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Type-2 Fuzzy Soft Sets on Fuzzy Decision Making Problems

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Abstract. Making decision is one of the most fundamental activities of human being. Decision making is a study of how decisions are actually made better. Applications of fuzzy sets within the field of decision making consisted of fuzzifications of the classical theories of decision making. Decisions are made under conditions of uncertainty is the prime domain for fuzzy decision making. In this paper, we have applied the notion of similarity measure and inclusion measure between Type-2 fuzzy soft sets to verify their relationships. This relation is used to obtain a solution of a decision making problem.

Keywords: Type-2 fuzzy soft sets, complement of type-2 fuzzy soft sets, similarity measure, inclusion measure

AMS Mathematics Subject Classification (2010): 05C72

1. Introduction

Decision making is broadly defined as include any choice or selection of alternatives and is therefore of importance in many fields in both “soft” social sciences and the “hard” disciplines of natural sciences. Soft set theory [6], firstly proposed by Molodtsov, is a general mathematical tool for dealing with uncertainty. The advantage of soft set theory is that it is free from the inadequacy of the parametrization tools of existing theories. It has been demonstrated that soft set theory brings about a rich potential for applications in many fields. It was growing rapidly over the years.

The concept of type-2 fuzzy set was introduced by Zadeh [8] in 1975. Type-2 fuzzy sets can improve certain kinds of inference better than do fuzzy sets with increasing imprecision, uncertainty, fuzziness in information. However, in the practical applications, we are often faced with the situation in which the evaluation of parameters is a fuzzy concept. Obviously, it is very difficult for the classical soft set and its existing extensions to deal with the evaluation of parameters of the object as a fuzzy concept. Hence, it is necessary to extend soft set theory to accommodate the situations in which the evaluation of parameters is a fuzzy concept.

To represent the fuzziness of evaluation of parameters directly, Zhang and Shouhua Zhang [9] present the concept of type-2 fuzzy soft sets. The similarity concept introduced by Zadeh, is extremely important for it provides the degree of similarity between two fuzzy concepts. In the same way the inclusion measure also provides the degree between two fuzzy concepts. Some researchers concentrated the relationship among these two measures in fuzzy concepts.

In this paper, we have applied the relationship between the similarity measure and inclusion measure to the type-2 fuzzy soft sets. This paper is organized as follows: In section 2, some basic definitions and some operations of Type-2 fuzzy soft sets are reviewed. In section 3, the relationship between similarity and inclusion measures of Type-2 fuzzy soft set is discussed. In section 4, a numerical example is given.

2. Basic definitions

Definition 2.1. A soft set (*P*, *E*) is a set of all parameterized family of subsets of the non-empty universe X. For every $e \in E$ there exists $P(e)$ such that $P : E \to \rho(X)$, where $\rho(X)$ is a power set of X.

Definition 2.2. Let X be a non-empty finite set, which is referred as the universal set. A Type-2 fuzzy set A, is characterized by a type-2 membership function $\mu_A(x,u) : X \times I \to I$ where $x \epsilon X$, I = [0, 1] and $u \in J_x \subseteq I$ that is $A = \{((x,u); \mu_A(x,u) / x \in X, u \in J_x \subseteq I)\}$, where $0 \le \mu_A(x,u) \le 1$.

A can also be expressed as $A = \int_{x \in X} \int_{u \in J_x} \frac{\mu_A(x,u)}{(x,u)} = \int_{x \in U} \frac{p_x(u)/u}{x}, J_x \subseteq I$, where $p_x(u) = \mu_A(x,u)$. The class of all Type-2 fuzzy set of the universe *X* is denoted by $P_{T2}(X)$.

Definition 2.3. A type-2 fuzzy soft set ($\mathscr{P}$, *A*) over the universal set X is a set of all parameterized family of subsets of the type-2 fuzzy set A. For every $e \in A, A \subseteq E$ there exists $\mathscr{P}(e)$ such that $\mathscr{P} : A \to P_{T2}(A)$ where $P_{T2}(X)$ denotes the set of all type-2 fuzzy set.

It can be written as $\mathscr{P}(\varepsilon) = \int_{x \in X} \int_{u \in J_x} \frac{\mu_{\mathscr{P}(\varepsilon)}(x,u)}{(x,u)} = \int_{x \in X} \frac{\int_{u \in J_x} p_x(u)/u}{x}, J_x \subseteq I$.

Here u, $\mu_{\mathscr{P}(\varepsilon)}(x,u)$ are respectively the primary membership degree and secondary membership degree that object *x* holds on parameter ε.

Definition 2.4. The union of two type-2 fuzzy soft sets ($\mathscr{P}$, *A*) and ($\mathscr{Q}$, *B*) be over the same universe *X* is the type-2 fuzzy soft sets ($\mathscr{R}$, C), where $C = A \cup B$ and for all $\varepsilon \in C$,

$$\mathscr{R}\left(\varepsilon\right)=\begin{cases}\mathscr{P}(\varepsilon) & ,if\,\varepsilon\in A-B\\ \mathscr{Q}(\varepsilon) & ,if\,\varepsilon\in B-A\\ \mathscr{P}(\varepsilon)\vee\mathscr{Q}(\varepsilon) & ,if\,\varepsilon\in A\cap B.\end{cases}$$

It is denoted by $\left(\mathscr{P},A\right)\cup\left(\mathscr{Q},B\right)=\left(\mathscr{R},C\right)$.

Definition 2.5. The intersection of two type-2 fuzzy soft sets ($\mathscr{P}$, A) and ($\mathscr{Q}$, B) be over the same universe X is the type–2 fuzzy soft sets ($\mathscr{S}$, C), where $C = A\cup B$ and for all $\varepsilon\in C$,

$$\mathscr{S}\left(\varepsilon\right)=\begin{cases}\mathscr{P}(\varepsilon) & ,if\,\varepsilon\in A-B\\ \mathscr{Q}(\varepsilon) & ,if\,\varepsilon\in B-A\\ \mathscr{P}(\varepsilon)\wedge\mathscr{Q}(\varepsilon) & ,if\,\varepsilon\in A\cap B.\end{cases}$$

It is denoted by $\left(\mathscr{P},A\right)\cap\left(\mathscr{Q},B\right)=\left(\mathscr{S},C\right)$.

Definition 2.6. The complement of a type-2 fuzzy soft set ($\mathscr{P}$, A) is denoted by $(\mathscr{P}, A)^c$, and it is defined by $(\mathscr{P}, A)^c = (\mathscr{P}^c, \neg A)$, $\mathscr{P}$ is a mapping given by $\mathscr{P}^c:\neg A\rightarrow P_{T2}\left(X\right)$ where $\mathscr{P}\left(\varepsilon\right)=\sim\mathscr{P}\left(\neg\varepsilon\right)$ for all $\varepsilon \in \neg A$.

Definition 2.7. If A is a type-2 fuzzy set $A\subseteq X$ in discrete case, the centroid type reduction can be defined as follows:

$$C_A=\frac{\int\limits_{u_1\in J_{x_1}}\cdot\int\limits_{u_2\in J_{x_2}}\cdots\int\limits_{u_n\in J_{x_n}}\left[p_{x_1}(u_1).p_{x_2}(u_2)...p_{x_n}(u_n)\right]}{\sum\limits_{j=1}^{n}x_j\mu_A\left(x_j\right)\Big/\sum\limits_{j=1}^{n}\mu_A\left(x_j\right)}$$

where $A=\sum\limits_{j=1}^{n}\left[\sum\limits_{u\in J_{x_j}}p_{x_j}\left(u\right)/u\right]\Big/x_j$.

3. Similarity measure and inclusion measure and their relationship

3.1. Similarity measure for type-2 fuzzy soft set

Let $M_1\left(\mathscr{P},\mathscr{Q}\right)$ denotes the similarity measure between the two ε_j-thapproximations of $\mathscr{P}\left(\varepsilon_j\right)$ and $\mathscr{Q}\left(\varepsilon_j\right)$. Then we define

$$M_1(\mathscr{P},\mathscr{Q}) = \max_j \left\{ M_{1j}(\mathscr{P},\mathscr{Q}) = \begin{cases} \dfrac{\sum_{i=1}^{m}\left(\mathscr{P}_{ij} \cap \mathscr{Q}_{ij}\right)}{\sum_{i=1}^{m}\left(\mathscr{P}_{ij} \cup \mathscr{Q}_{ij}\right)} & if\ \varepsilon_j \in A \cap B \\ 0 & otherwise \end{cases} \right\}$$

where $\mathscr{P}_{ij} = \mathscr{P}(\varepsilon_j)(x_i) \in I$ and $\mathscr{Q}_{ij} = \mathscr{Q}(\varepsilon_j)(x_i) \in I$

3.2. Inclusion measure for type – 2 fuzzy soft set

Let $M_2(\mathscr{P},\mathscr{Q})$ denotes the inclusion measure between the two ε_j-th approximations of $\mathscr{P}(\varepsilon_j)$ and $\mathscr{Q}(\varepsilon_j)$. Then we define

$$M_2(\mathscr{P},\mathscr{Q}) = \max_j \left\{ M_{2j}(\mathscr{P},\mathscr{Q}) = \begin{cases} \dfrac{\sum_{i=1}^{m}\left(\mathscr{P}_{ij} \cap \mathscr{Q}_{ij}\right)}{\sum_{i=1}^{m}\left(\mathscr{P}_{ij}\right)} & if\ \varepsilon_j \in A \cap B \\ 0 & otherwise \end{cases} \right\}$$

where $\mathscr{P}_{ij} = \mathscr{P}(\varepsilon_j)(x_i) \in I$ and $\mathscr{Q}_{ij} = \mathscr{Q}(\varepsilon_j)(x_i) \in I$

3.3. Relationship between similarity measure and inclusion measure

The relationship between similarity measure and the inclusion measure is given by, $M_1(\mathscr{P},\mathscr{Q}) = \min\left\{M_2(\mathscr{P},\mathscr{Q}), M_2(\mathscr{Q},\mathscr{P})\right\}$

4. Numerical example

Suppose that a software company desires to hire a system analysis engineer. After preliminary screening, three candidates A_1, A_2, A_3 remain for further evaluation. They considered five benefit criteria, are as follows:

(i) Emotional steadiness (C_1)
(ii) Oral communication skill (C_2)
(iii) Personality(C_3)
(iv) Past experience (C_4)
(v) Self-confidence(C_5)

The credits given by the two decision makers against the three candidates are given in Type -2 Fuzzy Soft Sets. They are tabulated as follows:

Among the three candidates, the company has to select one from the list and has to keep another one as a alternate person in the waiting list. For this reason the company has to neglect the remaining person from the list. To deal with this situation the concept of complement of type -2 fuzzy soft set is used.

$(\mathscr{P},A)$	A_1	A_2	A_3
C_1	$0.3/0.4+0.5/0.5$	$0.5/0.6$	$0.6/0.6$
C_2	$0.4/0.5+0.5/0.6$	$0.4/0.5+0.7/0.6$	$0.3/0.4+0.5/0.6$
C_3	$0.6/0.5$	$0.4/0.6$	$0.5/0.3+0.6/0.5$
C_4	$0.8/0.6$	$0.3/0.4+0.8/0.5$	$0.4/0.4+0.6/0.5$
C_5	$0.5/0.6$	$0.1/0.4$	$0.2/0.7$

Table 4.1: Tabular representation of the type – 2 fuzzy soft set $(\mathscr{P},A)$

$(\mathscr{P}^c,A)$	A_1	A_2	A_3
C_1	$0.3/0.6+0.5/0.5$	$0.5/0.4$	$0.6/0.4$
C_2	$0.4/0.5+0.5/0.4$	$0.4/0.5+0.7/0.4$	$0.3/0.6+0.5/0.4$
C_3	$0.6/0.5$	$0.4/0.4$	$0.5/0.7+0.6/0.5$
C_4	$0.8/0.4$	$0.3/0.6+0.8/0.5$	$0.4/0.6+0.6/0.5$
C_5	$0.5/0.4$	$0.1/0.6$	$0.2/0.3$

Table 4.2: Tabular representation of the complement of type – 2 fuzzy soft set $(\mathscr{P}^c,A)$

$(\mathscr{Q},A)$	A_1	A_2	A_3
C_1	$0.4/0.5+0.5/0.7$	$0.4/0.5+0.6/0.7$	$0.6/0.4+0.7/0.5$
C_2	$0.4/0.5+0.5/0.7$	$0.7/0.7$	$0.4/0.6$
C_3	$0.5/0.5+0.5/0.6$	$0.4/0.5+0.5/0.6$	$0.4/0.6+0.7/0.7$
C_4	$0.4/0.6+0.6/0.8$	$0.5/0.5+0.6/0.7$	$0.5/0.6$

C_5	$0.6/0.7$	$0.7/0.8$	$0.5/0.6 + 0.6/0.7$

Table 4.3: Tabular representation of the type – 2 fuzzy soft set ($\mathscr{Q}$,A)

($\mathscr{Q}^c$,A)	A_1	A_2	A_3
C_1	$0.4/0.5 + 0.5/0.3$	$0.4/0.5 + 0.6/0.3$	$0.6/0.6 + 0.7/0.5$
C_2	$0.4/0.5 + 0.5/0.3$	$0.7/0.3$	$0.4/0.4$
C_3	$0.5/0.5 + 0.5/0.4$	$0.4/0.5 + 0.5/0.4$	$0.4/0.4 + 0.7/0.3$
C_4	$0.4/0.4 + 0.6/0.2$	$0.5/0.5 + 0.6/0.3$	$0.5/0.4$
C_5	$0.6/0.3$	$0.7/0.2$	$0.5/0.4 + 0.6/0.3$

Table 4.4: Tabular representation of the complement of type–2 fuzzy soft set ($\mathscr{Q}$,A)

Using the above mentioned definitions we can find the similarity measure and inclusion measure between type -2 fuzzy soft sets. The calculated measure values are tabulated as below.

Parameters	$M_1(\mathscr{P},\mathscr{Q}$	$M_1\left(\mathscr{P}^c,\mathscr{Q}^c\right)$	$M_2(\mathscr{P},\mathscr{Q})$	$M_2(\mathscr{Q},\mathscr{P})$	$M_2\left(\mathscr{P}^c,\mathscr{Q}^c\right)$	$M_2\left(\mathscr{Q}^c,\mathscr{P}^c\right)$
A_1	2.5	1.6667	1	2.5	1.6667	1
A_2	0.1	0.1	0.1	0.02	0.1	0.04
A_3	0.2	25	0.2	0.25	25	1.25
Max(A_1,A, A_3)	2.5	25	1	2.5	25	1.25
Selected Parameter	A_1	A_3	A_1	A_1	A_3	A_3

Table 4.5: Calculated values of similarity measure and inclusion measure

From the table we observe that

1. The measure values of Type-2 fuzzy soft set and its complement are same when its membership values of the parameters are having median value.
2. $\min\left\{M_{1j}(\mathscr{P},\mathscr{Q})\right\} \neq M(\mathscr{P}^c,\mathscr{Q}^c)$ and

   $\min\left\{M_{2j}(\mathscr{P},\mathscr{Q})\right\} \neq M_2(\mathscr{P}^c,\mathscr{Q}^c)$

3. The following relationship between similarity measure and the inclusion measure is verified from the table 4.5

$$\min\left\{M_2\left(\mathscr{P},\mathscr{Q}\right),M_2\left(\mathscr{Q},\ \mathscr{P}\right)\right\}=M_1\left(\mathscr{P},\mathscr{Q}\right)$$

$$\min\left\{M_2\left(\mathscr{P}^c,\mathscr{Q}^c\right),M_2\left(\mathscr{Q}^c,\ \mathscr{P}^c\right)\right\}=M_1\left(\mathscr{P}^c,\mathscr{Q}^c\right)$$

The parameter $A_1$is having maximum value. So that the candidate A_1 is selected from the decision list. The parameter A_3 is having maximum value in the case of complement of type-2 fuzzy soft set. So that the candidate A_3 is neglected from the decision list.

4. Conclusion

In this paper, we have proved the relationship between similarity measure and inclusion measure on type -2 fuzzy soft sets. A numerical example is solved to illustrate this relationship to obtain a solution of a decision making problem.

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