A^C X_3 B^C$. (*iii*) $(AX_3B)^C \le A^C X_4 B^C$. (*iv*) $(AX_4B)^C \le A^C X_4 B^C$. **Proof:** (*i*) $(AX_3B)^C = ((a'_{ij}b'_{ij}, \min(a_{ij}, b_{ij})))$ $= A^C X_4 B^C$. Hence, $(AX_3B)^C = A^C X_4 B^C$. (*ii*) $(AX_4B)^C = ((\min(a'_{ij}, b'_{ij}), a_{ij}b_{ij})))$ $= A^C X_3 B^C$. Hence, $(AX_4B)^C = A^C X_3 B^C$. (*iii*) Since $a'_{ij}b'_{ij} \le \min(a'_{ij}, b'_{ij})$, min $(a_{ij}, b_{ij}) \ge a_{ij}b_{ij}$ and from the Definition (2.4) $(AX_3B)^C \le A^C X_3 B^C$. The proof (*iv*) is similar to that of (*iii*).

Property 4.2. For any IFM *A*, we have (*i*) $AX_3A^C \neq 0$.

(*ii*) $AX_4A^C \neq J$. **Proof:** (*i*) $AX_1A^C = (\langle \min(a_{ij}, a'_{ij}), a'_{ij}a_{ij} \rangle)$ $\neq \langle 0, 1 \rangle$. Hence, $AX_3A^C \neq O$. (*ii*) $AX_4A^C = (\langle a_{ij}a'_{ij}, \min(a_{ij}, a'_{ij}) \rangle)$ $= \langle 1, 0 \rangle$. Hence, $AX_4A^C \neq J$.

Property 4.3. For any IFM *A*, we have (*i*) $(AX_3A^c)^c = AX_4A^c$. (*ii*) $(AX_4A^c)^c = AX_3A^c$. Proof: $AX_3A^c = (\langle \min(a_{ij}, a_{ij}), a_{ij}^{'}a_{ij} \rangle)$ and $AX_4A^c = (\langle a_{ij}a_{ij}^{'}, \min(a_{ij}, a_{ij}^{'}) \rangle)$. (*i*) $(AX_3A^c)^c = (\langle a_{ij}^{'}a_{ij}, \min(a_{ij}, a_{ij}) \rangle)$ $= (\langle a_{ij}a_{ij}^{'}, \min(a_{ij}^{'}, a_{ij}) \rangle)$ $= AX_4A^c$. Hence, $(AX_3A^c)^c = AX_4A^c$. (*ii*) $(AX_4A^c)^c = (\langle \min(a_{ij}^{'}, a_{ij}), a_{ij}a_{ij}^{'} \rangle)$ $= AX_3A^c$. Hence, $(AX_4A^c)^c = AX_3A^c$.

Property 4.4. For any two IFMs A and B of same size, we have $(i)(AX_3A^C)X_3(AX_3A^C) \neq AX_3A^C$. $(ii)(AX_4A^C)X_4(AX_4A^C) \neq AX_4A^C$. **Proof:** $(i)(AX_3A^C)X_3(AX_3A^C) = (\langle \min(\min(a_{ij}, a'_{ij}), \min(a_{ij}, a'_{ij})), (a_{ij}a'_{ij})^2 \rangle)$. $AX_3A^C = (\langle \min(a_{ij}, a'_{ij}), a'_{ij}a_{ij}) \rangle$. Since $(a_{ij}a'_{ij})^2 \leq a'_{ij}a_{ij}$, for all i, j. Hence, $(AX_3A^C)X_3(AX_3A^C) \neq AX_3A^C$. The proof (ii) is similar to that of (i).

5. Results on X_1 and X_2 combined with max-min and min-max compositions of IFMs

We shall discuss the absorption property in the case where the operations X_3, X_4 , max-min and min-max are combined each other.

Property 5.1. Let *A* and *B* are any two IFMs of same size, we have (*i*) $AX_3(A \lor B) \neq A$. (*ii*) $AX_3(A \land B) \neq A$. **Proof:** (*i*) $AX_3(A \lor B) = (\langle \min(a_{ij}, \max(a_{ij}, b_{ij}), a_{ij}'\min(a_{ij}', b_{ij}') \rangle)$ $= (\langle a_{ij}, a_{ij}'a_{ij}' \rangle)$ $= (\langle a_{ij}, a_{ij}'^2 \rangle)$

$$\neq \left(\langle a_{ij}, a_{ij}'\rangle\right)$$

Hence, $AX_3(A \lor B) \neq A$. The proof (*ii*) is similar to that of (*i*).

Property 5.2. Let A and B are any two IFMs of same size, we have (i) $AX_4(A \lor B) \neq A$. (ii) $AX_4(A \land B) \neq A$. **Proof:** (i) $AX_4(A \lor B) = (\langle a_{ij} \max(a_{ij}, b_{ij}), \min(a_{ij}' \min(a_{ij}', b_{ij}')) \rangle)$ $= (\langle \max(a_{ij}^2, a_{ij}b_{ij}), \min(a_{ij}' \min(a_{ij}', b_{ij}')) \rangle)$ $\neq (\langle a_{ij}, a_{ij}' \rangle)$.

Hence, $AX_4(A \lor B) \neq A$. The proof (*ii*) is similar to that of (*i*).

Property 5.3. Let A and B are any two IFMs of same size, we have (i) $A \lor (AX_3B) = AX_1B$. (ii) $A \land (AX_3B) = A$. Proof: (i) $A \lor (AX_3B) = (\langle \max(a_{ij}, \min(a_{ij}, b_{ij})), \min(a_{ij}^{'}, a_{ij}^{'}b_{ij}^{'}) \rangle)$ $= (\langle a_{ij}, a_{ij}^{'}b_{ij}^{'} \rangle)$ $\neq (\langle a_{ij}, a_{ij}^{'} \rangle)$. Hence, $A \lor (AX_3B) \neq A$. (ii) $A \land (AX_3B) = (\langle \min(a_{ij}, \min(a_{ij}, b_{ij})), \max(a_{ij}^{'}, a_{ij}^{'}b_{ij}^{'}) \rangle)$ $= (\langle b_{ij}, a_{ij}^{'} \rangle)$ $\neq (\langle a_{ij}, a_{ij}^{'} \rangle)$. Hence, $A \land (AX_3B) \neq A$.

Similarly, we can prove the following property

Property 5.4. Let *A* and *B* are any two IFMs of same size, we have (*i*) $A \land (AX_4B) \neq A$. (*ii*) $A \lor (AX_4B) \neq A$.

Next, we shall discuss the distributivity in the case where the operations X_3 , X_4 , max-min and min-max are combined each other.

Property 5.5. Let *A*, *B* and *C* are any three IFMs of same size, we have $(i)AX_3(B \lor C) = (AX_3B) \lor (AX_3C).$ $(ii)AX_3(B \land C) = (AX_3B) \land (AX_3C).$ **Proof:** $(i) AX_3(B \lor C) = (\langle \min(a_{ij}, (\max(b_{ij}, c_{ij})), a'_{ij}\min(b'_{ij}, c'_{ij})) \rangle)$ $= (\langle \max(\min(a_{ij}, b_{ij}), \min(a_{ij}, c_{ij}), \min(a'_{ij}b'_{ij}, a'_{ij}c'_{ij}) \rangle)$ $= (AX_3B) \lor (AX_3C).$ Hence, $AX_3(B \lor C) = (AX_3B) \lor (AX_3C).$ The proof (ii) is similar to that of (i).

Property 5.6. Let A, B and C are any three IFMs of same size, we have

 $\begin{aligned} (i)AX_4(B \lor C) &= (AX_4B) \lor (AX_4C).\\ (ii)AX_4(B \land C) &= (AX_4B) \land (AX_4C).\\ \mathbf{Proof:} \ (i) AX_4(B \lor C) &= \left(\langle a_{ij}(\max(b_{ij}, c_{ij}), \min(a_{ij}', \min(b_{ij}', c_{ij}')) \right) \\ &= \left(\langle \max(a_{ij}b_{ij}, a_{ij}c_{ij}), \min(\min(a_{ij}', b_{ij}'), \min(a_{ij}', c_{ij}') \rangle \right) \\ &= (AX_4B) \lor (AX_4C).\\ \text{Hence, } AX_4(B \lor C) &= (AX_4B) \lor (AX_4C).\\ \text{The proof } (ii) \text{ is similar to that of } (i). \end{aligned}$

Property 5.7. Let *A*, *B* and *C* are any three IFMs of same size, we have $(i)A \lor (BX_3C) \neq (A \lor B)X_3(A \lor C).$ $(ii)A \land (BX_3C) \neq (A \land B)X_3(A \land C).$ **Proof:** $(i)A \lor (BX_3C) = (\langle \max(a_{ij}, \min(b_{ij}, c_{ij})), \min(a'_{ij}, b'_{ij}c'_{ij}) \rangle)$ $= (\langle \min(\max(a_{ij}, b_{ij}), \max(a_{ij}, c_{ij})), \min(a'_{ij}, b'_{ij}c'_{ij}) \rangle)$ $\neq (A \lor B)X_3(A \lor C).$ Hence, $A \lor (BX_3C) \neq (A \lor B)X_3(A \lor C)$. (By using Lemma 2.8) The proof (*ii*) is similar to that of (*i*).

Property 5.8. Let *A*, *B* and *C* are any three IFMs of same size, we have $(i)A \lor (BX_4C) \neq (A \lor B)X_4(A \lor C).$ $(ii)A \land (BX_4C) \neq (A \land B)X_4(A \land C).$ **Proof:** $(i)A \lor (BX_4C) = (\langle \max(a_{ij}, b_{ij}c_{ij}), \min(a'_{ij}, \min(b'_{ij}, c'_{ij})) \rangle)$ $= (\langle \max(a_{ij}, b_{ij}c_{ij}), \min(\min(a'_{ij}, b'_{ij}), \min(a'_{ij}, c'_{ij})) \rangle)$ $\neq (A \lor B)X_4(A \lor C).$ Hence, $A \lor (BX_4C) \neq (A \lor B)X_4(A \lor C).$ (By using Lemma 2.8) The proof (ii) is similar to that of (i).

Property 5.9. If A and B are any two IFMs of same size, we have $(i)AX_3B \neq A \wedge B.$ $(ii)AX_4B \neq A \lor B.$ $(iii)AX_4B \ge A \land B.$ $(iv)AX_3B \ge A \lor B.$ **Proof:** (*i*) $AX_3B = (\langle \min(a_{ij}, b_{ij}), a'_{ij}b'_{ij} \rangle)$ and $A \wedge B = \left(\langle \min(a_{ij}, b_{ij}), \max(a_{ij}^{'}, b_{ij}^{'}) \rangle \right).$ Since $a'_{ij}b'_{ij} \le \max(a'_{ij}, b'_{ij})$, for all i, j. Hence, $AX_3B \neq A \wedge B$. $(ii)AX_4B = (\langle a_{ij}b_{ij}, \min(a'_{ij}, b'_{ij})\rangle)$ and $A \lor B = (\langle \max(a_{ij}, b_{ij}), \min(a'_{ij}, b'_{ij}) \rangle).$ Since $a_{ij}b_{ij} \leq \max(a_{ij}, b_{ij})$, for all i, j. Hence, $AX_4B \neq A \lor B$. (*iii*) Since $a_{ij}b_{ij} \le \min(a_{ij}, b_{ij})$ and $\min(a_{ij}, b_{ij}) \ge \max(a_{ij}, b_{ij})$, Hence, $AX_4B \ge A \land B$. (*iv*) Since min $(a_{ij}, b_{ij}) \ge \max(a_{ij}, b_{ij})$ and $a'_{ij}b'_{ij} \le \min(a'_{ij}, b'_{ij})$,

Hence, $AX_3B \neq A \lor B$.

Property 5.10. If *A* and *B* are any two IFMs of same size, we have $(i)(A \lor B) \lor (AX_3B) = AX_1B.$ $(ii)(A \land B) \land (AX_4B) = AX_2B.$ **Proof:** $(i)(A \lor B) \lor (AX_3B) =$ $(\langle \max(\max(a_{ij}, b_{ij}), \min(a_{ij}, b_{ij})), \min(\min(a_{ij}^{'}, b_{ij}^{'}), a_{ij}^{'}b_{ij}^{'})\rangle)$ $= (\langle \max(a_{ij}, b_{ij}), a_{ij}^{'}b_{ij}^{'}\rangle)$ $= AX_1B.$ Hence, $(A \lor B) \lor (AX_3B) = AX_1B.$ The proof (*ii*) is similar to that of (*i*).

Property 5.11. If *A* and *B* are any two IFMs of same size, we have $(i)(A \land B)X_3(A \lor B) = AX_3B.$ $(ii)(A \land B)X_4(A \lor B) = AX_4B.$ **Proof:** $(i)(A \land B)X_3(A \lor B) =$ $(\langle \min(\min(a_{ij}, b_{ij}), \max(a_{ij}, b_{ij})), \max(a'_{ij}, b'_{ij}) \min(a'_{ij}, b'_{ij}) \rangle)$ $= (\langle \min(a_{ij}, b_{ij}), a'_{ij}b'_{ij} \rangle)$ $= AX_3B.$ Hence, $(A \land B)X_3(A \lor B) = AX_3B.$ The proof (*ii*) is similar to that of (*i*).

6. Conclusions

The set of all IFMs with respect to the operations X_3 and X_4 form a commutative monoid. The operations X_3 and X_4 of IFMs are satisfy the De Morgan's laws. Distributive laws max-min and min-max composition over X_3 and X_4 are proved and established some algebraic properties.

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