Intern. J. Fuzzy Mathematical Archive
Vol. 13, No. 2, 2017, 113-121
ISSN: 2320 –3242 (P), 2320 –3250 (online)
Published on 26 October 2017
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/ijfma.v13n2a2

International Journal of
Fuzzy Mathematical Archive

Some Notes on Pentagonal Fuzzy Numbers

Avinash J. Kamble

Department of Applied Sciences and Humanities
Pillai HOC College of Engineering and Technology, Panvel(E)-410207
Maharashtra, India.
Email: avinashmath@yahoo.co.in

Received 2 October 2017; accepted 22 October 2017

Abstract. The aim of this paper is to discuss the basic concept pentagonal fuzzy number (PFN). Canonical pentagonal fuzzy numbers are discussed by means of internal arithmetic operations by using α-cut operations.

Keywords: Fuzzy number, fuzzy arithmetic, pentagonal fuzzy number, canonical pentagonal fuzzy number.

AMS Mathematics Subject Classification (2010): 03E72

1. Introduction

Fuzzy sets were introduced by Zadeh [15] in 1965 to represent the information possessing non-statistical certainties. Different types of fuzzy sets are defined in order to clear the vagueness of the existing problems. A fuzzy number [6] is a quantity whose values are imprecise, rather than exact as in the case with single-valued function. The usual arithmetic operations on real numbers can be extended to the ones defined on fuzzy numbers by means of Zadeh's Extension Principle [15] and then some of the noteworthy contributions on fuzzy numbers and its applications have been made by Dubois and Prade [6], Mizumoto and Tanaka [9] internal arithmetic was first suggested by Dwyer [7]. The various operations on fuzzy numbers were also available in the literature [6, 7, 9, 15].

Due to error in measuring technique, instrumental faultiness etc., some data in our observation cannot be accurately determined. This leads to introduce a new type of fuzzy number called the Pentagonal fuzzy number (PFN). Generally, a pentagonal fuzzy number is a 5-tuple subset of a real number R having five parameters. A pentagonal fuzzy number A is denoted as $A=(a_1,a_2,a_3,a_4,a_5)$, where a_3 is the middle point, (a_1,a_2) and (a_4,a_5) are the left and right side points of a_3 respectively. A fuzzy number $A=(a_1,a_2,a_3,a_4,a_5)$ is called a Canonical Pentagonal fuzzy number, if it is closed and bounded pentagonal fuzzy number and its membership function is strictly increasing on the interval $[a_2,a_3]$ and strictly decreasing on the interval $[a_3,a_4]$. In this paper, we discuss the basics of Pentagonal fuzzy number (PFN) with their arithmetic operations. Canonical Pentagonal fuzzy numbers are discussed by means of internal arithmetic operations by using α-cut operations with relevant examples.

The paper is organized as follows: In section 2, basic concepts of fuzzy number, triangular and trapezoidal fuzzy numbers are discussed. Section 3 deals with the concept of Pentagonal fuzzy number (PFN) with their arithmetic properties. In section 4, canonical pentagonal fuzzy number is discussed with their internal arithmetic operations.

2. Preliminaries

This section recalls some basic definitions of fuzzy numbers, triangular fuzzy number and trapezoidal fuzzy number.

Definition 2.1. A **fuzzy set** A in R (real line)is defined to be a set of ordered pairs, $A = \{x, \mu_A(x) / x \in R\}$ where $\mu_A(x)$ is called the membership function for the fuzzy set.

Definition 2.2. The **α-cut** of α-level set of fuzzy set A is a set consisting of those elements of the universe X whose membership values exceed the threshold level α.
That is $A_\alpha = \{x \in X / \mu_A(x) \geq \alpha\}$.

Definition 2.3. A fuzzy set A is called **normal,** if there is at least one point $x \in R$ with $\mu_A(x) = 1$.

Definition 2.4. A fuzzy set A on R is **convex,** if for any $x, y \in R$ and for any $\lambda \in [0,1]$ we have $\mu_A(\lambda x + (1-\lambda)y) \geq \min\{\mu_A(x), \mu_A(y)\}$.
In general, a concave function is the negative of a convex function.

Definition 2.5. A **fuzzy number** A is a fuzzy set on the real line that satisfies the conditions of normality and convexity.

Definition 2.6. If a fuzzy set is convex and normalized and its membership function is defined in R and piecewise continuous, it is called as fuzzy number.
Fuzzy number represents a real number whose boundary is fuzzy.

Definition 2.7. (Triangular fuzzy number)
A triangular fuzzy number is a fuzzy set $A = (a_1, a_2, a_3)$ having membership function

$$\mu_A(x) = \begin{cases} 0, & for \quad x < a_1, a_3 \le x \\ \dfrac{x-a_1}{a_2-a_1}, & for \quad a_1 \le x < a_2 \\ \dfrac{a_3-x}{a_3-a_2}, & for \quad a_2 \le x < a_3 \end{cases}$$

Definition 2.8. (Trapezoidal fuzzy number)
A trapezoidal fuzzy number is fuzzy subset $A = (a_1, a_2, a_3, a_4)$ having membership function

$$\mu_A(x) = \begin{cases} 0, & for \quad x < a_1, a_4 \le x \\ \dfrac{x-a_1}{a_2-a_1}, & for \quad a_1 \le x \le a_2 \\ 1, & for \quad a_2 \le x \le a_3 \\ \dfrac{a_4-x}{a_4-a_3}, & for \quad a_3 \le x \le a_4 \end{cases}$$

where $a_1 \le a_2 \le a_3 \le a_4$

3. Pentagonal fuzzy number (PFN)

Definition 3.1. A fuzzy number $A = (a_1, a_2, a_3, a_4, a_5)$ is called a pentagonal fuzzy number, if its membership function is given by

$$\mu_A(x) = \begin{cases} 0, & for \quad x < a_1, a_5 \le x \\ \dfrac{x-a_1}{a_2-a_1}; & for \quad a_1 \le x \le a_2 \\ \dfrac{x-a_2}{a_3-a_2}; & for \quad a_2 \le x \le a_3 \\ 1, & for \quad x = a_3 \\ \dfrac{a_4-x}{a_4-a_3}; & for \quad a_3 \le x \le a_4 \\ \dfrac{a_5-x}{a_5-a_4}; & for \quad a_5 \le x \le a_4 \end{cases}$$

Here for PFN $A = (a_1, a_2, a_3, a_4, a_5), a_3$ is the middle point and (a_1, a_2) and (a_4, a_5) are the left and right side points of a_3 respectively. The middle point a_3 has the grade of membership 1 and w_1, w_2 are the grades of points a_2, a_4.
Note that, every PFN is associated with two weights w_1 and w_2.

We now look at the pentagonal fuzzy number (PFN) in a generalized way, so that, two special fuzzy numbers, namely triangular fuzzy number and trapezoidal fuzzy number can be visualized as follows.

Definition 3.2. A fuzzy number $A=(a_1,a_2,a_3,a_4,a_5)$ is a pentagonal fuzzy number (PFN) having membership function

$$\mu_{(x;w_1,w_2)}=\begin{cases} w_1\left(\dfrac{x-a_1}{a_2-a_1}\right); & for \quad a_1\le x\le a_2 \\ 1-(1-w_1)\left(\dfrac{x-a_2}{a_3-a_2}\right); & for \quad a_2\le x\le a_3 \\ 1; & for \quad x=a_3 \\ 1-(1-w_2)\left(\dfrac{x-a_3}{a_4-a_3}\right); & for \quad a_3\le x\le a_4 \\ w_2\left(\dfrac{x-a_5}{a_4-a_5}\right); & for \quad a_4\le x\le a_5 \\ 0; & for \quad x>a_5 \end{cases}$$

Case (i): When $w_1=w_2=0$, then the pentagonal fuzzy number is reduced to a triangular fuzzy number. That is, $A=(a_1,a_2,a_3,a_4,a_5)\approx(a_2,a_3,a_4)$ where membership function is given by

$$\mu_A(x)=\begin{cases} 0; & for \quad x\le a_2 \\ 1-\left(\dfrac{x-a_2}{a_3-a_2}\right); & for \quad a_2\le x\le a_3 \\ 1, & for \quad x=a_3 \\ 1-\left(\dfrac{x-a_3}{a_4-a_3}\right); & for \quad a_3< x\le a_4 \\ 0; & for \quad x\ge a_4 \end{cases}$$

Case (ii): When $w_1=w_2=1$, then the pentagonal fuzzy number becomes a trapezoidal fuzzy number. That is, the membership function is given by

$$\mu_A(x)=\begin{cases} 0, & for \quad x \le a_1 \\ \left(\dfrac{x-a_1}{a_2-a_1}\right); & for \quad a_1 \le x \le a_2 \\ 1, & for \quad a_2 \le x \le a_5 \\ \left(\dfrac{a_4-x}{a_5-a_4}\right); & for \quad a_4 \le x \le a_5 \\ 0, & for \quad x > a_5 \end{cases}$$

3.3. Arithmetic operation on pentagonal fuzzy number (PFN)

3.3.1. Addition of two PFNs

Let $A=(a_1,a_2,a_3,a_4,a_5)$ and $B=(b_1,b_2,b_3,b_4,b_5)$ be two PFNs, then addition of two PFNs is given by,

$A+B=(a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4,a_5+b_5)$.

3.3.2. Subtraction of two PFNs,

Let $A=(a_1,a_2,a_3,a_4,a_5)$ and $B=(b_1,b_2,b_3,b_4,b_5)$ be two PFNs, then subtraction of two PFNs is given by,

$A-B=(a_1-b_5,a_2-b_4,a_3-b_3,a_4-b_2,a_5-b_1)$

3.3.3. Multiplication of two PFNs

The multiplication of two PFNs $A=(a_1,a_2,a_3,a_4,a_5)$ and $B=(b_1,b_2,b_3,b_4,b_5)$ is given by,

$AB=(a_1b_1,a_2b_2,a_3b_3,a_4b_4,a_5b_5)$.

3.3.4. Inverse of a PFN

The inverse of a PFN is defined when all its components are non-zero.

Let $A=(a_1,a_2,a_3,a_4,a_5)$ be a PFN, then $A^{-1} \approx \dfrac{1}{A} \approx \left(\dfrac{1}{a_5},\dfrac{1}{a_4},\dfrac{1}{a_3},\dfrac{1}{a_2},\dfrac{1}{a_1}\right)$.

If one of the components of a PFN becomes zero, then we cannot find its inverse.

3.3.5. Division of two PFNs

The division of two PFNs $A=(a_1,a_2,a_3,a_4,a_5)$ and $B=(b_1,b_2,b_3,b_4,b_5)$ is approximated as the multiplication with inverse, given by $\dfrac{A}{B} \approx AB^{-1} \approx \left(\dfrac{a_1}{b_5},\dfrac{a_2}{b_4},\dfrac{a_3}{b_3},\dfrac{a_4}{b_2},\dfrac{a_5}{b_1}\right)$.

Definition 3.4. (Construction of PFN)

The pentagonal fuzzy number is represented by the five parameters a_1, a_2, a_3, a_4 and a_5 where a_1 and a_2 denote the smallest possible values, a_3 the most promising value and a_4, a_5 the largest possible value.

The pentagonal fuzzy number can be generated by using the following formula.

$A = (a-2, a-1, a, a+1, a+2)$ for all $a = 3,4,5,6,7$. Since the fuzzy number scale is defined between 1 to 9.

Definition 3.5. (Positive PFN)

A PFN $A = (a_1, a_2, a_3, a_4, a_5)$ is said to be positive, if all its entries are positive.

Similarly, $A = (a_1, a_2, a_3, a_4, a_5)$ is negative, if all of its entries are negative.

Definition 3.6. (Null PFN)

A PFN A is called a null PFN, if all of its entries are zero. That is, $A = (0,0,0,0,0)$.

Definition 3.7. (Null equivalent PFN)

A PFN $A = (a_1, a_2, a_3, a_4, a_5)$ is said to be null equivalent, if its middle entry is at the point 0, that is, of the form $(\alpha_1, \beta_1, 0, \alpha_2, \beta_2)$ where $\alpha_1, \beta_1 \neq 0$ $\alpha_2, \beta_2 \neq 0$.

Definition 3.8. (Unit equivalent PFN)

A PFN A is said to be a unit equivalent PFN, when its middle entry is at 1. That is, of the form $(\alpha_1, \beta_1, 1, \alpha_2, \beta_2)$ where $\alpha_1, \beta_1 \neq 0$ $\alpha_2, \beta_2 \neq 0$.

Remark 1. The subtraction of two PFNs with a common middle entry gives a null equivalent PFN, while their division yields another unit equivalent PFN.

4. Canonical pentagonal fuzzy number

A fuzzy number $A = (a_1, a_2, a_3, a_4, a_5)$ is called a canonical Pentagonal fuzzy number, if it is closed and bounded pentagonal fuzzy number and its membership function is strictly increasing on the interval $[a_2, a_3]$ and strictly decreasing on the interval $[a_3, a_4]$.

Definition 4.1. The classical alpha-cut set is the set of elements whose degree of membership in $A = (a_1, a_2, a_3, a_4, a_5)$ is not less than α and is defined as $A_\alpha = \{x \in X / \mu_A(x) \geq \alpha\}$.

4.2. α **-cut operations**

The α -cut operations on PFNs is given by,

$A_\alpha = [2\alpha(a_2 - a_1) + a_1, -2\alpha(a_5 - a_4) + a_5]$ for $\alpha \in [0,0.5]$

$A_\alpha = [2\alpha(a_3 - a_2) + 2a_2 - a_3, 2\alpha(a_4 - a_3) - a_4 + 2a_3]$ for $\alpha \in [0.5,1]$.

4.3. Arithmetic operations on canonical pentagonal fuzzy number

4.3.1. Addition of two canonical PFNs

Let $A = (a_1, a_2, a_3, a_4, a_5)$ and $B = (b_1, b_2, b_3, b_4, b_5)$ be two canonical PFNs for all $\alpha \in [0,1]$.

Let A_α and B_α be the α -cuts of A and B then

$A_\alpha + B_\alpha = [2\alpha(a_2 - a_1) + a_1, -2\alpha(a_5 - a_4) + a_5] + [2\alpha(b_2 - b_1) + b_1, -2\alpha(b_5 - b_4) + b_5]$
$+ [2\alpha(b_3 - b_2) + 2b_2 - b_3, 2\alpha(b_4 - b_3) - b_4 + 2b_3]$

for $\alpha \in [0,0.5]$

$A_\alpha + B_\alpha = [2\alpha(a_3 - a_2) + 2a_2 - a_3, 2\alpha(a_4 - a_3) - a_4 + 2a_3]$ for $\alpha \in [0.5,1]$.

For example, Let $A = (1,2,3,4,5)$ and $B = (6,7,8,9,10)$ be two canonical PFNs, then we have

$$A_\alpha + B_\alpha = \begin{cases} 4\alpha + 7, -4\alpha + 15; & \alpha \in [0,0.5] \\ 4\alpha + 7, 4\alpha + 9; & \alpha \in [0.5,1] \end{cases}$$

When $\alpha = 0$, $A_0 + B_0 = [7,15]$

When $\alpha = 0.5$, $A_{0.5} + B_{0.5} = [9,13]$

When $\alpha = 0.5$, $A_{0.5} + B_{0.5} = [9,11]$

When $\alpha = 1$, $A_1 + B_1 = [11,13]$

Therefore, $A_\alpha + B_\alpha = [7,9,11,13,15]$. All the points of $(A_\alpha + B_\alpha)$ coincides with the sum of the two canonical PFNs.

Therefore, the addition of two α -cuts lies within the interval.

4.3.2. Subtraction of two canonical PFNs

Let $A = (a_1, a_2, a_3, a_4, a_5)$ and $B = (b_1, b_2, b_3, b_4, b_5)$ be two canonical PFNs for all $\alpha \in [0,1]$.

$A_\alpha - B_\alpha = [2\alpha(a_2 - a_1) + a_1, -2\alpha(a_5 - a_4) + a_5] - [2\alpha(b_2 - b_1) + b_1, -2\alpha(b_5 - b_4) + b_5]$

for $\alpha \in [0,0.5]$.

$$\begin{aligned} A_\alpha - B_\alpha = &[2\alpha(a_3 - a_2) + 2a_2 - a_3, 2\alpha(a_4 - a_3) - a_4 + 2a_3] \\ &- [2\alpha(b_3 - b_2) + 2b_2 - b_3, 2\alpha(b_4 - b_3) - b_4 + 2b_3] \end{aligned}$$

for $\alpha \in [0.5,1]$.

For $A = (1,2,3,5,6)$ and $B = (2,4,6,10,12)$

$$A_\alpha - B_\alpha = \begin{cases} -2\alpha - 1, 2\alpha - 6; & \alpha \in [0,0.5] \\ -2\alpha - 1, -4\alpha - 1; & \alpha \in [0.5,1] \end{cases}$$

When $\alpha = 0$, $A_0 - B_0 = [-1,-6]$

When $\alpha = 0.5$, $A_{0.5} - B_{0.5} = [-2,-5]$

When $\alpha = 0.5$, $A_{0.5} - B_{0.5} = [-2,-3]$

When $\alpha = 1$, $A_1 - B_1 = [-3,-5]$.

Hence, $A_\alpha - B_\alpha = [-1,-2,-3,-5,-6]$.

All the points coincide with the differences of two canonical PFNs.

4.3.3. Multiplication of two canonical PFNs

Let $A = (a_1, a_2, a_3, a_4, a_5)$ and $B = (b_1, b_2, b_3, b_4, b_5)$ be two canonical PFNs for all $\alpha \in [0,1]$.

$$A_\alpha * B_\alpha = [2\alpha(a_2 - a_1) + a_1, -2\alpha(a_5 - a_4) + a_5] * [2\alpha(b_2 - b_1) + b_1, -2\alpha(b_5 - b_4) + b_5]$$

for $\alpha \in [0,0.5]$

$$\begin{aligned} A_\alpha * B_\alpha &= [2\alpha(a_3 - a_2) + 2a_2 - a_3, 2\alpha(a_4 - a_3) - a_4 + 2a_3] \\ &\quad * [2\alpha(b_3 - b_2) + 2b_2 - b_3, 2\alpha(b_4 - b_3) - b_4 + 2b_3] \end{aligned}$$

for $\alpha \in [0.5,1]$.

For $A = (1,2,3,5,6)$ and $B = (2,4,6,10,12)$, we have

$$A_\alpha * B_\alpha = [2\alpha + 1, -2\alpha + 6] * [4\alpha + 2, -4\alpha + 12] \text{ for } \alpha \in [0,0.5]$$

$$A_\alpha * B_\alpha = [2\alpha + 1, 4\alpha + 1] * [4\alpha + 2, 8\alpha + 2] \text{ for } \alpha \in [0.5,1].$$

When $\alpha = 0$, $A_0 * B_0 = [2,72]$

When $\alpha = 0.5$, $A_{0.5} * B_{0.5} = [8,50]$

When $\alpha = 0.5$, $A_{0.5} * B_{0.5} = [8,18]$

When $\alpha = 1$, $A_1 * B_1 = [18,50]$.

Hence, $A_\alpha * B_\alpha = [2,8,18,50,72]$ all the points coincide with the product of two canonical PFNs.

5. Conclusion

In this paper, the concept of pentagonal fuzzy number has been discussed with internal arithmetic properties. Special attention is paid to the canonical pentagonal fuzzy numbers along with their arithmetic operations.

REFERENCES

1. A.Panda and M. Pal, A study on pentagonal fuzzy number and its corresponding matrices, *Pacific Science Review B : Humanities and Social Sciences,* 1 (2015) 131-139.
2. B.Abhinav, Trapezoidal fuzzy numbers (a,b,c,d), arithmetic behavior, *International Journal of Physics and Mathematical Sciences,* Vol.- 02 (2011) 39-44.
3. B. J. Lee and Y. S. Yun, The Pentagonal Fuzzy Numbers, *Journal of Chungcheong Mathematical Society*, 27(2) (2014) 277 - 286.
4. M.Bhowmik and M.Pal, Some results on generalized interval-valued intuitionistic fuzzy sets, *Int. J. Fuzzy Syst.*, 14 (2) (2012) 193-203.
5. C.Parvathi and C.Malathi, Arithmetic operations on symmetric trapezoidal intuitionisticfuzzy numbers, *International Journal of Soft Computing and Engineering*, 2 (2012) 268-273.
6. D.Dubois and H.Prade, Operation on fuzzy numbers, *International Journal of Systems Science*, 9(6) (1978) 613-626.
7. P.S.Dwyer, *Linear Computation*, NewYork, (1951).
8. P.Dutta and A.Ali Tazid, Fuzzy arithmetic with and without α-cut method, A comparative study, *Journal of Latest Trends in Computing,* 3 (2011) 528-534.
9. M.Mizumoto and K.Tanaka, The four operations of arithmetic on fuzzy numbers, *Systems Computer Controls*., 7(5) (1977) 73-80.
10. G.S.Mahapatra and T.K.Roy, IFN and its arithmetic operations with application on system failure, *Journal of Uncertain Systems,* 7 (2013) 92-107.
11. K.Ponnivalavan and T.Pathinathan, Intuitionistic pentagonal fuzzy number, *ARPN Journal of Engineering and Applied Sciences*, 10(12) (2015), 5446 -5450.
12. K.Ponnivalavan and T.Pathinathan, Pentagonal fuzzy numbers, *International Journal of Computing Algorithm,* 3 (2014) 1003-1005.
13. R.Helen and G.Uma, A new operation and ranking on pentagon fuzzy numbers, Int. *Jr. of Mathematical Sciences and Applications,* 5(2) (2015) 341 – 346..
14. A.Vigin Raj and S.Karthik, Application of pentagonal fuzzy number in neural network, *International Journal of Mathematics and its Applications*, 4(4) (2016) 149-154.
15. L.A.Zadeh, Fuzzy set, *Information and Control*, 8 (1965) 338-353.