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A Fuzzy Inventory Model for Deteriorating Items with Linear Price Dependent Demand in a Supply Chain

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***Abstract*.** In this paper we have developed a supply chain production inventory model for deteriorating items under fuzzy environment. Demand is taken as linear price dependent. In reality it is seen that we cannot define all parameters precisely due to imprecision or uncertainty in the environment. So we have defined the inventory parameters, such as set up cost, holding cost and deteriorating cost as triangular fuzzy numbers. The signed distance method and graded mean integration method have been used for defuzzification. To illustrate the proposed model a numerical example and sensitivity analysis with respect to different associated parameter has been presented.

***Keywords*:** Inventory, deterioration, linear price dependent demand, supply chain, fuzzy, triangular fuzzy numbers, defuzzification, signed distance method, graded mean integration method.

AMS Mathematics Subject Classification (2010): 90B05

1. Introduction

The control and maintenance of any inventory of deteriorating items plays an important role in any supply chain management system as most physical goods such as food products and beverages, pharmaceuticals, radioactive substances, gasoline etc. deteriorate over time. Various researchers have investigated these issues over time. Misra first studied optimum production lot size model for a system with deteriorating inventory (Misra 1975). Goyal and Giri in 2003, considered a production–inventory problem of a product with time varying demand, production and deterioration rates (Goyal &Giri 2003). In the same year, Yang and Wee, considered a multi-lot-size production-inventory system for deteriorating items with constant production and demand rates (Yang & Wee 2003). In the following year, Sana et al. developed a production-inventory model for deteriorating item with trended demand and shortages (Sana et al. 2004). In 2010 Manna and Chiang developed an economic production quantity model for deteriorating items with ramp type demand rate (Manna & Chiang 2010).

In the crisp environment, all parameters associated with the model such as holding cost, set-up cost, purchasing price, rate of deterioration, demand rate, production rate etc. are known and have definite value without uncertainty. Although some of the business situations fit such condition, in reality most of the situations and in the rapidly changing market scenario the parameters and variables are highly uncertain. In such situations, these parameters and variables are described as fuzzy parameters. The fuzzification admits authenticity to the model by allowing vagueness in the whole setup which brings it closer to reality. Several researchers like Jaggi et al., Yao & Chiang, Wang et al., Yao & Lee, Wang et al, Yao & Lee, Kao & Hsu and Saha have studied inventory models under fuzzy environment.(Jaggi et al. 2013), (Yao & Chiang 2003),(Wang et al. 2007), (Yao & Lee 1999), (Kao & Hsu 2002), (Dutta et al. 2005), (Saha 2017).

Various authors have developed inventory models assuming various types of demand, such as constant, time dependent, stock dependent and price dependent. Kumar & Rajput, Mishra et al., Khurana, and Kar et al. studied the model with time dependent (Kumar & Rajput 2015), (Mishra et al. 2015), (Khurana 2015), (Kar et al. 2006). Whereas, Mondal et al., Mahata & De and Singh & Vishnoi investigated the price dependent inventory models (Mondal et al. 2003), (Mahata & De 2016), (Singh &Vishnoi 2013). Other related models on inventory systems with stock-dependent consumption rate were developed by He et al., Datta & Paul, Wang, Tripathi & Mishra and Rani Chaudhary et al. (He et al. 2013), (Datta & Paul 2001), (Wang 2011), (Tripathi & Mishra 2014), (Chaudhary et al. 2013).

This paper has presented a supply chain production inventory model with constant rate of deterioration, where we considered various costs, such as setup cost, holding cost and cost of deteriorating items is taken as triangular fuzzy numbers and demand rate is linear price dependent. Later on, the fuzzy total cost is defuzzified by using signed distance method and graded mean integration method.

The rest of this paper is organized as follows. In section 2, the assumption and notations are given. In section 3, we developed the mathematical models. In section 4, we provided numerical examples to illustrate the results. In addition, the sensitivity analysis of the optimal solution with respect to parameters of the system is carried out in section 5. Finally, we drew the conclusions in section 6 and references in section 7.

2. Assumptions and notations

2.1. Assumptions

The model is based on the following assumptions:

i) The inventory system involves production of single item.
ii) Lead time is zero and shortages are not allowed.
iii) The set-up cost, deterioration rate, holding cost are fuzzy.
iv) Demand rate is linear price dependent.
v) Replenishment is instantaneous.

2.2. Notations

We have used the following notations to develop the model-

i) $D = a – bp$ is the demand rate, where a, b are constants and p is the selling price.
ii) k= production rate.
iii) C_0 = setup cost.
iv) $\widetilde{C_0}$ = fuzzy setup cost.
v) C_1 = holding cost per unit per unit time.

vi) $\widetilde{C_1}$ = fuzzy holding cost per unit per unit time.
vii) θ = deterioration rate, $0 < \theta << 1$.
viii) C_2 = deterioration cost per unit per unit time.
ix) $\widetilde{C_2}$ = fuzzy deterioration cost per unit per unit time.
x) T = cycle length.
xi) T_s = cycle length when signed distance method of defuzzification is used.
xii) T_G = cycle length when graded mean integration method of defuzzification is used.
xiii) t_1 = duration of production.
xiv) $I_1(t)$ = inventory level at time t, $0 \le t \le t_1$.
xv) $I_2(t)$ = inventory level at any time t, $t_1 \le t \le T$.
xvi) TC = total cost per unit time.
xvii) $\widetilde{TC}$ = fuzzified value of TC.
xviii) TC_s= defuzzified value of $\widetilde{TC}$ when signed distance method of defuzzification is used.
xix) TC_G = defuzzified value of $\widetilde{TC}$ when graded mean integration method of defuzzification is used.

3. Mathematical model

The inventory level is zero at time t = 0. It accumulates in the time period [0, t_1] due to production at the constant rate k. After that inventory level decreases due to demand and deterioration and reaches to zero at t = T. The change in the inventory level can be described by the following differential equations:

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = k - (a - bp)\ , \ 0 \le t \le t_1 \tag{1}$$

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -(a - bp)\ , \ 0 \le t \le T \tag{2}$$

With boundary conditions $I_1(0) = 0$, $I_1(t_1) = I_2(t_1)$ and $I_2(T) = 0$
Solving these equations and using boundary conditions we have

$$I_1(t) = \frac{1}{\theta}\{k - (a - bp)\}(1 - e^{-\theta t}) \tag{3}$$

And $$I_2(t) = \frac{(a-bp)}{\theta}[e^{\theta(T-t)} - 1] \tag{4}$$

Now we find t_1 by using $I_1(t_1) = I_2(t_1)$

- $\frac{1}{\theta}\{k - (a - bp)\}(1-e^{-\theta t_1}) = \frac{(a-bp)}{\theta}[e^{\theta(T-t_1)} - 1]$

$$\Rightarrow k - ke^{-\theta t_1} + (a - bp)\, e^{-\theta t_1} = (a - bp)\, e^{\theta(T-t_1)}) \tag{5}$$

$$\Rightarrow ke^{\theta t_1} = \{k - (a - bp)\} + (a - bp)\, e^{\theta T}$$

$$\Rightarrow t_1 = \frac{1}{\theta}\ln[1 + \frac{(a-bp)}{k}(e^{\theta T} - 1)] \tag{6}$$

Holding cost

$= C_1 [\int_0^{t_1} I_1(t)dt + \int_{t_1}^{T} I_2(t)dt]$

$= C_1 [\int_0^{t_1} \frac{1}{\theta}\{k - (a - bp)\}(1 - e^{-\theta t})\ dt + \int_{t_1}^{T} \frac{(a-bp)}{\theta}[e^{\theta(T-t)} - 1]\ dt]$

$= \frac{C_1}{\theta^2}[k(\theta t_1 + e^{-\theta t_1} - 1) + (a - bp)\{e^{\theta(T-t_1)} - e^{-\theta t_1}\} - (a - bp)\theta T]$

$= \frac{C_1}{\theta^2}[k(\theta t_1 + e^{-\theta t_1} - 1) + (k - ke^{-\theta t_1}) - (a - bp)\theta T]$ (Using (5))

$= \frac{C_1}{\theta}\,[kt_1 - (a - bp)\,T]$ (7)

The deterioration cost per cycle-

$C_2\,[\int_0^{t_1} \theta I_1(t)dt + \int_{t_1}^{T} \theta I_2(t)dt]$

$= C_2\;[kt_1 - (a - bp)T]$ (8)

Therefore, total cost per unit time-

TC = set up cost + holding cost + deterioration cost

$= \frac{C_0}{T} + \frac{C_1}{\theta T}[\,kt_1 - (a - bp)T] + \frac{C_2}{T}\,[kt_1 - (a - bp)T]$

$= \frac{C_0}{T} + [\,kt_1 - (a - bp)T]\,\frac{(C_1+C_2\theta)}{\theta T}$

$= \frac{C_0}{T} + [\,\frac{k}{\theta}\ln\{1 + \frac{(a-bp)}{k}(e^{\theta T} - 1)\} - (a - bp)\,T]\,\frac{(C_1+C_2\theta)}{\theta T}$ (using (6))

$= \frac{C_0}{T} + \frac{(C_1+C_2\theta)}{\theta T}\,[\frac{k}{\theta}\ln\{1 + \frac{(a-bp)}{k}(\theta T + \frac{\theta^2T^2}{2})\} - (a - bp)T]$

(Neglecting higher power of θ)

$= \frac{C_0}{T} + \frac{(C_1+C_2\theta)}{\theta T}\,[\frac{k}{\theta}\,\{\frac{(a-bp)}{k}(\theta T + \frac{\theta^2T^2}{2}) - \frac{(a-bp)^2}{2k^2}(\theta T + \frac{\theta^2T^2}{2})^2\} - (a - bp)T]$

(Neglecting higher power of θ)

$= \frac{C_0}{T} + \frac{(C_1+C_2\theta)}{\theta T}\,[\frac{k}{\theta}\,\{\frac{(a-bp)}{k}(\theta T + \frac{\theta^2T^2}{2}) - \frac{(a-bp)^2}{2k^2}\theta^2T^2\} - (a - bp)T]$

(Neglecting higher power of θ)

$= \frac{C_0}{T} + \frac{(C_1+C_2\theta)}{\theta T}\,[\frac{(a-bp)\theta T^2}{2} - \frac{(a-bp)^2}{2k}\theta T^2]$

$= \frac{1}{T}\,[A + \frac{1}{2}(h + d\theta)(a - bp)T^2 - \frac{1}{2}(h + d\theta)(a - bp)^2\,\frac{T^2}{k}]$ (9)

Now, $\frac{\partial(TC)}{\partial T} = 0$ gives-

$$T = \sqrt{\frac{2C_0}{(C_1+C_2\theta)(a-bp)\{1-\frac{(a-bp)}{k}\}}}$$

3.1 Fuzzy model

Next we fuzzify the parameters C_0, C_1 and C_2.

Let $C_0 = (a_1, b_1, c_1)$, $C_1 = (a_2, b_2, c_2)$ and $\widetilde{C_2} = (a_3, b_3, c_3)$.

Then $\widetilde{TC} = \frac{1}{T}\,[\widetilde{C_0} + \frac{1}{2}(\widetilde{C_1} + \widetilde{C_2}\theta)\,(a - bp)\,T^2 - \frac{1}{2}((\widetilde{C_1} + \widetilde{C_2}\theta)\,(a - bp)^2\frac{T^2}{k}]$

$= (TC_1, TC_2, TC_3)$ (say)

Where, $TC_1 = \frac{1}{T}\,[a_1 + \frac{1}{2}(a_2 + a_3\theta)\,(\,(a - bp)\,T^2 - \frac{1}{2}(a_2 + a_3\theta)\frac{(a-bp)^2T^2}{k}]$

$TC_2 = \frac{1}{T}\,[b_1 + \frac{1}{2}(b_2 + b_3\theta)\,(\,(a - bp)\,T^2 - \frac{1}{2}(b_2 + b_3\theta)\frac{(a-bp)^2T^2}{k}]$

and $TC_3 = \frac{1}{T}\,[c_1 + \frac{1}{2}(c_2 + c_3\theta)\,(\,(a - bp)\,T^2 - \frac{1}{2}(c_2 + c_3\theta)\frac{(a-bp)^2T^2}{k}]$

i) Signed distance method

$TC_s = \frac{1}{4}(TC_1 + 2TC_2 + TC_3)$

$= \frac{1}{4T}$ [{$a_1 + \frac{1}{2}(a_2 + a_3\theta)$ ((a – bp) T^2 - $\frac{1}{2}(a_2 + a_3\theta)\frac{(a-bp)^2T^2}{k}$} + 2{$b_1 + \frac{1}{2}(b_2 + b_3\theta)$ ((a – bp) T^2 - $\frac{1}{2}$ $(b_2 + b_3\theta)\frac{(a-bp)^2T^2}{k}$} +{ c_1 + $\frac{1}{2}$ $(c_2 + c_3\theta)$ ((a – bp) T^2 - $\frac{1}{2}$ $(c_2 + c_3\theta)\frac{(a-bp)^2T^2}{k}$}]

$= \frac{1}{4T}$ [$(a_1 + 2b_1 + c_1)$ + $\frac{1}{2}$ {$(a_2 + 2b_2 + c_2)$ + $(a_3 + 2b_3 + c_3)\theta$} ((a – bp)T^2 - $\frac{1}{2}${$(a_2 + 2b_2 + c_2)$ + $(a_3 + 2b_3 + c_3)\theta$ }$\frac{(a-bp)^2T^2}{k}$]

$= \frac{(a_1+2b_1+c_1)}{4T} + \frac{1}{8}$ {$(a_2 + 2b_2 + c_2)$ + $(a_3 + 2b_3 + c_3)\theta$} (a – bp)T - $\frac{1}{8}${$(a_2 + 2b_2 + c_2)$ + $(a_3 + 2b_3 + c_3)\theta$} $\frac{(a-bp)^2T}{k}$

Now, $\frac{d(TC_s)}{dT}$ = 0 gives,

$$T_s = \sqrt{\frac{2(a_1+2b_1+c_1)}{\{(a_2+2b_2+c_2) + (a_3+2b_3+c_3)\theta\}(a-bp)\{1-\frac{(a-bp)}{k}\}}}$$

ii) Graded mean integration method:

$TC_G = \frac{1}{6}$ (TC$_1$ + 4TC$_2$ + TC$_3$)

$= \frac{1}{6T}$[{$a_1 + \frac{1}{2}(a_2 + a_3\theta)$ ((a – bp) T^2 - $\frac{1}{2}(a_2 + a_3\theta)\frac{(a-bp)^2T^2}{k}$}+ 4{ $b_1 + \frac{1}{2}(b_2 + b_3\theta)$ ((a – bp) T^2 - $\frac{1}{2}$ $(b_2 + b_3\theta)\frac{(a-bp)^2T^2}{k}$} + {$c_1$ + $\frac{1}{2}$ $(c_2 + c_3\theta)$ ((a – bp) T^2 - $\frac{1}{2}$ $(c_2 + c_3\theta)\frac{(a-bp)^2T^2}{k}$}]

$= \frac{1}{6T}$[$(a_1 + 4b_1 + c_1)$ + $\frac{1}{2}$ {$(a_2 + 4b_2 + c_2)$ + $(a_3 + 4b_3 + c_3)\theta$}(a – bp) T^2 - $\frac{1}{2}$ {$(a_2 + 4b_2 + c_2)$ + $(a_3 + 4b_3 + c_3)\theta$ }$\frac{(a-bp)^2T^2}{k}$]

$= \frac{(a_1+4b_1+c_1)}{6T} + \frac{1}{12}${$(a_2 + 4b_2 + c_2)$ + $(a_3 + 4b_3 + c_3)\theta$}(a – bp) T - $\frac{1}{12}${$(a_2 + 4b_2 + c_2)$ + $(a_3 + 4b_3 + c_3)\theta$}$\frac{(a-bp)^2T}{k}$

Now, $\frac{d(TC_G)}{dT}$ = 0 gives,

$$T_G = \sqrt{\frac{2(a_1+4b_1+c_1)}{\{(a_2+4b_2+c_2)+ (a_3+4b_3+c_3)\theta\}(a-bp)\{1-\frac{(a-bp)}{k}\}}}$$

4. Numerical example

We consider the following numerical values of the parameters in appropriate units to analyze the model:

$\widetilde{C_0}$ = (490,495, 500), $\widetilde{C_1}$ = (5, 6, 7), $\widetilde{C_2}$ = (10, 12, 14), k = 150, a = 145, θ = 0.01, b = 0.5, p = 125.

We obtain TC_s = 577.405 and total time T_s = 1.092 for signed distance method.
TC_G = 624.811 and total time T_G = 0.960 for graded mean integration method.

4.1. Sensitivity analysis

Table 1. Sensitivity on k

Change value	Signed distance method		Graded mean integration method	
K	TC_s	T_s	TC_G	T_G
150	577.405	1.092	624.811	0.960
155	588.677	1.071	637.009	0.941
160	599.052	1.052	648.236	0.925
165	608.638	1.036	658.608	0.910

Table 2. Sensitivity on θ

Change value	Signed distance method		Graded mean integration method	
Θ	TC_s	T_s	TC_G	T_G
0.1	577.405	1.092	624.811	0.960
0.3	584.008	1.086	630.753	0.956
0.5	590.559	1.080	636.657	0.952
0.7	597.057	1.075	642.524	0.948

Table 3. Sensitivity on $\widetilde{C_0}$

Change value	Signed distance method		Graded mean integration method	
$\widetilde{C_0}$	TC_s	T_s	TC_G	T_G
(490,495,500)	577.405	1.092	624.811	0.960
(486,491,496)	575.067	1.087	622.281	0.956
(482,487,492)	572.720	1.083	619.741	0.952
(478,483,488)	570.363	1.078	617.191	0.948

Table 4. Sensitivity on $\widetilde{C_1}$

Change value	Signed distance method		Graded mean integration method	
$\widetilde{C_1}$	TC_s	T_s	TC_G	T_G
(5, 6,7)	577.405	1.092	624.811	0.960
(6, 7, 8)	647.094	0.949	704.473	0.833
(7, 8, 9)	716. 369	0.839	783.824	0.736
(8, 9, 10)	785.351	0.752	862.956	0.659

5.1. Observations

The following are noted on the basis of the sensitivity analysis-

i) From table-1 and table-2 it is observed that, an increase in production rate and deterioration rate causes anupliftment in total cost for both the models. In contrast, the rise in these two parameters results in decrease in cycle time for both the developed models.

ii) As the set up cost decreases hence the costs TC_s and TC_G and the optimum cycle times $T_s$and $T_G$decrease.

iii) The total cost (for both the models) increases as the holding cost per unit time increases.

6. Conclusion

In this paper we have developed a supply chain inventory model for deteriorating items under fuzzy environment. In our real life we generally find the trend that consumers' consumption rate varies drastically depending on the selling price of the items, so demand rate is assumed to be linear price dependent. We observed that the total cost is minimum with corresponding value of T when signed distance method of defuzzification is used. On the other hand, the cycle time(T) is minimum with corresponding total cost when graded mean integration method is used.

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