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Some Common Fixed Point Theorems in Fuzzy 2-Banach Space

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Abstract. In this paper we proved a common fixed point theorem for two, four and six mappings in fuzzy 2-banach space by using compatibility of type A, compatibility of type P and implicit relations. Our result is an extension of existing results in fuzzy 2-banach space.

Keywords: Fuzzy 2-Banach space, contractive mapping, implicit relations, fixed point.

AMS Mathematics Subject Classification (2010): 47H10

1. Introduction

A theory of fuzzy sets was introduced by Zadeh [10] in 1965, which plays a major role in almost all branches of science and Engineering. The definition of fuzzy norms was introduced by Wu and Fang [3] and they studied the comparison between two definitions of fuzzy normed spaces. The concept of 2-norm in linear spaces was initiated by Gahler [4] and White [9] introduced the concept of Cauchy sequences and convergent sequences in a 2-normed spaces. Also, he introduced the concept of linear 2-functional on a fuzzy 2-normed space. Many authors have studied common fixed point theorems in fuzzy banach space. In this paper, we proved some common fixed point theorems for two, four and six mappings in fuzzy 2-banach space.

2. Preliminaries

In this section, we study 2-normed linear spaces, fuzzy normed spaces its convergence and completeness of sequences in a fuzzy 2-normed linear space. Also, we need some basic definitions required for proving the common fixed point theorems in fuzzy 2-Banach spaces.

Definition 2.1. Let X be a vector space over a field K (where K is R or C) and * be a continuous t-norm. A fuzzy set N in $X \times [0, \infty]$ is called a fuzzy norm on X if it satisfies the following conditions:

(i) $N(x,0) = 0 \; \forall \, x \in X$
(ii) $N(x,t) = 1, \; \forall \, t > 0$ iff $x = 0$
(iii) $N(\lambda x, t) = N\left(x, \frac{t}{|\lambda|}\right), \; \forall \, x \in X, \; t \geq 0 \; and \; \lambda \in K.$
(iv) $N(x+y, t+s) \geq N(x,t) * N(y,s), \; \forall x, y \in X \; and \; t, s \geq 0.$
(v) $for \; every \; x \in X, N(x,.)$ is left continuous and $\lim_{t \to \infty} N(x,t) = 1.$

The triple $(X, N, *)$will be called fuzzy normed linear space (FNLS).

Definition 2.2. Let X be a vector space over a field K (where K is R or C) and * be a continuous t-norm. A fuzzy set N in $X^2 \times [0, \infty]$ is called a fuzzy 2-norm on X if it satisfies the following conditions:

(i) $N(x,y,0) = 0 \; \forall \, x, y \in X$
(ii) $N(x,y,t) = 1, \forall \, t > 0$ and atleast two among the three points are equal.
(iii) $N(x,y,t) = N(y,x,t)$
(iv) $N(x+y+z, t_1+t_2+t_3) \geq N(x,y,t_1) * N(x,z,t_2) * N(y,z,t_3),$
$\forall x, y, z \in X \; and \; t_1, t_2, t_3 \geq 0.$
(v) $for \; every \; x, y \in X, N(x,y,.)$ is left continuous and
$\lim_{t \to \infty} N(x,y,t) = 1.$

The triple $(X, N, *)$will be called fuzzy 2-normed linear space (F2-NLS).

Definition 2.3. A sequence $\{x_n\}$ in a F2-NLS $(X, N, *)$ is converge to $x \in X$ if and only if $\lim_{n \to \infty} N(x_n, x, t) = 1, \forall \, t > 0.$

Definition 2.4. Let $(X, N, *)$ be a F2-NLS. A sequence $\{x_n\}$ in X is called a fuzzy Cauchy sequence if and only if $\lim_{m,n \to \infty} N(x_m, x_n, t) = 1 \,, \forall \, p, t > 0.$

Definition 2.5. A linear fuzzy 2-normed space in which every Cauchy sequence is convergent is called a fuzzy 2-Banach space.

Definition 2.6. Self mappings S and T of a fuzzy 2-Banach space $(X, N, *)$ are said to be weakly commuting if $N(STx, TSx, t) \geq N(Sx, Tx, t), \forall x \in X \;\&\; t > 0.$

Definition 2.7. Self mappings S and T of a fuzzy 2-Banach space $(X, N, *)$ are said to be compatible if and only if $\lim_{n \to \infty} N(STx_n, TSx_n, t) = 1, \forall \, t > 0$ whenever $\{x_n\}$ is a sequence in X such that $Tx_n, Sx_n \to p$ for some $p \in X \; as \; n \to \infty.$

Definition 2.8. Self mappings S and T of a fuzzy 2-Banach space $(X, N, *)$ are said to be compatible type (A) if and only if $\lim_{n \to \infty} N(STx_n, TTx_n, t) = 1$ and $\lim_{n \to \infty} N(TSx_n, SSx_n, t) = 1, \forall \, t > 0$ whenever $\{x_n\}$ is a sequence in X such that $Tx_n, Sx_n \to p$ for some $p \in X \; as \; n \to \infty.$

Definition 2.9. Self mappings S and T of a fuzzy 2-Banach space $(X, N, *)$ are said to be compatible type (P) if and only if $\lim_{n \to \infty} N(SSx_n, TTx_n, t) = 1 \; \forall \, t > 0$ whenever $\{x_n\}$ is a sequence in X such that $Tx_n, Sx_n \to p$ for some $p \in X \; as \; n \to \infty$.

Definition 2.10. Suppose S and T be self mappings of fuzzy 2-Banach space$(X, N, *)$. A point $x \in X$ is called a coincidence point of S and T if and only if $Sx = Tx$, then $w = Sx = Tx$ is called a point of coincidence of S and T.

Definition 2.11. Self-maps S and T of a fuzzy 2-Banach space $(X, N, *)$ are said to be weakly compatible if they commute at their coincidence points. That is, if $Sp = Tp$ for some $p \in X$ then $STp = TSp$.

Definition 2.12. Self-maps S and T of a fuzzy 2-Banach space $(X, N, *)$ are said to be occasionally weakly compatible (owc) if and only if there is a point $x \in X$ which is the coincidence point of S and T at which they commute.

Definition 2.13. Self-maps S and T of a fuzzy 2-Banach space $(X, N, *)$ are said to be sub compatible if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$, $z \in X$ and $\lim_{n\to\infty} N(STx_n, TSx_n, t) = 1$.

Definition 2.14. Self-maps S and T of a fuzzy 2-Banach space $(X, N, *)$ are said to be sub compatible of type (A) if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z, z \in X$ and satisfy $\lim_{n\to\infty} N(STx_n, TTx_n, t) = 1$ and $\lim_{n\to\infty} N(TSx_n, SSx_n, t) = 1$.

Implicit Relation: Let $\{\emptyset\}$ be the set of all real continuous function $\varphi: (R^+)^6 \to R^+$ satisfying the following condition: $\varphi(u, u, v, v, u, u) \geq 0$ imply $u \geq v\ \forall\ u, v \in [0,1]$.

3. Material and method

Lemma 3.1. Let $(X, N, *)$ be a fuzzy 2-Banach space. If there exists $k \in (0,1)$ such that $N(x, y, kt) \geq N(x, y, t)$ for all $x, y \in X$ and $t > 0$ then $x = y$.

Lemma 3.2. Let X be a set, f, g owc self maps of X. If f and g have a unique point of coincidence, w = f*x* = g*x*; then w is the unique common fixed point of f and g.

Theorem 3.1. Let $(X, N, *)$ be a fuzzy 2-Banach space with continuous t-norm. Let A, B be two self mappings of X satisfying

1. The pair (A, S) be owc.
2. For some $\varphi \in \emptyset$ and for all $x, y, z \in X$ and every t>0,

$$\varphi\begin{Bmatrix} N(Ax, Ay, t), N(Ax, Sy, t), N(Ax, Sx, t), N(Ay, Sy, t), \\ N(Ay, Sx, t), N(Sx, Sy, t) \end{Bmatrix} \geq 0$$

Then there exists a unique fixed point $w \in X$ such that $Aw = Sw = w$.

Proof: Since the pair (A, S) be owc, there are points $x, y, z \in X$ such that $Ax = Sx$.
We claim that $Ax = Ay$.
Suppose, $Ax \neq Ay$.
Then by (2),

$$\varphi\{N(Ax, Ay, t), N(Ax, Ay, t), N(Ax, Ax, t), N(Ay, Ay, t), N(Ay, Ax, t), N(Ax, Ay, t)\} \geq 0$$

$$That\ is, \varphi\{N(Ax, Ay, t), N(Ax, Ay, t), 1, 1, N(Ay, Ax, t), N(Ax, Ay, t)\} \geq 0$$

$$That\ is, \varphi\{N(Ax, Ay, t), N(Ax, Ay, t), 1, 1, N(Ax, Ay, t), N(Ax, Ay, t)\} \geq 0$$

In view of $\emptyset$ we get $Ax = Ay$.
That is, $Ax = Sx = Ay = Sy$.
Suppose that $w \in X$ is another fixed point such that $Aw = Sw$.
They by (1), $Aw = Sw = By = Ty$.
So, $Ax = Aw$ and $w = Ax = Sx$ is the unique point of coincidence of A and S.
$Therefore$, w is a common fixed point of A and S. [By Lemma 3.2].

Uniqueness:
Let w_1 and w_2 be two common fixed points of A and S.
Assume that $w_1 \neq w_2$

$$\varphi\left\{\begin{matrix} N(Aw_1, Aw_2, t), N(Aw_1, Aw_2, t), N(Aw_1, Aw_1, t), N(Aw_2, Aw_2, t), N(Aw_2, Aw_1, t), \\ N(Aw_1, Aw_2, t) \end{matrix}\right\} \geq 0$$

i.e.,

$$\varphi\{N(Aw_1, Aw_2, t), N(Aw_1, Aw_2, t), 1, 1, N(Aw_2, Aw_1, t), N(Aw_1, Aw_2, t)\} \geq 0$$

i.e.,

$$\varphi\{N(Aw_1, Aw_2, t), N(Aw_1, Aw_2, t), 1, 1, N(Aw_1, Aw_2, t), N(Aw_1, Aw_2, t)\} \geq 0$$

Therefore, $w_1 = w_2$.
Thus w is the unique fixed point of A and S.

Theorem 3.2 Let $(X, N, *)$ be a fuzzy 2-Banach space with continuous t-norm. Let A, B, S, T be four self mappings of X satisfying

1. The pairs (A, S) and (B, T) are owc.
2. For some $\varphi \in \emptyset$ and for all $x, y, z \in X$ and every t>0,

$$\varphi\left\{\begin{matrix} N(Ax, By, t), N(Sx, Ty, t), N(Sx, Ax, t), N(Ax, Ty, t), \\ N(Sx, By, t), N(Ty, By, t) \end{matrix}\right\} \geq 0$$

Then there exists a unique fixed point $w \in X$ such that $Aw = Sw = w$ and a unique point $z \in X$ such that $Bz = Tz = z$. Moreover $z = w$ is a unique common fixed point of A, B, S and T.

Proof : Let the pairs (A, S) and (B, T) be owc.
So, there are points $x, y, z \in X$ such that $Ax = Sx$ and $By = Ty$.
We claim that $Ax = By$.
If $Ax \neq By$, then by the inequality (2) we have,

$$\varphi\left\{\begin{matrix} N(Ax, By, t), N(Sx, Ty, t), N(Sx, Ax, t), N(Ax, Ty, t), \\ N(Sx, By, t), N(Ty, By, t) \end{matrix}\right\} \geq 0$$

i.e.,

$$\varphi\left\{\begin{matrix} N(Ax, By, t), N(Ax, By, t), N(Ax, Ax, t), N(Ax, By, t), \\ N(Ax, By, t), N(By, By, t) \end{matrix}\right\} \geq 0$$

i.e.,

$$\varphi\{N(Ax, By, t), N(Ax, By, t), 1, N(Ax, By, t), N(Ax, By, t), 1\} \geq 0$$

In view of $\emptyset$ we get $Ax = By$.
That is, $Ax = Sx = By = Ty$.
Suppose that there is another point $w \in X$ such that $Aw = Sw$.
Then we have, $Aw = Sw = By = Ty$.
So $Ax = Aw$ and $w = Ax = Sx$ is the unique point of coincidence of A and S.

Therefore , w is a common fixed point of A and S.
Suppose that there is another point $u \in X$ such that $Bu = Tu$.
Thus we have $Ax = Sx = Bu = Tu$.
So, $By = Bu$ and $u = By = Ty$ is the unique point of coincidence of B and T.
Therefore, u is a common fixed point of B and T.
Assume that $w \neq u$.
Then we have,

$$\varphi\begin{Bmatrix} N(Aw, Bu, t), N(Sw, Tu, t), N(Sw, Aw, t), N(Tu, Bu, t), \\ N(Aw, Tu, t), N(Sw, Bu, t) \end{Bmatrix} \geq 0$$

That is, $\varphi\{N(w, u, t), N(w, u, t), N(w, w, t), N(u, u, t), N(w, u, t), N(w, u, t)\} \geq 0$
That is, $\varphi\{N(w, u, t), N(w, u, t), 1,1, N(w, u, t), N(w, u, t)\} \geq 0$
In view of $\emptyset$ we get $w = u$.
Therefore, z is a common fixed point of A, B, S and T.

Uniqueness:
Let w_1 and w_2 be two common fixed points of A, B, S and T.
Assume that $w_1 \neq w_2$

$$\varphi\begin{Bmatrix} N(Aw_1, Bw_2, t), N(Sw_1, Tw_2, t), N(Sw_1, Aw_1, t), N(Tw_2, Bw_2, t), N(Aw_2, Tw_1, t), \\ N(Sw_1, Bw_2, t) \end{Bmatrix}$$
$$\geq 0$$

i.e.,
$\varphi\{N(w_1, w_2, t), N(w_1, w_2, t), N(w_1, w_1, t), N(w_2, w_2, t), N(w_1, w_2, t), N(w_1, w_2, t)\} \geq 0$
i.e., $\varphi\{N(w_1, w_2, t), N(w_1, w_2, t), 1, 1, N(w_1, w_2, t), N(w_1, w_2, t)\} \geq 0$
Therefore, $w_1 = w_2$.
Hence, the fixed point is unique.

Theorem 3.3. Let $(X, N, *)$ be a fuzzy 2-Banach space and let A *and* Bbe continuous self mappings of X, S and T satisfying the following conditions:
(i) $A(X) \subset T(X)$ and $B(X) \subset S(X)$
(ii) $N(Ax, By, t) \geq$
$min\{N(Sx, Ty, t), N(Ax, Sx, t), N(By, Ty, t), N(Ax, Ty, t), N(By, Sx, t)\}$
If the pairs (A, T) and (B, S) are compatible mappings of type (P), for any $x, y, z \in X$ and $t > 0$, there exists point $u \in X$ such that u is a coincidence point of A, B, S and T.
Proof: Let x_0 be any arbitrary point in X.
Define a sequence $\{x_n\}$ in X by $r_{2n} = Tx_{2n+1} = Ax_{2n}$, $r_{2n+1} = Bx_{2n+1} = Sx_{2n+2}$
Then $N(Ax_{2n}, Bx_{2n+1}, t) = N(r_{2n}, r_{2n+1}, t)$

$$\geq \min\begin{Bmatrix} N(Sx_{2n}, Tx_{2n+1}, t), N(Ax_{2n}, Sx_{2n}, t), N(Bx_{2n+1}, Tx_{2n+1}, t), \\ N(Ax_{2n}, Tx_{2n+1}, t), N(Bx_{2n+1}, Sx_{2n}, t) \end{Bmatrix}$$
$$= \min\begin{Bmatrix} N(r_{2n-1}, r_{2n}, t), N(r_{2n}, r_{2n-1}, t), N(r_{2n+1}, r_{2n}, t), \\ N(r_{2n}, r_{2n}, t), N(r_{2n+1}, r_{2n-1}, t) \end{Bmatrix}$$

Thus, $N(r_{2n}, r_{2n+1}, t) \geq \min\begin{Bmatrix} N(r_{2n-1}, r_{2n}, t), N(r_{2n}, r_{2n-1}, t), N(r_{2n+1}, r_{2n}, t), \\ N(r_{2n}, r_{2n}, t), N(r_{2n+1}, r_{2n-1}, t) \end{Bmatrix}$

$$= \min\left\{\begin{matrix} N(r_{2n-1}, r_{2n}, t), N(r_{2n}, r_{2n-1}, t), N(r_{2n+1}, r_{2n}, t), \\ 1, N(r_{2n+1}, r_{2n-1}, t) \end{matrix}\right\}$$
$$\geq N(r_{2n-1}, r_{2n}, t)$$
$$= N(Ax_{2n-1}, Bx_{2n}, t)$$

That is, $N(Ax_{2n}, Bx_{2n+1}, t) \geq N(Ax_{2n-1}, Bx_{2n}, t)$.
But $\{Ax_n\}$ and $\{Bx_{n+1}\}$ are Cauchy sequences in X.
Therefore, $\lim_{n\to\infty} N(Ax_{2n-1}, Bx_{2n}, t) = 1$.
Hence, $\lim_{n\to\infty} N(Ax_{2n}, Bx_{2n+1}, t) = 1$.
Since the pair (A, T) is compatible mapping of type (P),

$$1 = \lim_{n\to\infty} N(AAx_{2n}, TTx_{2n+1}, t)$$
$$= \lim_{n\to\infty} N(Ar_{2n}, Tr_{2n+1}, t) = N(Au, Tu, t)$$

That is, $Au = Tu$.
Also, the pair (B, S) is compatible mapping of type (P),

$$1 = \lim_{n\to\infty} N(BBx_{2n+1}, SSx_{2n+2}, t)$$
$$= \lim_{n\to\infty} N(Br_{2n+1}, Sr_{2n+1}, t)$$
$$= N(Bu, Su, t)$$

So that, $Bu = Su$.
Hence, $Au = Bu = Tu = Su$.
Therefore, u is a coincidence point of A, B, S and T.

Theorem 3.4. Let A, B, P, Q, S and T be six self-maps of a fuzzy 2-Banach space $(X, N,*)$ with continuous t-norm defined by $t * t \geq t$ for all $t \in [0,1]$. If the pairs (AB, S) and (PQ, T) are sub compatible of type A having the same coincidence point and $AB = BA$, $BS = SB, AS = SA, PQ = QP, TQ = QT, PT = TP$, then for all $, y \in X, k \in (0,1), t > 0$ $N(Sx, Ty, kt) \geq \{N(Sx, PQy, t) * N(Sx, ABx, t) * N(PQy, Ty, t) * N(ABx, PQy, t) * N(ABx, Ty, t)\}$. Then A, B, P, Q, S and T have a unique common fixed point in X.
Proof: Since the pairs (AB,S) and (PQ,T) are sub compatible of type A, then there exist two sequences $\{x_n\}, \{y_n\}$ in X such that $\lim_{n\to\infty} ABx_n = \lim_{n\to\infty} Sx_n = a, a \in X$ and satisfy $\lim_{n\to\infty} N(ABSx_n, SSx_n, t) = 1$ and $\lim_{n\to\infty} N(SABx_n, ABABx_n, t) = 1$.
Thus we have, $\lim_{n\to\infty} N(ABa, Sa, t) = 1$ and $\lim_{n\to\infty} N(Sa, ABa, t) = 1$.
Also, $\lim_{n\to\infty} PQy_n = \lim_{n\to\infty} Ty_n = b, b \in X$ and satisfy $\lim_{n\to\infty} N(PQTy_n, TTy_n, t) = 1$ and $\lim_{n\to\infty} N(TPQy_n, PQPQy_n, t) = 1$.
Thus we have, $\lim_{n\to\infty} N(PQb, Tb, t) = 1$ and $\lim_{n\to\infty} N(Tb, TQb, t) = 1$.
Therefore, $ABa = Sa$ and $PQb = Tb$.
Thus we have 'a' is coincidence point of AB and S and 'b' is coincidence point of PQ and T.
Now we prove $a = b$.
For this, take $x = x_n$ and $y = y_n$.

$$N(Sx_n, Ty_n, kt) \geq \left\{\begin{matrix} N(Sx_n, PQy_n, t) * N(Sx_n, ABx_n, t) * N(PQx_n, Ty_n, t) * \\ N(ABx_n, PQy_n, t) * N(ABx_n, Ty_n, t) \end{matrix}\right\}$$

Take the limit as $n \to \infty$, we get

$$N(a, b, kt) \geq \{N(a, b, t) * N(a, a, t) * N(b, b, t) * N(a, b, t) * N(a, b, t)\}$$

This implies $N(a, b, kt) \geq N(a, b, t)$ for all $t > 0$.
Thus by Lemma (3.1), $a = b$.
Thus AB, S, PQ and T have the same coincidence point.

Now we prove $Aa = Ba = Pa = Qa = Sa = Ta = a$.

Case (i) Take $x = a$ and $y = y_n$.

$$N(Sa, Ty_n, kt) \geq \left\{\begin{matrix} N(Sa, PQy_n, t) * N(Sa, ABa, t) * N(PQy_n, Ty_n, t) * \\ N(ABa, PQy_n, t) * N(ABa, Ty_n, t) \end{matrix}\right\}$$

Take the limit as $n \to \infty$, we get

$$N(Sa, a, kt) \geq \{N(Sa, b, t) * N(Sa, a, t) * N(b, b, t) * N(a, b, t) * N(a, b, t)\}$$

As $a = b$, we get $N(Sa, a, kt) \geq N(Sa, a, t)$

which gives $Sa = a$.

Case (ii) Take $x = x_n$ and $y = a$

$$N(Sx_n, Ta, kt) \geq \left\{\begin{matrix} N(Sx_n, PQa, t) * N(Sx_n, ABx_n, t) * N(PQa, Ta, t) * \\ N(ABx_n, PQa, t) * N(ABx_n, Ta, t) \end{matrix}\right\}$$

Take the limit as $n \to \infty$, we get

$$N(a, Ta, kt) \geq \{N(a, Ta, t) * N(a, a, t) * N(Ta, Ta, t) * N(a, Ta, t) * N(a, Ta, t)\}$$

We get,

$$N(a, Ta, kt) \geq N(a, Ta, t)$$

which gives $Ta = a$.

Next we prove $Aa = Ba = a$

Case (iii) Put $x = Ba$ and $y = y_n$

$$N(SBa, Ty_n, kt) \geq \left\{\begin{matrix} N(SBa, PQy_n, t) * N(SBa, ABBa, t) * N(PQy_n, Ty_n, t) * \\ N(ABBa, PQy_n, t) * N(ABBa, Ty_n, t) \end{matrix}\right\}$$

As A, B and S commute, $ABBa = BABa = BSa = Ba$ and $SBa = BSa = a$.

$$N(Ba, a, kt) \geq \left\{\begin{matrix} N(Ba, a, t) * N(Ba, Ba, t) * N(a, a, t) * \\ N(Ba, a, t) * N(Ba, a, t) \end{matrix}\right\}$$

That is, $N(Ba, a, kt) \geq N(Ba, a, t)$

Therefore, $Ba = a$.

Case (iv) Now put $x = Aa$ and $y = y_n$

$$N(SAa, Ty_n, kt) \geq \left\{\begin{matrix} N(SAa, PQy_n, t) * N(SAa, ABAa, t) * N(PQy_n, Ty_n, t) * \\ N(ABAa, PQy_n, t) * N(ABAa, Ty_n, t) \end{matrix}\right\}$$

As A, B and S commute, $ABAa = ASa = Aa$ and $SAa = ASa = Aa$.

$$N(Aa, a, kt) \geq \left\{\begin{matrix} N(Aa, a, t) * N(Aa, Aa, t) * N(a, a, t) * \\ N(Aa, a, t) * N(Aa, a, t) \end{matrix}\right\}$$

That is, $N(Aa, a, kt) \geq N(Aa, a, t)$

Therefore, $Aa = a$.

Thus $Aa = Ba = Sa = a$.

Next we prove $Pa = Qa = a$.

Case (v) Now put $x = x_n$ and $y = Qa$

$$N(Sx_n, TQa, kt) \geq \left\{\begin{matrix} N(Sx_n, PQQa, t) * N(Sx_n, ABx_n, t) * N(PQQa, TQa, t) * \\ N(ABx_n, PQQa, t) * N(ABx_n, TQa, t) \end{matrix}\right\}$$

As P, Q, S and T commute, $PQQa = QPQa = Qa$ and $TQa = QTa = Qa$.

$$N(a, Qa, kt) \geq \begin{Bmatrix} N(a, Qa, t) * N(a, a, t) * N(Qa, Qa, t) * \\ N(a, Qa, t) * N(a, Qa, t) \end{Bmatrix}$$

That is, $N(a, Qa, kt) \geq N(a, Qa, t)$
Therefore, $Qa = a$.

Case (vi) Take $x = x_n$ and $y = Pa$

$$N(Sx_n, TPa, kt) \geq \begin{Bmatrix} N(Sx_n, PQPa, t) * N(Sx_n, ABPa, t) * N(PQPa, TPa, t) * \\ N(ABx_n, PQPa, t) * N(ABx_n, TPa, t) \end{Bmatrix}$$

As P, Q and T commute, $PQPa = TPa = PTa = Pa$ and T$Pa = PTa = Pa$.

$$N(a, Pa, kt) \geq \begin{Bmatrix} N(a, Pa, t) * N(a, a, t) * N(Pa, Pa, t) * \\ N(a, Pa, t) * N(a, Pa, t) \end{Bmatrix}$$

That is, $N(a, Pa, kt) \geq N(a, Pa, t)$
Therefore, $Pa = a$.
This $implies$ $Aa = Ba = Pa = Qa = Sa = Ta = a$.
Thus, A, B, P, Q, S and T have a unique common fixed point in X.

4. Conclusions

In this paper, we have adapted the concepts of fuzzy 2-Banach space. Many fixed point theorems holds good for 2-Banach space are extended to fuzzy 2-Banach space. As a result this paper paves way to extend the theorems to fuzzy n-Banach spaces using fuzzy Banach space.

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