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M-N Fuzzy Normal Soft Groups

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Abstract. In this paper, we have discussed the concept of M-N fuzzy normal soft group, we then define the M-N level subsets of a fuzzy normal soft subgroup and its some elementary properties are also discussed. The presented method in this manuscript is more sensible and also reliable in solving the problems. This method can solve the decision making problems.

Keywords: Fuzzy group, M-N fuzzy group, M-N fuzzy soft subgroup, M-N level subset, M-N fuzzy soft normalize.

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1. Introduction

There are various types of uncertainties in the real world, but few classical mathematical tools may not be suitable to model these uncertainties. Many intricate problems in economics, social science, engineering, medical science and many other fields involve undefined data. These problems which one comes face to face with in life cannot be solved using classical mathematic methods. In classical mathematics, a mathematical model of an object is devised and the concept of the exact solution of this model is not yet determined. Since, the classical mathematical model is too complex, the exact solution cannot be found. There are several well-renowned theories available to describe uncertainty. For instance, Rosenfeld [7] introduced the concept of fuzzy subgroup in 1971 and the fuzzy normal subgroup was revealed by Wu [10] during 1981. Further, the theory of fuzzy sets was inspired by Zadoh [11] in addition to this, Molodtsov [5] have introduced the concept of softsets in 1999. Furthermore, Maji et al., [4] as well introduction the concept of fuzzy soft sets in 2001 and Jacobson [3] introduced the concept of m-group M-subgroup.

In 2015, Patel et al. [6] were developed three variables on normal fuzzy soft subgroup. Sarala and Suganya [8] beunraveled the three variables on normal fuzzy soft subgroup in 2004.In addition, Kandasamy [9] have introduced the fuzzy algebra during 2003. An introduction to the new definition of Soft sets and soft groups depending on inclusion relation and intersection of sets were exposed by Akta and Cagman [1]. In 1981, Das [2] studied the Fuzzy groups and level subgroups. Moreover, Maij et al. [4] were introduced the fuzzy soft set in 2001.

In the present manuscript, we have discussed the concept of M-N fuzzy normal soft group based on the concept of Normal fuzzy soft group [6, 8]. In section 2, we presented the basic definition, notations on M-N fuzzy normal soft group and required results on fuzzy normal soft group. In section 3, we define the M-N fuzzy soft set, normal fuzzy soft set and also define the M-N level subsets of a normal fuzzy soft subgroup. We have also discussed the concept of M-N fuzzy normal soft group and some of its elementary properties.

2. Preliminaries

In this section, some basic definitions and results needed are given. For the sake of convenience we set out the former concepts which will be used in this paper.

Definition 2.1. Let G be any non-empty set. A mapping α : G → [0, 1] is called fuzzy set in G.

Definition 2.2. Let x be a non-empty set. A fuzzy subset α of X is a function

$$\alpha : X \rightarrow [0, 1].$$

Definition 2.3. Let G be a group. A fuzzy subset α of G is called a fuzzy subgroup if for x, y $\in G$

(1) α (x y) ≥ min {α (x),α (y)}
(2) α (x^{-1}) =α (x)

Definition 2.4. A pair (F, A) is called a soft set over U, where F is a mapping given b F: A → $P(U)$

Definition 2.5. Let (F, A) be a soft set over G. Then (F, A) is called a soft group over G if F (α) is a group G for all α ∈ A.

Definition 2.6. A pair (F, A) is called a fuzzy soft set over U , where F: A → I^U is a mapping I = [0, 1], F(α)is a fuzzy subset of U for all α ∈ A.

Definition 2.7. Let (F, A) be a fuzzy soft set over G. Then (F, A) is a called a fuzzy soft group if F (α) is a fuzzy subgroup G for all α ∈ A.

Definition 2.8. Let (F, A) and (G, B) be two fuzzy soft set over U. Then (F, A) is called a fuzzy soft subset of (G, B) denoted by (F, A) ⊆ (G, B) if

(1) A ⊆ B
(2) F (α) is a fuzzy subset of G (α) for each α ∈ A.

Definition 2.9. A fuzzy set α is called a fuzzy soft subgroup of a group G, if for x, y ∈ G

(1) α (xy) ≥ min{α (x), α(y)}
(2) α (x^{-1}) ≥ α (x)

Definition 2.10. A subgroup H of G is called a normal subgroup of a G if aH = Ha for all a∈ G

Definition 2.11. Let G be a group. A fuzzy subgroup α of G is said be normal if for all x, y $\in G$, $\alpha(Xyx^{-1}) = \alpha$ (y) (or) α (xy) $\geq \alpha$ (yx)

Definition 2.12. Let G be a group. A fuzzy soft subgroup α of G is said be fuzzy normal soft subgroup, if for all x, y $\in G$ and α (xyx^{-1}) = α (y) (or) α (xy)$\geq \alpha$ (yx)

Definition 2.13. Let $\alpha \cap \beta$ be a fuzzy soft subgroup of a group G, for any t $\in$ [0, 1], we define the level subset of $\alpha \cap \beta$ is the set

$$(\alpha \cap \beta)_t = \{x \in X / (\alpha \cap \beta)(x) \geq t\}$$

Definition 2.14. Let G be a group and $\alpha \cap \beta$ be a fuzzy normal soft subgroup of G. Let N $(\alpha \cap \beta)$ = {y $\in$ G / $(\alpha \cap \beta)$ (y x y^{-1}) = $(\alpha \cap \beta)$ (x) for all x $\in$ G}, then N $(\alpha \cap \beta)$ is called the fuzzy soft Normalize of $\alpha \cap \beta$.

Definition 2.15. Let M, N be left and right operator sets of group G respectively if (m x) n = m (x n) for all x $\in$ G, m $\in$ M, n$\in N$. Then G is said be an M – N group.

Definition 2.16. If α is an M – N fuzzy subgroup of an M – N group G. Then the following statement holds for all x, y$\in$ G, m $\in$ M, and n $\in$ N

(1) α (m(x y)n)$\geq$ min{ α (x), α (y)}
(2) α (m x^{-1} n) $\geq \alpha$ (x)

Definition 2.17. Let G be an M –N group . α is said be an M- N fuzzy normal subgroup of G if α is not only an M- N fuzzy subgroup of G, but also fuzzy normal subgroup of G.

3. M-N fuzzy normal soft group

In this section, we shall define M-N fuzzy soft group, fuzzy normal soft group, discussed the concept of M-N fuzzy normal soft group based on the concept of fuzzy normal soft group [6, 8], also define the M-N level subsets of a fuzzy normal soft subgroup and its some elementary properties are discussed.

Definition 3.1. Let G be an M – N group and (F, A) be a fuzzy soft subgroup of G if

(1) F {m(x y) n} $\geq$ min {F(x), F(y)}
(2) F {(m x^{-1}) n} $\geq$ F(x) hold for any x, y $\in$ G, m $\in$ M, n $\in$ N, then (F, A) is said be an M – N fuzzy soft subgroup of G. Here F: A $\rightarrow$ P (G)

Definition 3.2. Let G be an M – N group and (F, A) be a fuzzy soft subgroup of G if

(1) F(m x) $\geq$ F(x)
(2) F(x n) $\geq$ F(x) hold for any x $\in$ G, m $\in$ M, and n $\in$ N, then (F, A) is said be an M – N fuzzy soft subgroup of G.

Definition 3.3. Let G be an M – N group. (F, A) is said be an M – N fuzzy normal soft subgroup of G if (F, A) is not only an M – N soft fuzzy subgroup of G, but also fuzzy normal soft subgroup of G.

Theorem 3.4. If $\alpha, \ \beta \ and \ \gamma$ are three M – N fuzzy soft subgroup of G, then $\alpha \cap \beta \cap \gamma$ is a M- N fuzzy soft subgroup of G

Proof: Let $\alpha, \ \beta$ and γ be three M - N fuzzy soft subgroup of G.

(1) $(\alpha \cap \beta \cap \gamma)(m(xy^{-1})n) \geq \min\{(\alpha \cap \beta \cap \gamma)(m\,x), (\alpha \cap \beta \cap \gamma)(y^{-1}n)\}$

$(\alpha \cap \beta \cap \gamma)(m\,(xy^{-1})\,n) \geq \min\{(\alpha \cap \beta)(m\,(xy^{-1})\,n), \gamma\,(m\,(xy^{-1})\,n)\}$

$\geq \min\{\min\{(\alpha \cap \beta)(m\,x), (\alpha \cap \beta)(y^{-1}n)\}, \min\{\gamma\,(m\,x), \gamma\,(y^{-1}n)\}\}$

$\geq \min\{\min\{(\alpha \cap \beta)(m\,x), \gamma\,(m\,x)\}, \min\{(\alpha \cap \beta)(y^{-1}n), \gamma\,(y^{-1}n)\}$

$\geq \min\{(\alpha \cap \beta \cap \gamma)(m\,x), (\alpha \cap \beta \cap \gamma)(y^{-1}n)\}$.

(2) $(\alpha \cap \beta \cap \gamma)(m\,x\,n) = (\alpha \cap \beta \cap \gamma)(mx^{-1}n)$

$(\alpha \cap \beta \cap \gamma)(m\,x\,n) = \{(\alpha \cap \beta)(m\,x\,n), \gamma\,(m\,x\,n)\}$

$= \{[\alpha\,(m\,x\,n),, \ \beta(m\,x\,n)], \gamma\,(m\,x\,n)\}$

$= \{[\alpha\,(mx^{-1}n),, \ \beta\,(m\,x\,n)], \gamma\,(m\,x\,n)\}$

$= \{(\alpha \cap \beta)(m\,x^{-1}n), \gamma\,(mx^{-1}n)\}$

$= (\alpha \cap \beta \cap \gamma)(mx^{-1}n)$

Hence, $\alpha \cap \beta \cap \gamma$ is an M – N fuzzy soft subgroup of G.

Theorem 3.5. The intersection of any three M – N fuzzy normal soft subgroup of G is also an M – N fuzzy normal soft subgroup G.

Proof: Let $\alpha, \ \beta \ and \ \gamma$ are three M – N fuzzy normal soft subgroup of G.

By the previous theorem we know that, $\alpha \cap \beta \cap \gamma$ is an M – N fuzzy soft subgroup of G.

Let x, y ∈ G, m ∈ M, and n ∈ N

To prove that $(\alpha \cap \beta \cap \gamma)(m\,y\,x\,y^{-1}\,n) = (\alpha \cap \beta \cap \gamma)(m\,x\,n)$

Now $(\alpha \cap \beta \cap \gamma)(m\,y\,x\,y^{-1}\,n) = \min\{(\alpha \cap \beta)(m\,(y\,x\,y^{-1})n), \ \gamma\,(m(y\,x\,y^{-1})n)\}$

$= \min\{[\alpha\,(m\,(y\,x\,y^{-1})n), \ \beta\,(m(y\,x\,y^{-1})n), \ \gamma(m\,(y\,x\,y^{-1})n)]\}$

$= \min\{[\alpha\,(m\,x\,n), \ \beta\,(m\,xn), \ \gamma(m\,xn)]\}$

$= \min\{(\alpha \cap \beta)(m\,x\,n), \ \gamma\,(m\,xn)\}$

$= (\alpha \cap \beta \cap \gamma)(m\,x\,n)$

Therefore, $(\alpha \cap \beta \cap \gamma)(m\,y\,x\,y^{-1}\,n) = (\alpha \cap \beta \cap \gamma)(m\,x\,n)$.

Hence $\alpha \cap \beta \cap \gamma$ is an M – N fuzzy normal soft subgroup G.

Note 3.6. If $(\alpha \cap \beta)_i$, $i \in \Delta$ are M – N fuzzy normal soft subgroup of G, then $\bigcap_{i \in \Delta} (\alpha \cap \beta)_i$ is a M – N fuzzy normal soft subgroup of G.

Definition 3.7. Let G be a group, α is a M – N fuzzy soft subgroup of G is said be a M – N fuzzy normal soft subgroup if

$\alpha\,(m\,(xyx^{-1})n) = \alpha\,(m\,y\,n)$ (or)

$\alpha\,(m\,(x\,y)n) \geq \alpha\,(m\,(y\,x)\,n)$ for all x, y ∈ G, m ∈ M, and n ∈ N.

Theorem 3.8. Let α is an M – N fuzzy normal soft subgroup of G, then for any y ∈G we have $\alpha\,(m(y^{-1}xy)\,n) = \alpha\,(m\,(yxy^{-1})\,n)$

Proof: Let α is an M- N fuzzy normal soft subgroup G, then for any y ∈ G.

Now $\alpha\,(m(y^{-1}xy)\,n = \alpha\,(m\,(xy^{-1}\,y)\,n)$

$= \alpha\,(m(x)\,n)$

$= \alpha\,(m\,(yy^{-1}x)\,n)$

$= \alpha\,(m\,(yxy^{-1})\,n)$

Therefore α (m(y^{-1}xy) n) = α (m (yxy^{-1}) n).
Hence the theorem.

Theorem 3.9. If α is an M- N fuzzy normal soft subgroup G, then $g\alpha g^{-1}$ is also M- N fuzzy normal soft subgroup G, for all $g \in$ G.
Proof: Let α be an M- N fuzzy normal soft subgroup G, then $g\alpha\, g^{-1}$ is an M- N subgroup G, for all $g \in$ G.

$$\text{Now } g\,\alpha\, g^{-1}\ (\text{m (y x y}^{-1)}\text{ n}) = \alpha(\, g^{-1}\text{m (y x y}^{-1)}\text{ n)}\ g\,)$$
$$= \alpha(\text{ m (y x y}^{-1)}\text{ n)}$$
$$= \alpha(\text{m x n})$$
$$= \alpha(\, g(\text{m xn}))\, g^{-1}$$
$$= g\,\alpha g^{-1}\ (\text{m x n})$$

Therefore, $g\,\alpha\, g^{-1}$ (m (y x $y^{-1)}$ n) = $g\,\alpha g^{-1}$ (m x n).
Hence the theorem.

Theorem 3.10. Let $\alpha\ \cap \beta$ is a M – N fuzzy normal soft subgroup of G, then

$$(\alpha\ \cap \beta\ )\ (\text{m (y}^{-1}\text{ x y)n}) = (\alpha\ \cap \beta\ )\ (\text{m x n}).$$

Proof: Let $\alpha\ \cap \beta$ is an M – N fuzzy normal soft subgroup of G.
Let x, y G.

$$\text{Now}\quad (\alpha\ \cap \beta\ )\ (\text{m (y}^{-1}\text{ x y)n}) = (\alpha \cap \beta\ )\ (\text{m (xy}^{-1}\text{y) n})$$
$$= (\alpha \cap \beta\ )\ (\text{m (x) n})$$
$$= (\alpha \cap \beta\ )\ (\text{m (yy}^{-1}\text{ x) n})$$
$$= (\ \alpha \cap \beta\ )\ (\text{m(y xy}^{-1}\text{) n})$$
$$(\alpha\ \cap \beta\ )\ (\text{m (y}^{-1}\text{ x y)n}) = (\alpha \cap \beta\ )\ (\text{m(y xy}^{-1}\text{) n}).$$

Hence the proof.

Theorem 3.11. If $\alpha \cap \beta$ is a M- N fuzzy normal soft subgroup G, then $g(\ \alpha \cap \beta)\ g^{-1}$ is also a M- N fuzzy normal soft subgroup G, for all $g \in$ G .
Proof: If $\alpha \cap \beta$ is a M- N fuzzy normal soft subgroup G, then $g(\ \alpha \cap \beta)\ g^{-1}$ is also a M- N fuzzy normal soft subgroup G, for all $g \in$ G .

$$\text{To prove that}\qquad g(\ \alpha \cap \beta)\ g^{-1}(\text{m (y x y}^{-1}\text{) n}) = g(\ \alpha \cap \beta)\ g^{-1}\ (\text{m x n}).$$
$$\text{Now } g(\ \alpha \cap \beta)\ g^{-1}\ (\text{m (y x y}^{-1}\text{) n}) = (\ \alpha \cap \beta)(\ g^{-1}\text{m (y x y}^{-1}\text{) n)}\ g\ )$$
$$= (\ \alpha \cap \beta)(\text{ m (y x y}^{-1}\text{) n)}$$
$$= (\ \alpha \cap \beta)(\text{m x n})$$
$$= (\ \alpha \cap \beta)(\ g(\text{ m x n})g^{-1}$$
$$= g(\ \alpha \cap \beta g^{-1}(\text{m x n}).$$

Therefore, $g(\ \alpha \cap \beta)g^{-1}$(m (y x y^{-1}) n) = $g(\ \alpha \cap \beta)g^{-1}$ (m x n).
Hence the theorem.

Definition 3.12. Let $\alpha \cap \beta$ be an M – N fuzzy soft subgroup of a group G. For any t$\in$ [0, 1], we define the M – N level subset of $\alpha \cap \beta$ is the set

$$(\alpha \cap \beta)_t = \{x \in G\ /\ (\alpha \cap \beta\ )\ (\text{m x}) \geq t,\ (\ \alpha \cap \beta\ )\ (\text{x n}) \geq t \text{ for all m} \in M,\ n \in N\}$$

Theorem 3.13. Let G be a group and $\alpha \cap \beta$ be a fuzzy subset of G. Then $\alpha \cap \beta$ is a M – N fuzzy normal soft subgroup of G iff the level subset $(\alpha \cap \beta)_{t,}$ t$\in$ [0, 1]are M- N subgroup of G.

Proof: Let $\alpha \cap \beta$ be an M – N fuzzy normal soft subgroup of G and the level subset $(\alpha \cap \beta)_t$ = { x ∈ G / $(\alpha \cap \beta)$ (m x) ≥ t,($\alpha \cap \beta$) (x n) ≥ t, t ∈ [0,1] m∈ M, n∈ N }

Let x, y ∈ $(\alpha \cap \beta)_t$ then $(\alpha \cap \beta)$ (m x) ≥ t and ($\alpha \cap \beta$) (x n) ≥ t

Now $(\alpha \cap \beta)$ (m xy^{-1}n) ≥ min {($\alpha \cap \beta$) (m x), ($\alpha \cap \beta$) (y^{-1} n)}

= min {($\alpha \cap \beta$) (m x), ($\alpha \cap \beta$) (y n)}

≥ min { t , t }

$(\alpha \cap \beta)$ (m xy^{-1}n) ≥ t

m x y^{-1}n ∈ $(\alpha \cap \beta)_t$

Therefore $(\alpha \cap \beta)_t$ is a M – N subgroup of G.

Conversely, let us assume that $(\alpha \cap \beta)_t$ is an M – N subgroup G.

Let x, y ∈ $(\alpha \cap \beta)_t$ then $(\alpha \cap \beta)$ (m x) ≥ t and ($\alpha \cap \beta$) (x n) ≥ t

Also ($\alpha \cap \beta$) (m (xy^{-1}) n) ≥ t.

Since m (xy^{-}1) n ∈ $(\alpha \cap \beta)_t$ = min {t, t}

= min {$(\alpha \cap \beta)$ (m x), ($\alpha \cap \beta$) (y n)}.

Therefore ($\alpha \cap \beta$) (m (xy^{-1}) n) ≥ min {$(\alpha \cap \beta)$ (m x), ($\alpha \cap \beta$) (y n)}.

Hence $\alpha \cap \beta$ is an M – N fuzzy normal soft subgroup of G.

Definition 3.14. Let G be a group and $\alpha \cap \beta$ be an M – N fuzzy normal soft subgroup of G.

Let N $(\alpha \cap \beta)$ = {y ∈ G / $(\alpha \cap \beta)$ (m (y x y^{-1}) n) = $(\alpha \cap \beta)$ (m x n) for all x ∈ G, m ∈ M, n ∈ N}, then N $(\alpha \cap \beta)$ is called the M – N fuzzy soft normalizer of $\alpha \cap \beta$.

Theorem 3.15. Let G be a group and $\alpha \cap \beta$ be a fuzzy subset of G. Then $\alpha \cap \beta$ is a M– N fuzzy normal soft subgroup of G iff the level subset $(\alpha \cap \beta)_{t,}$ t ∈ [0, 1] are M- N normal subgroup of G.

Proof: Let $\alpha \cap \beta$ be a M – N fuzzy normal soft subgroup of G and level subset $(\alpha \cap \beta)_t$, t ∈ [0,1].

Let x ∈ G and y ∈ $(\alpha \cap \beta)_{t,}$ then $(\alpha \cap \beta)$ (m y n) ≥ t, for all m ∈ M, n ∈ N.

Now $(\alpha \cap \beta)$ (m (xyx^{-1}) n) = $(\alpha \cap \beta)$ (m y n) ≥ t.

Since $\alpha \cap \beta$ is an M – N fuzzy normal soft subgroup of G.

That is $(\alpha \cap \beta)$ (m (x y x^{-1}) n) ≥ t.

Therefore (m (x y x^{-1}) n ∈ $(\alpha \cap \beta)_{t.}$

Hence $(\alpha \cap \beta)_t$ is an M – N normal subgroup of G.

4. Conclusion

The main results in the present manuscript are based on the concept of fuzzy normal soft group [6 and 8]. We have also defined the M-N level subsets of a fuzzy normal soft subgroup and its some elementary properties are discussed.

REFERENCES

1. H.Akta and N.Cagman, Soft sets and soft group, *Information Sciences*, 177 (2007) 2726 – 2735.
2. P.S.Das, Fuzzy groups and level subgroups, *J. Math.Anal. Appl.*, 84 (1981) 264–269.
3. N.Jacobson, Lectures in Abstract Algebra, East-West Press, (1951).
4. P.K. Maij, R.Biswas and A.R.Ray, Fuzzy soft set, *J. Fuzzy Math.*, 9 (2001) 589-602.
5. D.Molodtsov, Soft set theory – first result, *Comput. Math. Appl.*, 37 (1999) 19-31.

6. R.Patel, R.Bhardwaj, S.Choudhary, S.Garge, On normal fuzzy soft group, *Mathematical Theory and Modeling*, 5(7) (2015) 26-32.
7. A. Rosenfield, Fuzzy groups, *J. Math. Anal. Appl.*, 35 (1971) 512 – 517.
8. N.Sarala and B.Suganya, On normal fuzzy soft group, *International Journal of Mathematics Trend and Technology*, 10(2) (2014) 70-75.
9. W.B.Vasantha Kandasamy, *Fuzzy Algebra*, American Research Press (2003).
10. W.M.Wu, Normal fuzzy subgroups, *Journal of Fuzzy Math.*, 1 (1981) 21–23.
11. L.A.Zadeh, Fuzzy sets, *Inform. and Control*, 8 (1965) 338 -353.