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Level Separation on Intuitionistic Fuzzy T_0 Spaces

***Md. Saiful Islam*[1], *Md. Sahadat Hossain*[2] and *Md. Asaduzzaman*[2]**

[1]Department of Computer Science and Engineering
Jatiya Kabi Kazi Nazrul Islam University, Trishal, Mymensingh
Bangladesh. E-mail: saifulmath@yahoo.com

[2]Department of Mathematics, University of Rajshahi, Rajshahi, Bangladesh.
E-mail: sahadat@ru.ac.bd, asad_math@ru.ac.bd

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Abstract. The purpose of this paper is to introduce and study the intuitionistic fuzzy T_0 spaces. We give eight new notions of intuitionistic fuzzy T_0 spaces and investigate some relations among them. Also we investigate some relations between our notions and other given notions of intuitionistic fuzzy T_0 spaces. Further, we study hereditary and productive property of such spaces. Moreover, under some conditions it is shown that image and preimage preserve intuitionistic fuzzy T_0 spaces.

Keywords: Fuzzy set, intuitionistic fuzzy set, intuitionistic topological space, intuitionistic fuzzy topological space, intuitionistic fuzzy T_0 space.

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1. Introduction

At first fuzzy sets were introduced by Zadeh [1] in 1965 as follows: a fuzzy set λ in a nonempty set X is a mapping from X to the closed unit interval $[0,1]$, and $\lambda(x)$ is interpreted as the degree of membership of x in λ where $x \in X$. Atanassov [2] generalized this concept and introduced intuitionistic fuzzy sets which take into account both the degrees of membership and nonmembership subject to the condition that their sum does not exceed 1. Coker et al.[3,4,5,6], Srivastava et al. [7,8], Lee et al. [9,10], Ahmed et al. [11,12,13,14,15] subsequently initiated a study of intuitionistic fuzzy topological spaces by using intuitionistic fuzzy sets. In this paper, we investigate the properties and features of intuitionistic fuzzy T_0 Spaces.

2. Notations and preliminaries

Through this paper, X will be a nonempty set, r and s are constants in $(0,1)$, T is a topology, t is a fuzzy topology, $\mathcal{T}$ is an intuitionistic topology and τ is an intuitionistic fuzzy topology. λ and μ are fuzzy sets, $A = (\mu_A, \nu_A)$ is intuitionistic fuzzy set. By $\underline{0}$ and $\underline{1}$ we denote constant fuzzy sets taking values 0 and 1 respectively.

Definition 2.1. [16] Let X be a non empty set. A family t of fuzzy sets in X is called a fuzzy topology on X if the following conditions hold.

(1) $\underline{0}, \underline{1} \in t$,

(2) $\lambda \cap \mu \in t$ for all $\lambda, \mu \in t$,
(3) $\cup \lambda_j \in t$ for any arbitrary family $\{\lambda_j \in t\ ,\ j \in J\}$.

Definition 2.2. [3] Suppose X is a non empty set. An intuitionistic set A on X is an object having the form $A = (X, A_1, A_2)$ where $A_1$and A_2 are subsets of X satisfying $A_1 \cap A_2 = \phi$. The set A_1 is called the set of member of A while A_2 is called the set of non-member of A. In this paper, we use the simpler notation $A = (A_1, A_2)$ instead of $A = (X, A_1, A_2)$ for an intuitionistic set.

Remark 2.1. Every subset A of a nonempty set X may obviously be regarded as an intuitionistic set having the form $A = (A, A^c)$ where $A^c = X \setminus A$.

Definition 2.3. [3] Let the intuitionistic sets A and B in X be of the forms $A = (A_1, A_2)$ and $B = (B_1, B_2)$ respectively. Furthermore, let $\{A_j, j \in J\}$ be an arbitrary family of intuitionistic sets in X, where $A_j = \left(A_j^{(1)}, A_j^{(2)}\right)$. Then

(a) $A \subseteq B$ if and only if $A_1 \subseteq B_1$ and $A_2 \supseteq B_2$,
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
(c) $\bar{A} = (A_2, A_1)$, denotes the complement of A,
(d) $\cap A_j = (\cap A_j^{(1)}, \cup A_j^{(2)})$,
(e) $\cup A_j = (\cup A_j^{(1)}, \cap A_j^{(2)})$,
(f) $\phi_\sim = (\phi, X)$ and $X_\sim = (X, \phi)$.

Definition 2.4. [5] Let X be a non empty set. A family $\mathcal{T}$ of intuitionistic sets in X is called an intuitionistic topology on X if the following conditions hold.

(1) $\phi_\sim , X_\sim \in \mathcal{T}$,
(2) $A \cap B \in \mathcal{T}$ for all $A, B \in \mathcal{T}$,
(3) $\cup A_j \in \mathcal{T}$ for any arbitrary family $\{A_j \in \mathcal{T}, j \in J\}$.

The pair $(X, \mathcal{T})$ is called an intuitionistic topological space (ITS, in short), members of $\mathcal{T}$ are called intuitionistic open sets (IOS, in short) in X and their complements are called intuitionistic closed sets (ICS, in short) in X.

Definition 2.5. [2] Let X be a non empty set. An intuitionistic fuzzy set A (IFS, in short) in X is an object having the form $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$, where μ_A and ν_A are fuzzy sets in X denote the degree of membership and the degree of non-membership respectively subject to the condition $\mu_A(x) + \nu_A(x) \leq 1$.
Throughout this paper, we use the simpler notation $A = (\mu_A, \nu_A)$ instead of $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$ for IFSs.

Definition 2.6. [2] Let X be a nonempty set and IFSs A, B in X be given by $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ respectively, then

(a) $A \subseteq B$ if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,
(b) $A = B$ if $A \subseteq B$ and $B \subseteq A$,
(c) $\bar{A} = (\nu_A, \mu_A)$,

(d) $A \cap B = (\mu_A \cap \mu_B,\ \nu_A \cup \nu_B)$,
(e) $A \cup B = (\mu_A \cup \mu_B,\ \nu_A \cap \nu_B)$.

Definition 2.7. [4] Let $\{A_j = \left(\mu_{A_j}, \nu_{A_j}\right),\ j \in J\}$ be an arbitrary family of IFSs in X. Then
(a) $\cap A_j = (\cap\, \mu_{A_j},\ \cup\, \nu_{A_j})$,
(b) $\cup A_j = (\cup\, \mu_{A_j},\ \cap\, \nu_{A_j})$,
(c) $0_\sim = (\underline{0}, \underline{1})$, $1_\sim = (\underline{1}, \underline{0})$.

Definition 2.8. [4] An intuitionistic fuzzy topology (IFT, in short) on a nonempty set X is a family τ of IFSs in X satisfying the following axioms:
(1) $0_\sim, 1_\sim \in \tau$,
(2) $A \cap B \in \tau$, for all $A, B \in \tau$,
(3) $\cup A_j \in \tau$ for any arbitrary family $\{A_j \in \tau,\ j \in J\}$.
The pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS, in short), members of τ are called intuitionistic fuzzy open sets (IFOS, in short) in X, and their complements are called intuitionistic fuzzy closed sets (IFCS, in short) in X.

Remark 2.2. [17] Let X be a non empty set and $A \subseteq X$, then the set A may be regarded as a fuzzy set in X by its characteristic function $1_A: X \to \{0,1\}$ which is defined by

$$1_A(x) = \begin{cases} 1 & if\ x \in A \\ 0 & if\ x \notin A, i.e., if\ x \in A^c \end{cases}$$

Again we know that a fuzzy set λ in X may be regarded as an intuitionistic fuzzy set by $(\lambda, 1-\lambda) = (\lambda, \lambda^c)$. So every sub set A of X may be regarded as intuitionistic fuzzy set by $(1_A, 1 - 1_A) = (1_A, 1_{A^c})$.

Theorem 2.1. Let (X, T) be a topological space. Then (X, τ) is an intuitionistic fuzzy topological space where $\tau = \left\{(1_{A_j}, 1_{A_j}^c), j \in J : A_j \in T\right\}$.

Note 2.1. Above τ is the corresponding IFT of T.

Theorem 2.2. Let (X, t) is a fuzzy topological space. Then (X, τ) is an intuitionistic fuzzy topological space where $\tau = \{(\lambda_j,\ \lambda_j^c), j \in J: \lambda_j \in t\}$.

Note 2.2. Above τ is the corresponding IFT of t.

Theorem 2.3. Let $(X, \mathcal{T})$ be an intuitionistic topological space. Then (X, τ) is an intuitionistic fuzzy topological space where $\tau = \left\{\left(1_{A_{j1}}, 1_{A_{j2}}\right), j \in J: A_j = \left(A_{j1}, A_{j2}\right) \in \mathcal{T}\right\}$.

Note 2.3. [11] Above τ is the corresponding IFT of $\mathcal{T}$.

Definition 2.9. [11] An intuitionistic topological space $(X, \mathcal{T})$ is called T_0 if for all $x, y \in X$ with $x \neq y$, there exists an intuitionistic set $A = (A_1, A_2) \in \mathcal{T}$ such that $x \in A_1, y \in A_2$ or $y \in A_1, x \in A_2$.

Definition 2.10. [2] Let X and Y be two nonempty sets and $f: X \to Y$ be a function. If $A = \{(x, \mu_A(x),\ \nu_A(x)): x \in X\}$ and $B = \{(y, \mu_B(y),\ \nu_B(y)): y \in Y\}$ are IFSs in X and Y respectively, then the pre image of B under f, denoted by $f^{-1}(B)$ is the IFS in X defined by

$f^{-1}(B) = \{(x,\ (f^{-1}(\mu_B))(x), (f^{-1}(\nu_B))(x)): x \in X\}$=$\{(x,\ \mu_B(f(x)), \nu_B(f(x))): x \in X\}$ and the image of A under f, denoted by $f(A)$ is the IFS in Y defined by $f(A)$= $\{(y, (f(\mu_A))(y), (f(\nu_A))(y)): y \in Y\}$, where for each $y \in Y$

$$(f(\mu_A))(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu_A(x) & if\ f^{-1}(y) \neq \phi \\ 0 & otherwise \end{cases}$$

$$(f(\nu_A))(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} \nu_A(x) & if\ f^{-1}(y) \neq \phi \\ 1 & otherwise \end{cases}$$

Definition 2.11. [18] Let $A = (x, \mu_A, \nu_A)$ and $B = (y, \mu_B, \nu_B)$ be IFSs in X and Y respectively. Then the product of IFSs A and B denoted by $A \times B$ is defined by $A \times B = \{(x, y), \mu_A \times \mu_B, \nu_A \times \nu_B)\}$ where $(\mu_A \times \mu_B)(x, y) = \min(\mu_A(x), \mu_B(y))$ and $(\nu_A \times \nu_B)(x, y) = \max(\nu_A(x), \nu_B(y))$ for all $(x, y) \in X \times Y$.
Obviously $0 \leq (\mu_A \times \mu_B) + (\nu_A \times \nu_B) \leq 1$. This definition can be extended to an arbitrary family of IFSs.

Definition 2.12. [18] Let (X_j, τ_j), $j = 1,2$ be two IFTSs. The product topology $\tau_1 \times \tau_2$ on $X_1 \times X_2$ is the IFT generated by $\{\rho_j^{-1}(U_j): U_j \in \tau_j, j = 1,2\}$, where $\rho_j: X_1 \times X_2 \to X_j$, $j = 1,2$ are the projection maps and IFTS $\{X_1 \times X_2, \tau_1 \times \tau_2\}$ is called the product IFTS of (X_j, τ_j), $j = 1,2$. In this case $\mathcal{S} = \{\rho_j^{-1}(U_j), j \in J: U_j \in \tau_j\}$ is a sub base and $\mathcal{B} = \{U_1 \times U_2: U_j \in \tau_j, j = 1,2\}$ is a base for $\tau_1 \times \tau_2$ on $X_1 \times X_2$.

Definition 2.13. [4] Let (X, τ) and (Y, δ) be IFTSs. A function $f: X \to Y$ is called continuous if $f^{-1}(B) \in \tau$ for all $B \in \delta$ and f is called open if $f(A) \in \delta$ for all $A \in \tau$.

Definition 2.14. [19] A topological space (X, T) is called T_0 if for all $x, y \in X$ with $x \neq y$, there exists $U \in T$ such that $x \in U, y \notin U$ or $y \in U, x \notin U$.

Definition 2.15. [7] A fuzzy topological space (X, t) is called T_0 if for all $x, y \in X$ with $x \neq y$, there exists $U \in t$ such that $U(x) = 1$, $U(y) = 0$ or $U(y) = 1$, $U(x) = O$ i.e., $x \in U, y \notin U$ or $y \in U, x \notin U$.

Definition 2.16. [8] Let $A = (\mu_A, \nu_A)$ be a IFS in X and U be a non empty subset of X. The restriction of A to U is a IFS in U, denoted by $A|U$ and defined by $A|U = (\mu_A|U, \nu_A|U)$.

Definition 2.17. Let (X, τ) be an intuitionistic fuzzy topological space and U is a non empty sub set of X then $\tau_U = \{A|U: A \in \tau\}$ is an intuitionistic fuzzy topology on U and (U, τ_U) is called sub space of (X, τ).

3. Intuitionistic fuzzy T_0 space

Definition 3.1. Let $r \in (0,1)$. An intuitionistic fuzzy topological space (X, τ) is called.

(1) IF-T_0(r-i) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A) \in \tau$ such that $\mu_A(x) > r, \nu_A(x) < r$; $\mu_A(y) < r, \nu_A(y) > r$ or $\mu_A(y) > r, \nu_A(y) < r$; $\mu_A(x) < r$, $\nu_A(x) > r$.

(2) IF-T_0(r-ii) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A) \in \tau$ such that $\mu_A(x) > r, \nu_A(x) < r$; $\mu_A(y) < r, \nu_A(y) > 0$ or $\mu_A(y) > r, \nu_A(y) < r$; $\mu_A(x) < r, \nu_A(x) > 0$.

(3) IF-T_0(r-iii) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A) \in \tau$ such that $\mu_A(x) > 0, \nu_A(x) < r$; $\mu_A(y) < r, \nu_A(y) > r$ or $\mu_A(y) > 0, \nu_A(y) < r$; $\mu_A(x) < r, \nu_A(x) > r$.

(4) IF-T_0(r-iv) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A) \in \tau$ such that $\mu_A(x) > 0, \nu_A(x) < r$; $\mu_A(y) < r, \nu_A(y) > 0$ or $\mu_A(y) > 0, \nu_A(y) < r$; $\mu_A(x) < r, \nu_A(x) > 0$.

(5) IF-T_0(r-v) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A) \in \tau$ such that $\mu_A(x) > r, \nu_A(x) < 1$; $\mu_A(y) < r, \nu_A(y) > r$ or $\mu_A(y) > r, \nu_A(y) < 1$; $\mu_A(x) < r, \nu_A(x) > r$.

(6) IF-T_0(r-vi) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A) \in \tau$ such that $\mu_A(x) > r, \nu_A(x) < r$; $\mu_A(y) < 1, \nu_A(y) > r$ or $\mu_A(y) > r, \nu_A(y) < r$; $\mu_A(x) < 1, \nu_A(x) > r$.

(7) IF-T_0(r-vii) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A) \in \tau$ such that $\mu_A(x) > r, \nu_A(x) < 1$; $\mu_A(y) < 1, \nu_A(y) > r$ or $\mu_A(y) > r, \nu_A(y) < 1$; $\mu_A(x) < 1, \nu_A(x) > r$.

(8) IF-T_0(viii) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A) \in \tau$ such that $\mu_A(x) > 0, \nu_A(x) < 1$; $\mu_A(y) < 1, \nu_A(y) > 0$ or $\mu_A(y) > 0, \nu_A(y) < 1$; $\mu_A(x) < 1, \nu_A(x) > 0$.

Theorem 3.1. Let (X, T) be a topological space and (X, τ) be its corresponding IFTS, where $\tau = \left\{(1_{A_j}, 1_{A_j^c}), j \in J : A_j \in T\right\}$. Then (X, T) is T_0 if and only if (X, τ) is IF-T_0(r-k), for any $k \in \{i, ii, iii, iv, v, vi, vii\}$ and (X, T) is T_0 if and only if (X, τ) is IF-T_0(viii).

Proof: The proofs of all implications are similar. For an example we shall prove this for $k=i$.

Suppose (X, T) is T_0. Let $x, y \in X$ with $x \neq y$. Since (X, T) is T_0, there exists $A \in T$ such that $x \in A, y \notin A$ or $y \in A, x \notin A$. We consider $x \in A, y \notin A$. Now $1_A(x) = 1$ and $1_A(y) = 0$.

By the definition of τ, we get $(1_A, 1_{A^c}) \in \tau$ as $A \in T$.

Now clearly $1_{A^c}(x) = 0$ and $1_{A^c}(y) = 1$.

That is, $1_A(x) > r$, $1_{A^c}(x) < r$; $1_A(y) < r$, $1_{A^c}(y) > r$. So (X, τ) is IF-T_0(r-i).

Conversely suppose (X, τ) is IF-T_0(r-i).

Let $x, y \in X$ with $x \neq y$. Since (X, τ) is IF-T_0(r-i), there exists $(1_A, 1_{A^c}) \in \tau$ such that $1_A(x) > r, 1_{A^c}(x) < r$; $1_A(y) < r$, $1_{A^c}(y) > r$ or $1_A(y) > r, 1_{A^c}(y) < r$; $1_A(x) < r, 1_{A^c}(x) > r$. Consider $1_A(x) > r, 1_{A^c}(x) < r$; $1_A(y) < r$, $1_{A^c}(y) > r$.

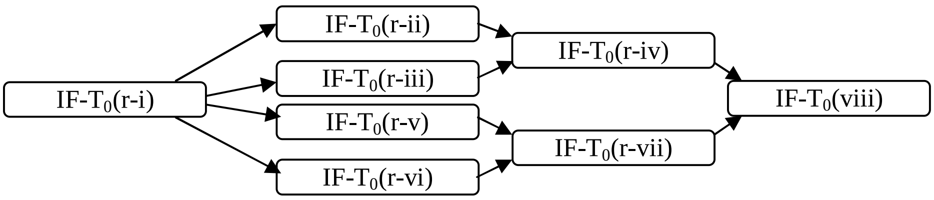
Since $r \in (0,1)$, we can write $1_A(x) = 1, 1_{A^c}(x) = 0$; $1_A(y) = 0, 1_{A^c}(y) = 1$

This implies $x \in A, y \notin A$. Clearly $A \in T$ as $(1_A, 1_{A^c}) \in \tau$. Therefore (X, T) is T_0.

Theorem 3.2. Let $(X, \mathcal{T})$ be an intuitionistic topological space and (X, τ) be its corresponding IFTS, where $\tau = \left\{\left(1_{A_{j1}}, 1_{A_{j2}}\right), j \in J : A_j = \left(A_{j1}, A_{j2}\right) \in \mathcal{T}\right\}$. Then $(X, \mathcal{T})$ is

T_0 if and only if (X,τ) is IF-T_0(r-k), for any $k \in \{i, ii, iii, iv, v, vi, vii\}$ and $(X,\mathcal{T})$ is T_0 if and only if (X,τ) is IF-T_0(viii).
Proof: The proof is obvious as theorem 3.1.

Theorem 3.3. Let (X,t) be a fuzzy topological space and (X,τ) be its corresponding IFTS where $\tau = \{(\lambda, \lambda^c), j \in J : \lambda \in t\}$. If (X,t) is T_0 then (X,τ) is IF-T_0(r-k) for any $k \in \{i, ii, iii, iv, v, vi, vii\}$ and if (X,t) is T_0 then (X,τ) is IF-T_0(viii).
Proof: The proof is obvious as theorem 3.1.



Theorem 3.4. Let (X,τ) be a IFTS. Then we have the following implications.
Proof: Suppose (X,τ) is IF-T_0(r-i). Let $x, y \in X$ with $x \neq y$. Since (X,τ) is IF-T_0(r-i), there exists $A = (\mu_A, \nu_A) \in \tau$ such that $\mu_A(x) > r, \nu_A(x) < r$; $\mu_A(y) < r, \nu_A(y) > r$ or $\mu_A(y) > r, \nu_A(y) < r$; $\mu_A(x) < r, \nu_A(x) > r$.
Consider $\mu_A(x) > r, \nu_A(x) < r$; $\mu_A(y) < r, \nu_A(y) > r$ ……………………..………...(1)
Now, from (1), we can write $\mu_A(x) > r, \nu_A(x) < r$; $\mu_A(y) < r, \nu_A(y) > 0$ ………..……...(2)
Again from (2), we can write $\mu_A(x) > 0, \nu_A(x) < r$; $\mu_A(y) < r$, $\nu_A(y) > 0$ …........….(3)
And finally from (3), we get $\mu_A(x) > 0, \nu_A(x) < 1$; $\mu_A(y) < 1$, $\nu_A(y) > 0$ ……...….(4)
Therefore IF-T_0(r-i) ⇒ IF-T_0(r-ii) ⇒ IF-T_0(r-iv) ⇒ IF-T_0(viii).
Similarly other implications may be proved.
The reverse implications are not true in general which can be seen as the following examples:

Example 3.1. Let $X = \{x, y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A = \{(x, 0.7, 0.1), (y, 0.2, 0.3)\}$. If $r = 0.5$, then clearly (X,τ) is IF-T_0(r-ii) but not IF-T_0(r-i).

Example 3.2. Let $X = \{x, y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A = \{(x, 0.2, 0.1), (y, 0.2, 0.3)\}$. If $r = 0.5$, then clearly (X,τ) is IF-T_0(r-iv) but not IF-T_0(r-i), IF-T_0(r-ii) and IF-T_0(r-iii).

Example 3.3. Let $X = \{x, y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A = \{(x, 0.2, 0.1), (y, 0.2, 0.6)\}$. If $r = 0.5$, then clearly (X,τ) is IF-T_0(r-iii) but not IF-T_0(r-i).

Example 3.4. Let $X = \{x, y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A = \{(x, 0.3, 0.6), (y, 0.1, 0.6)\}$. If $r = 0.2$, then clearly (X,τ) is IF-T_0(r-v) but not IF-T_0(r-i).

Example 3.5. Let $X = \{x, y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A = \{(x, 0.3, 0.1), (y, 0.4, 0.5)\}$. If $r = 0.2$, then clearly (X, τ) is IF-T_0(r-vi) but not IF-T_0(r-i).

Example 3.6. Let $X = \{x, y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A = \{(x, 0.3, 0.6), (y, 0.5, 0.3)\}$. If $r = 0.2$, then clearly (X, τ) is IF-T_0(r-vii) but not IF-T_0(r-i), IF-T_0(r-v) and IF-T_0(r-vi).

Example 3.7. Let $X = \{x, y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A = \{(x, 0.3, 0.6), (y, 0.2, 0.6)\}$. If $r = 0.5$, then clearly (X, τ) is IF-T_0(viii) but not IF-T_0(r-iv) and IF-T_0(r-vii).

Theorem 3.5. Let (X, τ) be a IFTS and $r, s \in (0,1)$ with $r < s$, then (X, τ) is T_0(r-iv) $\Rightarrow$ (X, τ) is T_0(s-iv) and (X, τ) is T_0(s-vii) $\Rightarrow$ (X, τ) is T_0(r-vii).

Proof: IF-T_0(r-iv) $\Rightarrow$IF-T_0(s-iv): Suppose (X, τ) is IF-T_0(r-iv).

Let $x, y \in X$ with $x \neq y$. Since (X, τ) is IF-T_0(r-iv), there exists an intuitionistic fuzzy set $A = (\mu_A, \nu_A) \in \tau$ such that $\mu_A(x) > 0, \nu_A(x) < r$; $\mu_A(y) < r, \nu_A(y) > 0$ or $\mu_A(y) > 0, \nu_A(y) < r$; $\mu_A(x) < r, \nu_A(x) > 0$. Consider $\mu_A(x) > 0, \nu_A(x) < r$; $\mu_A(y) < r$, $\nu_A(y) > 0$. Since $r < s$, we can write $\mu_A(x) > 0, \nu_A(x) < s$; $\mu_A(y) < s, \nu_A(y) > 0$. Therefore (X, τ) is IF-T_0(s-iv).

Similarly we can prove IF-T_0(s-vii) $\Rightarrow$IF-T_0(r-vii).

The reverse implications are not true in general which can be seen as the following examples:

Example 3.8. Let $X = \{x, y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A = \{(x, 0.6, 0.4), (y, 0.2, 0.3)\}$. If $r = 0.3$ and $s = 0.5$ then clearly (X, τ) is IF-T_0(s-iv) but not IF-T_0(r-iv).

Example 3.9. Let $X = \{x, y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A = \{(x, 0.4, 0.4), (y, 0.2, 0.4)\}$. If $r = 0.3$ and $s = 0.5$ then clearly (X, τ) is IF-T_0(r-vii) but not IF-T_0(s-vii).

Theorem 3.6. Let (X, τ) and (Y, δ) be IFTSs and $f: X \to Y$ is one-one and continuous. Then (Y, δ) is IF-T_0(r-k)$\Rightarrow$ (X, τ) is IF-T_0(r-k) for any $k \in \{i, ii, iii, iv, v, vi, vii\}$ and (Y, δ) is IF-T_0(viii)$\Rightarrow$ (X, τ) is IF-T_0(viii).

Proof: Suppose (Y, δ) is IF-T_0(r-i). Let_$x, y \in X$ with $x \neq y$. Since f is one-one, $f(x), f(y) \in Y$ with $f(x) \neq f(y)$. Since (Y, δ) is IF-T_0(r-i), there exists $B = (\mu_B, \nu_B) \in \delta$ such that $\mu_B(f(x)) > r, \nu_B(f(x)) < r$; $\mu_B(f(y)) < r, \nu_B(f(y)) > r$ or $\mu_B(f(y)) > r, \nu_B(f(y)) < r$; $\mu_B(f(x)) < r, \nu_B(f(x)) > r$.

We consider $\mu_B(f(x)) > r, \nu_B(f(x)) < r$; $\mu_B(f(y)) < r, \nu_B(f(y)) > r$.

Since f is continuous, $f^{-1}(B)$=$(f^{-1}(\mu_B), f^{-1}(\nu_B)) \in \tau$.

Now $f^{-1}(\mu_B)(x) = \mu_B(f(x)) > r, f^{-1}(\nu_B)(x) = \nu_B(f(x)) < r$

And $f^{-1}(\mu_B)(y) = \mu_B(f(y)) < r, f^{-1}(\nu_B)(y) = \nu_B(f(y)) > r$

Therefore (X, τ) is IF-T_0(r-i).

Similarly we can show others implications.

Theorem 3.7. Let (X,τ) and (Y,δ) be IFTSs and $f: X \to Y$ is one-one, onto and open. Then (X,τ) is IF-T_0(r-k) $\Rightarrow$ (Y,δ) is IF-T_0(r-k) for any $k \in \{i, ii, iii, iv, v, vi, vii\}$ and (X,τ) is IF-T_0(viii) $\Rightarrow$ (Y,δ) is IF-T_0(viii).

Proof: Suppose (X,τ) is IF-T_0(r-i). Let $x, y \in Y$ with $x \neq y$. Since f is onto, there exists some $p, q \in X$ such that $p \neq q$, $f(p) = x$ and $f(q) = y$. Again since f is one-one, these p and q are unique. i.e., $f^{-1}(x) = \{p\}$ and $f^{-1}(y) = \{q\}$. Again since (X,τ) is IF-T_0(r-i), there exists $A = (\mu_A, \nu_A) \in \tau$ such that $\mu_A(p) > r, \nu_A(p) < r$; $\mu_A(q) < r, \nu_A(q) > r$ or $\mu_A(q) > r, \nu_A(q) < r$; $\mu_A(p) < r, \nu_A(p) > r$.

Suppose $\mu_A(p) > r, \nu_A(p) < r$; $\mu_A(q) < r, \nu_A(q) > r$.

Since f is open, $f(A) = \big(f(\mu_A), f(\nu_A)\big) \in \delta$.

Now $f(\mu_A)(x) = \sup_{a \in f^{-1}(x)} \mu_A(a) = \mu_A(p) > r$, $f(\nu_A)(x) = \inf_{a \in f^{-1}(x)} \nu_A(a) = \nu_A(p) < r$.

And $f(\mu_A)(y) = \sup_{a \in f^{-1}(y)} \mu_A(a) = \mu_A(q) < r$, $f(\nu_A)(y) = \inf_{a \in f^{-1}(y)} \nu_A(a) = \nu_A(q) > r$.

Therefore (Y,δ) is IF-T_0(r-i).

Similarly we can show others implications.

From theorem 3.6 and theorem 3.7 we have the following corollary.

Corollary 3.1. If (X,τ) and (Y,δ) are IFTSs and $f: X \to Y$ is a homeomorphism then (X,τ) is IF-T_0(r-k) if and only if (Y,δ) is IF-T_0(r-k) for any $k \in \{i, ii, iii, iv, v, vi, vii\}$ and (X,τ) is IF-T_0(viii) if and only if (Y,δ) is IF-T_0(viii).

Remark 3.1. IF-T_0(r-k) for k= i, ii, iii, iv, v, vi, vii and IF-T_0(viii) are topological property.

Theorem 3.8. Let (X,τ) be an intuitionistic fuzzy topological space and U is a non empty sub set of X. Then (X,τ) is IF-T_0(r-k) $\Rightarrow (U,\tau_U)$ is IF-T_0(r-k) any $k \in \{i, ii, iii, iv, v, vi, vii\}$ and (X,τ) is IF-T_0(viii) $\Rightarrow$ (U,τ_U) is IF-T_0(viii).

Proof: Suppose (X,τ) is IF-T_0(r-i). Let $x, y \in U$ with $x \neq y$. So $x, y \in X$ with $x \neq y$ as $U \subseteq X$. Now since (X,τ) is IF-T_0(r-i), there exists $A = (\mu_A, \nu_A) \in \tau$ such that $\mu_A(x) > r, \nu_A(x) < r$; $\mu_A(y) < r, \nu_A(y) > r$ or $\mu_A(y) > r, \nu_A(y) < r$; $\mu_A(x) < r, \nu_A(x) > r$.

We consider $\mu_A(x) > r, \nu_A(x) < r$; $\mu_A(y) < r, \nu_A(y) > r$.

Clearly $A|U = (\mu_A|U, \nu_A|U) \in \tau_U$.

Now $\mu_A|U(x) = \mu_A(x) > r$, $\nu_A|U(x) = \nu_A(x) < r$.

and $\mu_A|U(y) = \mu_A(y) < r$, $\nu_A|U(y) = \nu_A(y) > r$.

Therefore (U,τ_U) is IF-T_0(r-i).

Similarly we can show others implications.

Remark 3.2. The properties IF-T_0(r-k) for k=i, ii, iii, iv, v, vi, vii and IF-T_0(viii) are hereditary.

Definition 3.2. [11] An intuitionistic fuzzy topological space (X,τ) is called

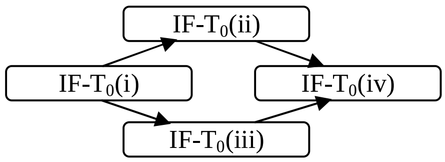
(1) IF-T_0(i) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A) \in \tau$ such that $\mu_A(x) = 1, \nu_A(x) = 0$; $\mu_A(y) = 0, \nu_A(y) = 1$ or $\mu_A(y) = 1, \nu_A(y) = 0$; $\mu_A(x) = 0, \nu_A(x) = 1$.

(2) IF-T_0(ii) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A) \in \tau$ such that $\mu_A(x) = 1, \nu_A(x) = 0$; $\mu_A(y) = 0, \nu_A(y) > 0$ or $\mu_A(y) = 1, \nu_A(y) = 0$; $\mu_A(x) = 0, \nu_A(x) > 0$.

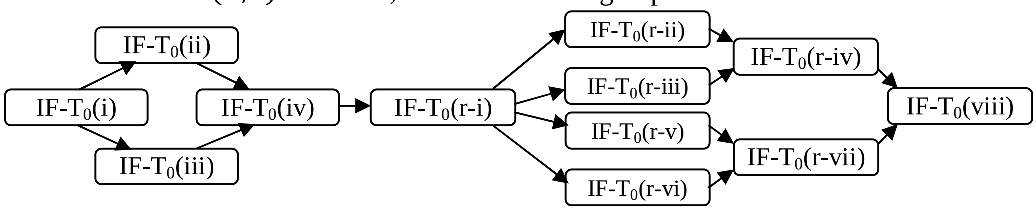
(3) IF-T_0(iii) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A) \in \tau$ such that $\mu_A(x) > 0, \nu_A(x) = 0$; $\mu_A(y) = 0, \nu_A(y) = 1$ or $\mu_A(y) > 0, \nu_A(y) = 0$; $\mu_A(x) = 0, \nu_A(x) = 1$.

(4) IF-T_0(iv) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A) \in \tau$ such that $\mu_A(x) > 0, \nu_A(x) = 0$; $\mu_A(y) = 0, \nu_A(y) > 0$ or $\mu_A(y) > 0, \nu_A(y) = 0$; $\mu_A(x) = 0, \nu_A(x) > 0$.

Theorem 3.9. [11] Let (X, τ) be a IFTS. Then the following implications hold.



Theorem 3.10. If (X, τ) is a IFTS, then the following implications hold.



Proof: To prove this theorem we only have to prove that (X, τ) is IF-T_0(iv) $\Rightarrow$ (X, τ) is IF-T_0(r-i).

Let (X, τ) is IF-T_0(iv) and $x, y \in X$ with $x \neq y$. Then there exists $A = (\mu_A, \nu_A) \in \tau$ such that $\mu_A(x) > 0$, $\nu_A(x) = 0$; $\mu_A(y) = 0, \nu_A(y) > 0$ or $\mu_A(y) > 0, \nu_A(y) = 0$; $\mu_A(x) = 0, \nu_A(x) > 0$ as (X, τ) is IF-T_0(iv). We consider $\mu_A(x) > 0$, $\nu_A(x) = 0$; $\mu_A(y) = 0, \nu_A(y) > 0$, then we can write $\mu_A(x) > r, \nu_A(x) < r$; $\mu_A(y) < r, \nu_A(y) > r$ as $r \in (0,1)$. So (X, τ) is IF-T_0(r-i).

Other implication clearly holds by theorem 3.4 and theorem 3.9.

The reverse implication is not necessarily true. For this we only have to show that (X, τ) is IF-T_0(r-i) $\not\Rightarrow$ (X, τ) is IF-T_0(iv), which is shown by the following example:

Example 3.10. Let $X = \{x, y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A = \{(x, 0.6, 0.4), (y, 0.2, 0.6)\}$. If $r = 0.5$ then clearly (X, τ) is IF-T_0(r-i) but not IF-T_0(iv).

Theorem 3.11. Let (X_j, τ_j), $j = 1{,}2$ be IFTSs and $(X, \tau) = \{X_1 \times X_2, \tau_1 \times \tau_2\}$. If each (X_j, τ_j), $j = 1{,}2$ are IF-T_0(r-k), then (X, τ) is IF-T_0(r-k) for any $k \in \{i, ii, iii, iv, v, vi, vii\}$ and If each (X_j, τ_j), $j = 1{,}2$ are IF-T_0(viii), then (X, τ) is IF-T_0(viii).

Proof: The proofs of all implications are similar. For an example we shall prove that if each (X_j, τ_j), $j = 1{,}2$ are IF-T_0(r-iv), then (X, t) is IF-T_0(r-iv).

Suppose each (X_j, τ_j), $j = 1{,}2$ are IF-T_0(r-iv).

Let $x, y \in X$ with $x \neq y$ where $x = (x_1, x_2)$ and $y = (y_1, y_2)$.

Then at least $x_1 \neq y_1$ or $x_2 \neq y_2$. Suppose $x_1 \neq y_1$. Clearly $x_1, y_1 \in X_1$

and $x_2, y_2 \in X_2$. Since (X_1, τ_1) is IF-T_0(r-iv), there exists $A_1 = (\mu_{A_1}, \nu_{A_1}) \in \tau_1$ such that $\mu_{A_1}(x_1) > 0, \nu_{A_1}(x_1) < r; \mu_{A_1}(y_1) < r, \nu_{A_1}(y_1) > 0$
or $\mu_{A_1}(y_1) > 0, \nu_{A_1}(y_1) < r; \mu_{A_1}(x_1) < r, \nu_{A_1}(x_1) > 0$.
Consider $\mu_{A_1}(x_1) > 0, \nu_{A_1}(x_1) < r; \mu_{A_1}(y_1) < r, \nu_{A_1}(y_1) > 0$.
Choose $A_2 = 1_\sim = (\underline{1}, \underline{0})$. Clearly $A_2 \in \tau_2$.
Let $A = A_1 \times A_2 = (\mu_{A_1} \times \underline{1}, \nu_{A_1} \times \underline{0}) = (\mu_A, \nu_A)$ (say)
By the definition of product IFT, $A = (\mu_A, \nu_A) \in \tau$
Now $\mu_A(x) = (\mu_{A_1} \times \underline{1})(x_1, x_2)$=$min(\mu_{A_1}(x_1), \underline{1}(x_2))$= $min(\mu_{A_1}(x_1), 1) > 0$ as $\mu_{A_1}(x_1) > 0$.
And $\nu_A(x) = (\nu_{A_1} \times \underline{0})(x_1, x_2)$=$max(\nu_{A_1}(x_1), \underline{0}(x_2))$= $max(\nu_{A_1}(x_1), 0) < r$ as $\nu_{A_1}(x_1) < r$.
Again
$\mu_A(y) = (\mu_{A_1} \times \underline{1})(y_1, y_2)$=$min(\mu_{A_1}(y_1), \underline{1}(y_2))$= $min(\mu_{A_1}(y_1), 1) < r$ as $\mu_{A_1}(y_1) < r$.
And $\nu_A(y) = (\nu_{A_1} \times \underline{0})(y_1, y_2)$=$max(\nu_{A_1}(y_1), \underline{0}(y_2))$= $max(\nu_{A_1}(y_1), 0) > 0$ as $\nu_{A_1}(y_1) > 0$.
i.e., for $x, y \in X$ with $x \neq y$ we get $A = (\mu_A, \nu_A) \in \tau$ such that $\mu_A(x) > 0$, $\nu_A(x) < r$; $\mu_A(y) < r, \nu_A(y) > 0$. Therefore (X, τ) is is IF-T_0(r-iv).

Remark 3.3. The properties IF-T_0(r-k) for k=i, ii, iii, iv, v, vi, vii and IF-T_0(viii) are productive

4. Conclusion

In this paper we see that our eight definitions are more general than that of Estiaq Ahmed et al. Also we see that our definitions satisfy hereditary and productive properties. Moreover, the definitions preserved under one-one and open mapping.

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