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# On the λ-Robustness of Fuzzy Neutrosophic Soft Matrix

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***Abstract.*** Let $(\mathcal{N},\leq)$ be a non-empty, bounded, linearly ordered set a $a\oplus b=max\{a,b\}$ , $a\otimes b=min\{a,b\}$ for $a,b\in\mathcal{N}$. $A$ fuzzy neutrosophic soft vector (FNSV) $\langle x^T,x^I,x^F\rangle$ is said to be a $\lambda$-fuzzy neutrosophic soft eigenvector (FNSEv) of a square fuzzy neutrosophic soft matrix (FNSM) $A$ if $A\otimes x=\lambda\otimes x$ for some $\lambda\in\mathcal{N}$ . $A$ given FNSM $A$ is called (strongly) $\lambda$-robust if for every $x$ the FNSV $A^k\otimes x$ is a (greatest ) FNSEv of A for some natural number k. We present a characterization of $\lambda$-robust and strongly $\lambda$-robust FNSMs. Building on this, an efficient algorithm for checking the $\lambda$-robustness and strong $\lambda$-robustness of a given FNSM is introduced.

***Keywords:*** Fuzzy neutrosophic soft set, fuzzy neutrosophic soft matrix, fuzzy neutrosophic soft eigenvector, $\lambda$-robust fuzzy neutrosophic soft matrix, strong $\lambda$-robust fuzzy neutrosophic soft matrix

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## 1. Introduction

In dealing with uncertainties many theories have been recently developed, the theory of probability, theory of fuzzy sets, Zadeh [38], theory of intuitionistic fuzzy sets, Atanaasov [2] and theory of rough sets and so on. Although many new techniques have been developed as a result of these theories, yet difficulties are still there. The major difficulties arise due to inadequacy of indeterminate and inconsistent information which exists in the belief system. Smarandache [33] introduced the concept of Neutrosophic set (NS) which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. In our regular everyday life we face situations which require procedures allowing certain flexibility information processing capacity. Matrices play a vital role in various areas of Science and Engineering. The classical matrix theory cannot solve the problems involving various types of uncertainties. These types of problems are solved by using Fuzzy matrix (FM)[34].

Practical application of graph theory, scheduling, knowledge engineering, cluster analysis, fuzzy systems and many other research areas can be formulated using the language of fuzzy algebras in which the addition and the multiplication of the vectors (Fuzzy vector, Intuitionistic Fuzzy vector) and matrices (Fuzzy matrices, Intuitionistic Fuzzy Matrix) are formally replaced by operations of the taking the maximum and minimum. In [29], the following question was posed: give a fuzzy relation $R$ between medical symptoms expressing the action of a drug on patients in a given therapy, what are the greatest invariants of the system?. The question leads to the problem of finding the greatest eigenvector of the matrix *A*, with elements of fuzzy algebra corresponding to the fuzzy relation *R*. The eigenproblem in fuzzy algebra and its connection to paths in digraphs were investigated in [7,18,19,20,21,37]. The eigenproblem in distributive lattices was studied in [10,22]. The interpretations of fuzzy eigenproblem of a matrix in cluster analysis and the generalized results on the problem can be found in [20]. Relations in fuzzy algebra are often studied using matrix operations. Convergence and periodicity of matrix powers and the relations between the matrix and orbit periods in fuzzy algebra have been studied in [8,9,14,15,23,30,31].

Soft set theory was initiated by Russian researcher Molodtov [26], he proposed soft set as a completely generic mathematical tool for modeling uncertainties. Maji et al., [27] applied this theory to several directions for dealing with the problems in uncertainty and imprecision. Yong et. al, [36] introduced a matrix representation of a fuzzy soft set and applied it in decision making problems. Borah et. al, [6] extended fuzzy soft matrix theory and its application. Chetia et.al, [12] proposed Intuitionistic fuzzy soft matrix theory, then Rajarajeswari and Dhanalakshmi [28] proposed new definition for Intuitionistic fuzzy soft matrices and its types. Sumathi and Arokiarani [1] introduced new operation on fuzzy neutrosophic soft matrices. Dhar et.al,[13] have also defined neutrosophic fuzzy matrices and studied square neutrosophic fuzzy matrices. Uma et.al, [35] introduced two types of fuzzy neutrosophic soft matrices.

The aim of this paper is to describe FNSM (denoted by $\mathcal{N}$ ) for which the (grestest) FNSEvs are reached with any start FNSV $\langle x^T, x^I, x^F \rangle$ these kinds of FNSMs are called (strongly) robust. The questions considered in this paper are analogous to those in [4,5], where robust FNSMs in a max-plus algebra are studied.

**2. Preliminaries**

In this section the basic definition of neutrosophic set (NS), fuzzy neutrosophic soft set (FNSS), fuzzy neutrosophic soft matrix (FNSM) and fuzzy neutrosophic soft matrixces of type-I are provided.

**Definition 2.1. [33]** A neutrosophic set $A$ on the universe of discourse $X$ is defined as $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X\}$ where $T, I, F : X \to ]^{-}0,1^{+}[$ and

$$^{-}0 \le T_A(x) + I_A(x) + F_A(x) \le 3^{+}. \tag{2.1}$$

From philosophical point of view the neutrosophic set takes the value from real standard or non-standard subsets of $]^{-}0,1^{+}[$. But in real life application especially in Scientific and Engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]^{-}0,1^{+}[$. Hence we consider the neutrosophic set which takes the value from the subset of [0, 1]

Therefore we can rewrite equation (2.1) as $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.. In short an element a in the neutrosophic set $A$ can be written as (Tex translation failed) where $a^T$ denotes degree of truth, $a^I$ denotes degree of indeterminacy, $a^F$ denotes degree of falsity such that $0 \le a^T + a^I + a^F \le 3$.

**Example 2.2.** Assume that the universe of discourse $X = \{x_1, x_2, x_3\}$ where $x_1, x_2$ and $x_3$ characterize the quality, reliability, and the price of the objects. It may be further assumed that the values of $\{x_1, x_2, x_3\}$ are in [0,1] and they are obtained from some investigations of some experts. The experts may impose their opinion in three components viz; the degree of goodness, the degree of indeterminacy and the degree of poorness to explain the characteristics of the objects. Suppose A is a Neutrosophic Set (NS) of $X$ such that $A = \{\langle x_1, 0.4, 0.5, 0.3\rangle, \langle x_2, 0.7, 0.2, 0.4\rangle, \langle x_3, 0.8, 0.3, 0.4\rangle\}$ where for $x_1$ the degree of goodness of quality is 0.4, degree of indeterminacy of quality is 0.5 and degree of falsity of quality is 0.3 etc,.

**Definition 2.3. [26]** Let $U$ be the initial universe set and $E$ be a set of parameter. Consider a non-empty set $A, A \subset E$. Let $P(U)$ denotes the set of all fuzzy neutrosophic sets of $U$. The collection $(F, A)$ is termed to the fuzzy neutrosophic soft set (FNSS) over $U$, where $F$ is a mapping given by $F : A \to P(U)$. Here after we simply consider $A$ as FNSS over $U$ instead of $(F, A)$.

**Definition 2.4. [1]** Let $U = \{c_1, c_2, \ldots c_m\}$ be the universal set and $E$ be the set of parameters given by $E = \{e_1, e_2, \ldots e_m\}$. Let $A \subset E$. A pair $(F, A)$ be a FNSS over $U$. Then the subset of $U \times E$ is defined by $R_A = \{(u, e); e \in A, u \in F_A(e)\}$ which is called a relation form of $(F_A, E)$. The membership function, indeterminacy membership function and non membership function are written by

$$T_{R_A} : U \times E \to [0,1], I_{R_A} : U \times E \to [0,1] \text{ and } F_{R_A} : U \times E \to [0,1]$$

where $T_{R_A}(u,e) \in [0,1], I_{R_A}(u,e) \in [0,1]$ and $F_{R_A}(u,e) \in [0,1]$ are the membership value, indeterminacy value and non membership value respectively of $u \in U$ for each $e \in E$. If $[(T_{ij}, I_{ij}, F_{ij})] = [T_{ij}(u_i, e_j), I_{ij}(u_i, e_j), F_{ij}(u_i, e_j)]$ we define a matrix

$$[\langle T_{ij}, I_{ij}, F_{ij}\rangle]_{m\times n} = \begin{bmatrix} \langle T_{11}, I_{11}, F_{11}\rangle & \cdots & \langle T_{1n}, I_{1n}, F_{1n}\rangle \\ \langle T_{21}, I_{21}, F_{21}\rangle & \cdots & \langle T_{2n}, I_{2n}, F_{2n}\rangle \\ \vdots & \vdots & \vdots \\ \langle T_{m1}, I_{m1}, F_{m1}\rangle & \cdots & \langle T_{mn}, I_{mn}, F_{mn}\rangle \end{bmatrix}.$$

This is called an $m \times n$ $FNSM$ of the FNSS $(F_A, E)$ over $U$
FNSMs of Type-I

**Definition 2.5. [35]** Let $A = (\langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle), B = \langle (b_{ij}^T, b_{ij}^I, b_{ij}^F \rangle) \in \mathcal{N}_{m\times n}$. The component wise addition and component wise multiplication is defined as

$$A \oplus B = (sup\{a_{ij}^T, b_{ij}^T\},\ sup\{a_{ij}^I, b_{ij}^I\},\ inf\{a_{ij}^F, b_{ij}^F\})$$

$$A \otimes B = (inf\{a_{ij}^T, b_{ij}^T\},\ inf\{a_{ij}^I, b_{ij}^I\},\ sup\{a_{ij}^F, b_{ij}^F\})$$

**Definition 2.6.** Let $A \in \mathcal{N}_{m\times n}, B \in \mathcal{N}_{n\times p}$, the composition of $A$ and $B$ is defined as

$$A^\circ B = \left( \sum_{k=1}^{n} (a_{ik}^T \wedge b_{kj}^T),\ \sum_{k=1}^{n} (a_{ik}^I \wedge b_{kj}^I),\ \prod_{k=1}^{n} (a_{ik}^F \vee b_{kj}^F) \right)$$

$$= \left( \sum_{k=1}^{n} (a_{ik}^T \wedge b_{kj}^T),\ \sum_{k=1}^{n} (a_{ik}^I \wedge b_{kj}^I),\ \prod_{k=1}^{n} (a_{ik}^F \vee b_{kj}^F) \right).$$

equivalently we can write the same as

The product $A^\circ B$ is defined if and only if the number of columns of $A$ is same as the number of rows of $B$. Then $A$ and $B$ are said to be conformable for multiplication. We shall use A B instead of $A^\circ B$. Where $\sum (a_{ik}^T \wedge b_{kj}^T)$ means max-min operation and

$\prod_{k=1}^{n} (a_{ik}^F \vee b_{kj}^F)$ means min-max operation.

### 3.1. Background of the problem

In this section, we discuss about some definition of digraph notion, then FNSV, FNSEVs, Fuzzy Neutrosophic soft eigenspace and Fuzzy Neutrosophic soft eigenvalue.

By a fuzzy Neutrosophic soft algebra $\mathcal{B} = (\mathcal{N}, \oplus, \otimes)$ we understand a bounded linearly ordered set $(\mathcal{N}, \leq)$ with the binary operations of taking the maximum and minimum, denoted by $\oplus, \otimes$.

FNSM operations over $\mathcal{B}$ are defined with respect to $\oplus$ and $\otimes$, formally in the same manner as Fuzzy Neutrosophic soft matrix operations over any field. The least element in $\mathcal{N}$ will be 0 denoted by $\langle 0,0,1 \rangle$ the greatest one by 1 denoted by $\langle 1,1,0 \rangle$. By $N^+$ we denote the set of all positive natural numbers. The greatest common divisor of a set $S \subseteq N^+$ is denoted by gcd S. For a given natural $n \in N^+$, we use the notation $N = \{1, 2, ..., n\}$, and the notation $\mathcal{N}_{(n)}, \mathcal{N}_{(n,n)}$ for the set of all n-dimensional column FNSVs (square FNSM) over $\mathcal{B}$.

Let $\langle x^T, x^I, x^F \rangle = (\langle x_1^T, x_1^I, x_1^F \rangle, ..., \langle x_n^T, x_n^I, x_n^F \rangle) \in \mathcal{N}_{(n)}$ and $\langle y^T, y^I, y^F \rangle = (\langle y_1^T, y_1^I, y_1^F \rangle, ..., \langle y_n^T, y_n^I, y_n^F \rangle) \in \mathcal{N}_{(n)}$ be FNSVs.

We write $\langle x^T, x^I, x^F \rangle \leq \langle y^T, y^I, y^F \rangle (\langle x^T, x^I, x^F \rangle < \langle y^T, y^I, y^F \rangle)$

if $\langle x_i^T, x_i^I, x_i^F \rangle \le \langle y_i^T, y_i^I, y_i^F \rangle (\langle x_i^T, x_i^I, x_i^F \rangle < \langle y_i^T, y_i^I, y_i^F \rangle)$ holds for each $i \in N$ .

If $A \in \mathcal{N}_{(n,n)}$ is a square FNSM and $\pi$ is a permutation on $N$ , then $A_{\pi\pi} \in \mathcal{N}_{(n,n)}$ denotes the result of applying the permutation $\pi$ to the rows and to the columns of the FNSM $A$ . We say that FNSMs A, A' are equivalent if there is a permutation $\pi$ such that $A^{'} = A_{\pi\pi}$, i.e. $\langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle = \langle a^{T'} \pi(i)\pi(j), a_{\pi(i)\pi(j)}^{I'}, a_{\pi(i)\pi(j)}^{F'} \rangle$ for every $i, j \in N$ . A FNSM $A \in \mathcal{N}_{(n,n)}$ is called upper triangular if its entries below the main diagonal and on the main diagonal are equal to 0. For $A \in \mathcal{N}_{(n,n)}, A^s$ stands for the iterated product $A \otimes ... \otimes A$ in which the symbol $A$ appears $s$ times. An ordered pair $G = (N, E)$ is called a **digraph** if $N$ is non-empty (set of nodes) and $E \subseteq N \times N$ (set of arcs). A path in a digraph $G = (N, E)$ is sequence of nodes $p = (i_1, ..., i_k)$ such that $(i_j, i_{j+1}) \in E$ for $j = 1, ..., k-1$. A path is elementary if all its nodes are mutually distinct. It is a cycle if $i_1 = i_k$; its length is $k-1$ and it is denoted by $l(p)$. A digraph $G = (N, E)$ that does not contain any cycle is called **acyclic**. The symbol $G(A) = (N, E)$ stands for a complete, arc-weighted digraph associated with the FNSM $A$. The node set of $G(A)$ is $N$ , and the capacity of any arc $(i, j)$ is $\langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle$ .

The capacity $c(p)$ of a path $p = (i_1, ..., i_k)$ in the digraph $G(A) = (N, E)$ is equal to

$$c(p) = \bigotimes_{j=1}^{k-1} \langle a_{i_j i_{j+1}}^T, a_{i_j i_{j+1}}^I, a_{i_j i_{j+1}}^F \rangle .$$

By a strongly connected component $\mathcal{K}$ of $G(A) = (N, E)$ we means a sub graph $\mathcal{K}$ generated by a non-empty subset $K \subseteq N$ such that any two distinct nodes $u, v \in K$ are contained in a common cycle and $K$ is a maximal subset with this property. $A$ strongly connected component $\mathcal{K}$ of a digraph is called non-trivial if there is a cycle of positive length in $\mathcal{K}$. For any strongly connected component $\mathcal{K}$, the period per $\mathcal{K}$ is defined as the gcd of the lengths of all cycles in $\mathcal{K}$. By $\mathcal{K}(G)$ we denote the set of all strongly connected components of $G$ . There is well-known connection between the entries in powers of FNSMs and paths in associated digraphs: the $(i, j)^{th}$ entry $\langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle^k$ in $A^k$ is equal to the maximum capacity of a path from $\mathcal{P}_{G(A)}^k(i, j)$, where $\mathcal{P}_{G(A)}^k(i, j)$ is the set of all paths in $G(A)$ of length $k$ beginning at node $i$ and ending at node $j$ . If $\mathcal{P}_{G(A)}(i, j)$ denotes the set of all paths from $i$ to $j$ , then

$$\langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle^* = \bigotimes_{k \ge 1} \langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle^k$$

is the maximum capacity of a path from $\mathcal{P}_{G(A)}(i, j)$ and $\langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle^*$ is the maximum capacity of a cycle containing node $j$ . For given $A \in \mathcal{N}_{(n,n)}, h \in \mathcal{N}$ , the threshold digraph $G(A, h)$ is the digraph $G(A) = (N, E)$, with the node set N and with the arc set

$E=\{(i,j);i,j\in N,\langle a_{ij}^T,a_{ij}^I,a_{ij}^F\rangle\geq\langle h^T,h^I,h^F\rangle\}$. For a given the FNSM $A\in\mathcal{N}_{(n,n)},\lambda\in\mathcal{N}$ and an n-tuple $\langle x^T,x^I,x^F\rangle\in\mathcal{N}_{(n)}$ are called an FNSE value of $A$ and an FNSEv of $A$, respectively, if $A\otimes\langle x^T,x^I,x^F\rangle=\lambda\otimes\langle x^T,x^I,x^F\rangle$.
in that case, $\lambda$ is the associated FNSE value. Note that some authors require the FNSEvs to be non-zero. In the case the results presented would be a little bit different. The structure of the set of all FNSEvs of a FNSM with elements of B has not been completely described.

Let $A\in\mathcal{N}_{(n,n)}$ and $\lambda\in\mathcal{N}$, that each $\lambda\in\mathcal{N}$ is an FNSE value of a given FNSM $A$ and the sequence $e_\lambda,A\otimes e_\lambda,A^2\otimes e_\lambda,...,$ where $e_\lambda=(\lambda,...,\lambda)^t$, converges to an FNSEv with associated FNSE value $\lambda$ in at most n steps. The eigenspace $V(A,\lambda)$ is defined as the set of all FNSEv of $A$ with associated FNSE value $\lambda$, i.e.
$V(A,\lambda)=\{\langle x^T,x^I,x^F\rangle\in\mathcal{N}_{(n)};A\otimes\langle x^T,x^I,x^F\rangle=\lambda\otimes\langle x^T,x^I,x^F\rangle\}$.

The next assertion describes the relationships among the eigenspaces of the powers of $A$.

**Lemma 3.1.1.** Let $A\in\mathcal{N}_{(n,n)}$ and $\lambda\in\mathcal{N}$. Then

1. $V(A,\lambda)\subseteq V(A^2,\lambda)\subseteq...\subseteq V(A^n,\lambda)\subseteq...$
2. If $\lambda>\max\limits_{i,j\in N}\langle a_{ij}^T,a_{ij}^I,a_{ij}^F\rangle$, then $V(A,\lambda)=V(A,I)$.

**Proof:**

1. Suppose $\langle x^T,x^I,x^F\rangle\in V(A,\lambda)$. Then we have $A^2\otimes\langle x^T,x^I,x^F\rangle$ $=A\otimes(A\otimes\langle x^T,x^I,x^F\rangle)=A\otimes(\lambda\otimes\langle x^T,x^I,x^F\rangle)=\lambda\otimes(A\otimes\langle x^T,x^I,x^F\rangle)$ because of the distributivity of $\otimes$ with respect to $\oplus$ and of the idempotency of $\otimes$.
2. Note that for $k\in N$ we have $(A\otimes\langle x^T,x^I,x^F\rangle)_k\leq\max\limits_{i,j\in N}\langle a_{ij}^T,a_{ij}^I,a_{ij}^F\rangle$. Now, let us suppose that an FNSE value $\lambda$ satisfies $\lambda>\max\limits_{i,j\in N}\langle a_{ij}^T,a_{ij}^I,a_{ij}^F\rangle$ and let $\langle x^T,x^I,x^F\rangle$ be a corresponding FNSEv of A; then we have $A\otimes\langle x^T,x^I,x^F\rangle=\lambda\otimes\langle x^T,x^I,x^F\rangle$ $=\langle x^T,x^I,x^F\rangle$.

**Lemma 3.1.2.** Let $A\in\mathcal{N}_{(n,n)}$ and $\lambda\in\mathcal{N}$. If $\langle x^T,x^I,x^F\rangle\in(V,I)$, then $\lambda\otimes\langle x^T,x^I,x^F\rangle\in V(A,\lambda)$.

### 3.2. $\lambda$-robust fuzzy neutrosophic soft matrices

In this section we shows that $\lambda$-robust fuzzy neutrosophic soft matrices with some examples.
Let $A=(\langle a_{ij}^T,a_{ij}^I,a_{ij}^F\rangle)\in\mathcal{N}_{(n,n)},\lambda\in\mathcal{N}$ and $\langle x^T,x^I,x^F\rangle\in\mathcal{N}_{(n)}$.
The orbit $O(A,\langle x^T,x^I,x^F\rangle)$ of a FNSM A with starting FNSV $\langle x^T,x^I,x^F\rangle=\langle x^T,x^I,x^F\rangle^{(0)}$ is the sequence

$\langle x^T, x^I, x^F\rangle^{(0)}, \langle x^T, x^I, x^F\rangle^{(1)}, \langle x^T, x^I, x^F\rangle^{(2)}, ..., \langle x^T, x^I, x^F\rangle^{(n)}, ...,$
where $\langle x^T, x^I, x^F\rangle^{(r)} = A^r \otimes x$ for $r = 1, 2, ....$ Let
$T(A,\lambda) = \{\langle x^T, x^I, x^F\rangle \in \mathcal{N}_{(n)}; O(A, \langle x^T, x^I, x^F\rangle) \cap V(A,\lambda) \neq \phi\}.$
it follows from the definitions of $V(A,\lambda)$ and $T(A,\lambda)$ that $\langle x^T, x^I, x^F\rangle \in V(A,\lambda)$ implies $A \otimes \langle x^T, x^I, x^F\rangle \in V(A,\lambda)$ and $V(A,\lambda) \subseteq T(A,\lambda) \subseteq \mathcal{N}_{(n)}$ is fulfilled for every FNSM $A \in \mathcal{N}_{(n,n)}$ and $\lambda \in \mathcal{N}$. It may be happen that $T(A,\lambda) = V(A,\lambda)$. For instance, consider $\mathcal{N} = [0,1] \subset R, \lambda = 1$ and the FNSM $A \in \mathcal{N}_{(2,2)}$ in the following form:

$$A = \begin{pmatrix} \langle 0\ 0\ 1\rangle & \langle 1\ 1\ 0\rangle \\ \langle 1\ 1\ 0\rangle & \langle 0\ 0\ 1\rangle \end{pmatrix},$$

then

$$\begin{pmatrix} \langle 0\ 0\ 1\rangle & \langle 1\ 1\ 0\rangle \\ \langle 1\ 1\ 0\rangle & \langle 0\ 0\ 1\rangle \end{pmatrix} \otimes \begin{pmatrix} \langle a^T\ a^I\ a^F\rangle \\ \langle b^T\ b^I\ b^F\rangle \end{pmatrix} = 1 \otimes$$

$$\begin{pmatrix} \langle a^T\ a^I\ a^F\rangle \\ \langle b^T\ b^I\ b^F\rangle \end{pmatrix} \Leftrightarrow \langle a^T\ a^I\ a^F\rangle = \langle b^T\ b^I\ b^F\rangle$$

and we have

$V(A,1) = \{\alpha \otimes (\langle 1,1,0\rangle\ \langle 1,1,0\rangle)^t; \alpha \in [0,1]\}.$

Since

$$A \otimes (A \otimes (\langle a^T\ a^I\ a^F\rangle\ \langle b^T\ b^I\ b^F\rangle)^t) = A \otimes (\langle a^T\ a^I\ a^F\rangle\ \langle b^T\ b^I\ b^F\rangle)^t \Leftrightarrow$$

$$\begin{pmatrix} \langle 1\ 1\ 0\rangle & \langle 0\ 0\ 1\rangle \\ \langle 0\ 0\ 1\rangle & \langle 1\ 1\ 0\rangle \end{pmatrix} \otimes \begin{pmatrix} \langle a^T\ a^I\ a^F\rangle \\ \langle b^T\ b^I\ b^F\rangle \end{pmatrix} = \begin{pmatrix} \langle 0\ 0\ 1\rangle & \langle 1\ 1\ 0\rangle \\ \langle 1\ 1\ 0\rangle & \langle 0\ 0\ 1\rangle \end{pmatrix} \otimes \begin{pmatrix} \langle a^T\ a^I\ a^F\rangle \\ \langle b^T\ b^I\ b^F\rangle \end{pmatrix}$$

$$we obtain $A \otimes (\langle a^T\ a^I\ a^F\rangle\ \langle b^T\ b^I\ b^F\rangle)^t \in V(A,1)$
if and only if $\langle a^T\ a^I\ a^F\rangle = \langle b^T\ b^I\ b^F\rangle$ that is $A \otimes \langle x^T, x^I, x^F\rangle \in V(A,1)$ if and only if $\langle x^T, x^I, x^F\rangle \in V(A,1)$. Hence $T(A,1) = V(A,1)$.
The result that we shall now formulate for a fuzzy algebra, as well as the method used to prove it, where the max-plus algebra was considered.

**Lemma 3.2.1.** Let $A = \langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle \in \mathcal{N}_{(n,n)}, \lambda \in \mathcal{N}$. Then $T(A,\lambda) = V(A,\lambda)$ if and only if for every $x \in \mathcal{N}_{(n)} : A \otimes \langle x^T, x^I, x^F\rangle \in V(A,\lambda) \Leftrightarrow \langle x^T, x^I, x^F\rangle \in V(A,\lambda).$

**Proof:** Let us notice first that $\langle x^T, x^I, x^F\rangle \in V(A,\lambda) \Rightarrow A \otimes \langle x^T, x^I, x^F\rangle \in V(A,\lambda)$ and $V(A,\lambda) \subseteq T(A,\lambda)$ holds true for every FNSM A and every $\lambda$. Suppose now that $V(A,\lambda) = T(A,\lambda)$ and $A \otimes \langle x^T, x^I, x^F\rangle \in V(A,\lambda)$. Then $\langle x^T, x^I, x^F\rangle \in T(A,\lambda)$ and hence $\langle x^T, x^I, x^F\rangle \in V(A,\lambda)$. For the converse implication, let us assume that $A \otimes \langle x^T, x^I, x^F\rangle \in V(A,\lambda) \Rightarrow \langle x^T, x^I, x^F\rangle \in V(A,\lambda)$ holds for every $\langle x^T, x^I, x^F\rangle$

$\in \mathcal{N}_{(n)}$ and $\langle x^T, x^I, x^F\rangle \in T(A,\lambda)$. Then $A^k \otimes \langle x^T, x^I, x^F\rangle \in V(A,\lambda)$ for some $k$ implies
$A^k \otimes \langle x^T, x^I, x^F\rangle \in V(A,\lambda), A^{k-1} \otimes \langle x^T, x^I, x^F\rangle \in V(A,\lambda), ..., \langle x^T, x^I, x^F\rangle \in V(A,\lambda).$
In general, $T(A,\lambda) \neq V(A,\lambda)$ and $T(A,\lambda) \neq \mathcal{N}_{(n)}$. Let us consider $\mathcal{N} = [0,1], \lambda = 1$ and the FNSM.

$$A = \begin{pmatrix} \langle 0.1\ 0.2\ 0.4\rangle & \langle 0.1\ 0.2\ 0.4\rangle & \langle 0.1\ 0.2\ 0.4\rangle \\ \langle 0.1\ 0.2\ 0.4\rangle & \langle 0.1\ 0.2\ 0.4\rangle & \langle 0.2\ 0.3\ 0.3\rangle \\ \langle 0.1\ 0.2\ 0.4\rangle & \langle 0.2\ 0.3\ 0.3\rangle & \langle 0.1\ 0.2\ 0.4\rangle \end{pmatrix}.$$

FNSV $\langle x^T, x^I, x^F\rangle = (\langle 0.5\ 0.6\ 0.1\rangle\ \langle 0.5\ 0.6\ 0.1\rangle\ \langle 0.5\ 0.6\ 0.1\rangle)^t$ does not belong to $V(A,1)$ but
$A \otimes \langle x^T, x^I, x^F\rangle = (\langle 0.1\ 0.2\ 0.4\rangle\ \langle 0.2\ 0.3\ 0.3\rangle\ \langle 0.2\ 0.3\ 0.3\rangle)^t \in V(A,1)$ which means that $T(A,1) \neq V(A,1).$
Moreover, if $\langle y^T, y^I, y^F\rangle = (\langle 0.1\ 0.2\ 0.4\rangle\ \langle 0.1\ 0.2\ 0.4\rangle\ \langle 0.2\ 0.3\ 0.3\rangle)^t$ then $A^k \otimes \langle y^T, y^I, y^F\rangle$ is $\langle y^T, y^I, y^F\rangle$ for $k$ even and
$(\langle 0.1\ 0.2\ 0.4\rangle\ \langle 0.2\ 0.3\ 0.3\rangle\ \langle 0.1\ 0.2\ 0.4\rangle)^t$ for k odd showing that
$\langle y^T, y^I, y^F\rangle \notin T(A,1)$

**Definition 3.2.2.** Let $A = (\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle) \in \mathcal{N}_{(n,n)}, \lambda \in \mathcal{N}$. A FNSM A is called $\lambda$-robust if $T(A,\lambda) = \mathcal{N}_{(n)}$. Let us call the $\lambda$-robust FNSM with $\lambda = 1$ the robust FNSM.
Let a FNSM $A = (\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle) \in \mathcal{N}_{(n,n)}$. The FNSM A is ultimately periodic if there is a natural number $p$ such that the following holds for some $\lambda \in \mathcal{N}$ and natural number R:
$A^{k+p} = \lambda \otimes A^k$ for all $k \geq R$ if $p$ is the minimal natural number with this property then we call $p$ the period of $A$, denoted by $per(A,\lambda).$ The least $R$ with this property is called the defect of A denoted by $def(A,\lambda).$

Let us call a FNSM A with $per(A,I) = 1$ the stationary FNSM. By the linearity of B, any element of any power of the FNSM A is equal to some element of A. Therefore, the sequence of powers of A contains only finitely many different FNSMs with entries of A. It is easy to see that if $A = (\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle)$ is ultimately periodic, then the inequality $\lambda \geq \max_{i,j\in N}(\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle^k)$ holds for all $k \geq def(A,\lambda),$ i.e. if the equality $A^{k+p} = \lambda \otimes A^k$ is fulfilled for all $k \geq def(A,\lambda),$ then all elements of $A^{k+p}$ have to be less than or equal to $\lambda.$ Moreover, $\lambda \geq \max_{i,j\in N}\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle$ implies $\lambda \geq \max_{i,j\in N}\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle^k$ and $A^{k+p} = \lambda \otimes A^k = A^k.$

These relations allow us to formulate the next assertion for $\lambda \geq \max_{i,j\in N}\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle$ using the known results on the ultimately periodic FNSMs.

**Lemma 3.2.3.** $A = (\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle) \in \mathcal{N}_{(n,n)}, \lambda \geq \max_{i,j\in N}\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle$. Then $A$ is $\lambda$-robust if and only if $per(A,\lambda) = 1$.

**Proof:** First let us suppose that $per(A,\lambda) = 1, \langle x^T, x^I, x^F\rangle \in \mathcal{N}_{(n)}$ is an arbitrary FNSV and $r \geq R$. Then $A^{r+1} \otimes \langle x^T, x^I, x^F\rangle = \lambda \otimes (A^r \otimes \langle x^T, x^I, x^F\rangle)$ implies $A^r \otimes \langle x^T, x^I, x^F\rangle \in V(A,\lambda)$. Hence A is $\lambda$-robust. For the converse implication let us suppose that $A$ is a $\lambda$-robust FNSM and $x$ is an arbitrary element of $\mathcal{N}_{(n)}$. Then there exists a natural number $r$ such that $A^r \otimes \langle x^T, x^I, x^F\rangle \in V(A,\lambda)$. In particular, for $\langle x^T, x^I, x^F\rangle = A_j$ (the $j^{th}$ column of A) there exists a natural number $r_j$ such that $A^{r_{j+1}} \otimes A_j = \lambda \otimes (A^{rj} \otimes A_j) = (\lambda \otimes A^{rj}) \otimes A_j$. For $R = \max_{j\in N} r_j$ we, obtain $A^{r+1} = \lambda \otimes A^r$ for all $r \geq R$.

**Theorem 3.2.4.** Let $A = \langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle \in \mathcal{N}_{(n,n)}, \lambda \geq \max_{i,j\in N}\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle$. Then there exists an algorithm $\mathcal{A}$ which for a given FNSM $A \in \mathcal{N}_{(n,n)}$ and $\lambda \in \mathcal{N}$ decides whether $A$ is a $\lambda$-robust FNSM in $O(n^3)$ time.

**Proof:** Let us suppose that $A = (\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle) \in \mathcal{N}_{(n,n)}$ and $\lambda \geq \max_{i,j\in N}\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle$ holds true. Then we get that the inequality $\lambda \geq \max_{i,j\in N}\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle$ is fulfilled for each natural number $k$; hence $\lambda \otimes A^k = A^k$ and $per(A,\lambda) = per(A,I)$. Now, it suffices to use the known $O(n^3)$ algorithm for computing $per(A,I)$ and the assertion follows.

**Theorem 3.2.5.** Let $A = \langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle \in \mathcal{N}_{(n,n)}, \lambda \geq \max_{i,j\in N}\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle$. Then A is a $\lambda$-robust FNSM if and only if $V(A,\lambda) = V(A^l,\lambda)$ for each $l \in N^+$.

**Proof:** Let us assume that $A$ is a $\lambda$-robust FNSM, i.e. $per(A,\lambda) = 1$ by Lemma 4.3 and $\lambda \geq \max_{i,j\in N}\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle$. Then there is a natural number $R$ such that for all $k \geq R$ we have $A^{k+1} = \lambda \otimes A^k = A^k$. We shall prove that the set $V(A^l,\lambda)$ is a subset of the set $V(A^{l-1},\lambda)$ for $l = 2,\ldots,R$.

Let $x$ be an arbitrary element of $V(A^l,\lambda)$ and $ls > R$ for some natural number $s$. Then we have

$A^{ls} \otimes \langle x^T, x^I, x^F \rangle = A^{ls-l} \otimes (A^l \otimes \langle x^T, x^I, x^F \rangle) = A^{ls-l}$
$\otimes(\lambda \otimes \langle x^T, x^I, x^F \rangle) = \lambda \otimes (A^{ls-l} \otimes \langle x^T, x^I, x^F \rangle) = \lambda$
$\otimes(A^{ls-2l} \otimes (A^{ls-} \otimes \langle x^T, x^I, x^F \rangle) = ... = \lambda \otimes (A^l \otimes \langle x^T, x^I, x^F \rangle) = \lambda$
$\otimes(\lambda \otimes \langle x^T, x^I, x^F \rangle) = \lambda \otimes \langle x^T, x^I, x^F \rangle$
and
$A^{ls} \otimes \langle x^T, x^I, x^F \rangle = A^{ls-1} \otimes \langle x^T, x^I, x^F \rangle = A^{ls-l-1} \otimes (A^l \otimes \langle x^T, x^I, x^F \rangle) = A^{ls-l-1}$
$\otimes(\lambda \otimes \langle x^T, x^I, x^F \rangle) = \lambda \otimes (A^{ls-l-1} \otimes \langle x^T, x^I, x^F \rangle) = \lambda$
$\otimes A^{ls-2l-1} \otimes (A^l \otimes \langle x^T, x^I, x^F \rangle) = ... = \lambda \otimes (A^{l-1}$
$\otimes \langle x^T, x^I, x^F \rangle) = (\lambda \otimes A^{l-1}) \otimes \langle x^T, x^I, x^F \rangle = A^{l-1} \otimes \langle x^T, x^I, x^F \rangle$
because of the idempotency of $\otimes$ and $\lambda \geq \max_{i,j \in N} \langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle$. From the above equalities we obtain $A^{l-1} \otimes \langle x^T, x^I, x^F \rangle = \lambda \otimes \langle x^T, x^I, x^F \rangle$ and moreover we get $V(A^l, \lambda) \subseteq V(A^{l-1}, \lambda)$. Now, the implication is a consequence of Lemma 4.3.

For a converse implication let us suppose that $V(A, \lambda) = V(A^l, \lambda)$ holds for each $l \in N^+$ and $per(A, \lambda) = p \geq 1$, i.e. there is a natural number $R$ such that for all $K \geq R$ we have $A^{k+p} = \lambda \otimes A^k = A^k$. Without loss of generality we can assume the case where $R \geq p$ (in the opposite case we put $R := R + p$). From number theory it is well known that for every $R, p \in N^+, R \geq p$, there exists a natural number $s \in \{0, 1, ..., p-1\}$ such that $p$ divides $R+s$, i.e. $R + s = vp$ for some $v \in N^+$. Then for the power of the FNSM $A^{R+s}$ we have
$(A^{R+s})^2 = (A^{vp})^2 = A^{2vp} = A^{vp+vp} = A^{R+s+vp} = A^{R+s}$
because of the periodicity of the FNSM A. As the FNSM $A^{R+s}$ is idempotent, each of its columns is an eigenvector, or equivalently,
$A^{R+s} \otimes A_j^{R+s} = A_j^{R+s} = \lambda \otimes A_j^{R+s}$ for every $j \in N$,
where $A_j^{R+s}$ is the jth column of $A^{R+s}$. In view of the assumption that $V(A^{R+s}, \lambda) = V(A^l, \lambda)$ for each $l \in N^+$, we get $A_j^{R+s} \in V(A^l, \lambda)$. Then the equality $A^l \otimes A_j^{R+s} = A_j^{R+s}$ implies $A^{R+s+l} = A^{R+s}$ for each $l \in N^+$ and hence $per(A, \lambda) = p = 1$
.

### 3.3. Strongly $\lambda$-robust FNSMs

In this section we discuss about the strongly $\lambda$-robust fuzzy neutrosophic soft matrices and Algorithm of strong robust.

Let a FNSM $A = \langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle \in \mathcal{N}_{(n,n)}$ and $\lambda \in \mathcal{N}$. Let us suppose that $\langle x^T, x^I, x^F \rangle, \langle y^T, y^I, y^F \rangle \in V(A, \lambda)$. Then $A \otimes \langle x^T, x^I, x^F \rangle = \lambda \otimes \langle x^T, x^I, x^F \rangle$ and

$A \otimes \langle y^T, y^I, y^F \rangle = \lambda \otimes \langle y^T, y^I, y^F \rangle$ imply $A \otimes (\langle x^T, x^I, x^F \rangle \oplus \langle y^T, y^I, y^F \rangle)$ $= (A \otimes \langle x^T, x^I, x^F \rangle) \oplus (A \otimes \langle y^T, y^I, y^F \rangle) =$ $(\lambda \otimes \langle x^T, x^I, x^F \rangle) \oplus (\lambda \otimes \langle y^T, y^I, y^F \rangle) = \lambda \otimes (\langle x^T, x^I, x^F \rangle \oplus \langle y^T, y^I, y^F \rangle)$ using the distributivity of $\otimes$ with respect to $\oplus$. Let a FNSM $A = \langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle \in \mathcal{N}_{(n,n)}$ and $\lambda \in \mathcal{N}$ .Let us define the greatest $\lambda$-FNSEv $\langle x^T, x^I, x^F \rangle^*(A, \lambda)$ corresponding to a FNSM $A$ and $\lambda$ $\langle x^T, x^I, x^F \rangle^*(A, \lambda) = \bigoplus_{\langle x^T, x^I, x^F \rangle \in V(A,\lambda)} \langle x^T, x^I, x^F \rangle$.

That for given eigenvalue $\lambda$ of $A$ the greatest $\lambda$-FNSEv exists, and the greatest I-FNSEv $\langle x^T, x^I, x^F \rangle^*(A, I)$ exists for every FNSM A and its entries are given by the formula $\langle x^T, x^I, x^F \rangle_i^*(A, I) = \bigoplus_j \langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle^* \otimes \langle a_{jj}^T, a_{jj}^I, a_{ij}^F \rangle^*$. The greatest I-FNSEv

$$\langle x^T, x^I, x^F \rangle^*(A, I)$$

can be computed by the following iterative procedure. Let us define $\langle x^T, x^I, x^F \rangle_i^{(1)}(A)$ $= \bigoplus_{j \in N} \langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle$ for each $i \in N$ and $\langle x^T, x^I, x^F \rangle^{k+1}(A) = A \otimes \langle x^T, x^I, x^F \rangle^k(A)$ for all $k \in \{1, 2, ...,\}$.

Then $\langle x^T, x^I, x^F \rangle^{k+1}(A) \le \langle x^T, x^I, x^F \rangle^k(A)$ and $\langle x^T, x^I, x^F \rangle^*(A, I) = \langle x^T, x^I, x^F \rangle^n(A)$

For every FNSM $A \in \mathcal{N}_{(n,n)}$ define $c(A) = \bigotimes_{i \in N} \bigoplus_{j \in N} \langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle$

and $c^*(A) = (c(A), ..., c(A))^t \in \mathcal{N}_{(n)}$.

**Theorem 3.3.1.** Let $A \in \mathcal{N}_{(n,n)}$ and $\lambda = 1$. Every constant vector $x = (\alpha, ..., \alpha)^t$ with $\alpha \le c(A)$ is an FNSEv of $A$, and no constant vector with entries $\alpha > c(A)$ is FNSEv of $A$.

**Example 5.2:**

$$A = \begin{pmatrix} \langle 0.3\ 0.4\ 0.5 \rangle & \langle 0.1\ 0.2\ 0.7 \rangle & \langle 0.8\ 0.9\ 0.1 \rangle \\ \langle 0.2\ 0.3\ 0.6 \rangle & \langle 0.1\ 0.2\ 0.7 \rangle & \langle 0.7\ 0.8\ 0.2 \rangle \\ \langle 0.2\ 0.3\ 0.6 \rangle & \langle 0.3\ 0.4\ 0.5 \rangle & \langle 0.8\ 0.9\ 0.1 \rangle \end{pmatrix}.$$

The greatest I-eigenvector is equal to $(\langle 0.8\ 0.9\ 0.1 \rangle, \langle 0.7\ 0.8\ 0.2 \rangle, \langle 0.8\ 0.9\ 0.1 \rangle)^t$ and it is easy to see that $\langle x^T, x^I, x^F \rangle^*(A, \langle 0.4\ 0.5\ 0.2 \rangle) = (\langle 1\ \ 1\ \ 0 \rangle, \langle 1\ \ 1\ \ 0 \rangle,$ $\langle 0.4\ 0.5\ 0.2 \rangle)^t$.

**Lemma 3.3.2.** Let $A \in \mathcal{N}_{(n,n)}$ and $\lambda \ge \max_{i,j \in N} \langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle$. Then $\langle x^T, x^I, x^F \rangle^*(A, \lambda) = \langle x^T, x^I, x^F \rangle^*(A, I)$.

**Proof:** The assertion follows from Lemma 3.1.

Define $c^{+}(A)=\min_{i,j\in N}\{\langle a_{ij}^{T},a_{ij}^{I},a_{ij}^{F}\rangle;\langle a_{ij}^{T},a_{ij}^{I},a_{ij}^{F}\rangle>c(A)\}$. We assume that the minimum of the empty set is equal to I.

**Theorem 3.3.3.** $A\in\mathcal{N}_{(n,n)}$ and $\lambda\in\mathcal{N},\lambda>c(A)$. Then $\langle x^{T},x^{I},x^{F}\rangle^{*}(A,\lambda)=c^{*}(A)$ if and only if $G(A,c^{+}(A))$ is an acyclic digraph.

**Proof:** Let us suppose that $\langle x^{T},x^{I},x^{F}\rangle_{i}^{*}(A,\lambda)=c(A)$ hold for all $i\in N$ and $G(A,c^{+}(A))$ contains a cycle $\sigma=(i_{1},i_{2},...,i_{k})$, i.e. the capacity $c(\sigma)$ of the cycle is greater than $c(A)$. By the definition of the greater $\lambda$-FNSEv

$\langle x^{T},x^{I},x^{F}\rangle_{i}^{*}(A,\lambda)=(\bigoplus_{\langle x^{T},x^{I},x^{F}\rangle\in V(A,\lambda)}\langle x^{T},x^{I},x^{F}\rangle)_{i}=c(A)$ we have

$\langle x^{T},x^{I},x^{F}\rangle_{i}\le c(A)$ for every $\langle x^{T},x^{I},x^{F}\rangle\in V(A,\lambda)$. Hence, for

$\langle x^{T},x^{I},x^{F}\rangle=\lambda\otimes\langle x^{T},x^{I},x^{F}\rangle^{*}(A,I)\in V(A,\lambda)$ we have

$\langle x^{T},x^{I},x^{F}\rangle_{i}=\lambda\otimes\bigoplus_{j}(\langle a_{jj}^{T},a_{jj}^{I},a_{jj}^{F}\rangle^{+}\otimes\langle a_{ij}^{T},a_{ij}^{I},a_{ij}^{F}\rangle^{*})>c(A)$ for every $i\in\{i_{1},...,i_{k}\}$,

a contradiction with the maximality of $\langle x^{T},x^{I},x^{F}\rangle_{i}^{*}(A,\lambda)$.

Conversely, if $G(A,c^{+}(A))$ is acyclic, $c^{*}(A)\in V(A,\lambda)$ is implied directly by Theorem 5.1. Inequality $\langle x^{T},x^{I},x^{F}\rangle_{i}\le c(A)$ for each $\langle x^{T},x^{I},x^{F}\rangle\in V(A,\lambda)$ and each $i$, can be shown as follows. Suppose that there exists a FNSV $\langle x^{T},x^{I},x^{F}\rangle\in V(A,\lambda)$ and each $i$ such that $\langle x^{T},x^{I},x^{F}\rangle_{i}\ge c^{+}(A)$.

Then, since $\bigoplus_{j}(\langle a_{ij}^{T},a_{ij}^{I},a_{ij}^{F}\rangle\otimes\langle x_{j}^{T},x_{j}^{I},x_{j}^{F}\rangle)=\lambda\otimes\langle x^{T},x^{I},x^{F}\rangle_{i}$, there must exist an index $j$ such that $\langle a_{ij}^{T},a_{ij}^{I},a_{ij}^{F}\rangle\ge c^{+}(A)$ and $\langle x_{j}^{T},x_{j}^{I},x_{j}^{F}\rangle\ge c^{+}(A)$. Let us denote $i$ by $i_{1}$ and $j$ by $i_{2}$ and repeat this argument for $i=i_{2}$. After a finite number of steps a repetition in the sequence $i_{1},i_{2},...$ must occur, that defines a cycle in $G(A,c^{+}(A))$, a contradiction.

**Lemma 3.3.4.** Let $A\in\mathcal{N}_{(n,n)},\langle x^{T},x^{I},x^{F}\rangle\in\mathcal{N}_{(n)},h\in\mathcal{N},k\in N^{+},i,j\in N.$ Then

(i). $\langle a_{ij}^{T},a_{ij}^{I},a_{ij}^{F}\rangle^{k}\ge h\Leftrightarrow\mathcal{P}_{G(A,h)}^{k}(i,j)\ne\phi$.

(ii). $\langle x^{T},x^{I},x^{F}\rangle_{i}^{(k)}\ge h\Leftrightarrow(\exists j\in N)(\langle x^{T},x^{I},x^{F}\rangle_{j}\ge h\wedge\mathcal{P}_{G(A,h)}^{k}(i,j)\ne\phi)$.

Let us denote the vector $e_{i}=(e_{1i},...,e_{ni})^{T}\in\mathcal{N}_{(n)}$ as follows:

$$\langle e_{ij}^{T},e_{ij}^{I},e_{ij}^{F}\rangle=\begin{cases}I, & for\, i=j\\ 0, & otherwise\end{cases}$$

**Lemma 3.3.5.** Let $A \in \mathcal{N}_{(n,n)}$. Then the following conditions are equivalent:

(i) $(\forall i \in N)(\exists r_i \in N^+)(\forall k \geq r_i)(e_i^k \geq c^*(A)).$

(ii) $G(A, c(A))$ is strongly connected with period equal to 1.

**Proof:** (ii) $\rightarrow$ (i). Let us suppose that $G(A, c(A))$ is strongly connected with period equal to 1, i.e. $G(A, c(A))$ contains cycles $c_1, ..., c_m$ with lengths $l_1, ..., l_m$ such that $gcd(l_1, ..., l_m) = 1$. By the known facts from number theory, there exists a natural number $k_0$ such that each integer $k \geq k_0$ can be expressed as a linear combination of $l_1, ..., l_m$, with non-negative coefficients. Therefore, for arbitrary but fixed $i \in N$ and each $j \in N$ there exists a number $r_{ij} \in N^+$ such that for each $k \geq r_{ij}$ there exists a path $p \in G(A, c(A))$ from $j$ to $i$ of length $k$ containing cycles $c_1, ..., c_m$ (each of them is used a suitable number of times). By Lemma 5.5(ii) the assertion follows for $r_i = \max_{j \in N} r_{ji}$

(i) $\rightarrow$ (ii) To prove the converse implication, let us assume that for each $i \in N$ there is $r_i \in N^+$ such that the inequality $e_i^k \geq c^*(A)$ is fulfilled for all $k \geq r_i$. Now take an arbitrary index $s \in N$ and apply Lemma 5.5(ii) for the vector $e_s^{(k)}$. The equivalence

$$e_{is}^{(k)} \geq c(A) \Leftrightarrow (\exists j \in N,\ j = s \text{ in the case}) (e_{ss}^{(k)} \geq c(A)) \wedge \mathcal{P}^k_{G(A,c(A))}(i, s) \neq \phi)$$

holds for each $i \in N$ and each $s \in N$ and it implies the strong connectivity of $G(A, c(A))$. In contrast, suppose that the gcd of all cycle lengths in $G(A, c(A))$ is $l > 1$. Then for $e_{ii}^{(k)} \geq c(A)$ there exists a cycle c from i to i in $G(A, c(A))$ of length $k \geq r_i$ and $l$ divides k. It is easily seen that $e_{ii}^{(k)} \geq c(A)$ only for $k$ a multiple of $l$ which is a contradiction.

From now, we will suppose that $\lambda > c(A)$.

Let $T^*(A, \lambda) = \{\langle x^T, x^I, x^F \rangle \in \mathcal{N}_{(n)}; \langle x^T, x^I, x^F \rangle^*(A, \lambda) \in O(A, \langle x^T, x^I, x^F \rangle)\}$.

The set $T^*(A, \lambda)$ allows us to describe FNSMs for which the greatest $\lambda$-FNSEv is reached with any start FNSV. It is easily seen that $\langle x^T, x^I, x^F \rangle^*(A, \lambda) \geq c^*(A)$ holds and $\langle x^T, x^I, x^F \rangle^*(A, \lambda)$ cannot be reached with a FNSV

$$\langle x^T, x^I, x^F \rangle \in \mathcal{N}_{(n)}, \langle x^T, x^I, x^F \rangle < c^*(A).$$

Let us denote the following set by $M(A) = \{\langle x^T, x^I, x^F \rangle \in \mathcal{N}_{(n)}; \langle x^T, x^I, x^F \rangle < c^*(A)\}$. It follows from the definitions of $T^*(A, \lambda)$ and $M(A)$ that $T^*(A, \lambda) \subseteq \mathcal{N}_{(n)} / M(A)$ is fulfilled for every FNSM $A \in \mathcal{N}_{(n,n)}$. It may happen that $T^*(A, \lambda) \neq \mathcal{N}_{(n)} / M(A)$. For instance consider $\mathcal{N} = [0,1] \subset R, \lambda = 1$ and the FNSM $A \in \mathcal{N}_{(3,3)}$ in the following from:

$$A = \begin{pmatrix} \langle 0\ \ 0\ \ 1\rangle & \langle 0\ \ 0\ \ 1\rangle & \langle 0.8\ 0.9\ 0.1\rangle \\ \langle 0\ \ 0\ \ 1\rangle & \langle 0\ \ 0\ \ 1\rangle & \langle 0.7\ 0.8\ 0.2\rangle \\ \langle 0\ \ 0\ \ 1\rangle & \langle 0.5\ 0.6\ 0.3\rangle & \langle 0\ \ 0\ \ 1\rangle \end{pmatrix}.$$

FNSV $\langle x^T, x^I, x^F\rangle = (\langle 0.5\ 0.6\ 0.3\rangle, \langle 0\ \ 0\ \ 1\rangle, \langle 0\ \ 0\ \ 1\rangle)^t$ does not belong to $T^*(A,1)$ since $A^k \otimes \langle x^T, x^I, x^F\rangle = (\langle 0\ \ 0\ \ 1\rangle, \langle 0\ \ 0\ \ 1\rangle, \langle 0\ \ 0\ \ 1\rangle)^t$ for each $k$, showing that $x^T, x^I, x^F \notin T^*(A,1)$ but $\langle x^T, x^I, x^F\rangle \in \mathcal{N}_{(n)} / M(A)$ which means that $T(A,1) \neq \mathcal{N}_{(n)} / M(A)$.

**Definition 3.3.6.** Let $A \in \mathcal{N}_{(n,n)}$ and $\lambda \in \mathcal{N}$. A FNSM $A$ is called a strongly $\lambda$-robust if $T^*(A,\lambda) = \mathcal{N}_{(n)} / M(A)$.

**Lemma 3.3.7.** Let $\lambda > c(A)$ and A be a strongly $\lambda$- robust FNSM.
Then $x^*(A,\lambda) = c^*(A)$.

**Proof:** Let us suppose that $A$ is a strongly $\lambda$-robust FNSM. In contrast we assume that $\langle x^T, x^I, x^F\rangle^*(A,\lambda) \neq c^*(A)$, i.e. there exists $i \in N$ such that $\langle x^T, x^I, x^F\rangle_i^*(A,\lambda) > c(A)$. Then for $\langle x^T, x^I, x^F\rangle = c^*(A) \in \mathcal{N}_{(n)} / M(A)$ we have $A^k \otimes c^*(A) = \lambda \otimes c^*(A) = c^*(A) \neq \langle x^T, x^I, x^F\rangle^*(A,\lambda)$ for all integer $k \geq 1$, a contradiction with the strong $\lambda$-robustness of A.
From the above result a characterization of the strongly $\lambda$-robust FNSMs follows.

**Theorem 3.3.8.** Let $A \in \mathcal{N}_{(n,n)}$ and $\lambda$ be greater than $c(A)$. Then $A$ is a strongly $\lambda$-robust FNSM if and only if $\langle x^T, x^I, x^F\rangle^*(A,\lambda) = c^*(A)$ and $G(A,c(A))$ is a strongly connected digraph with period equal to 1.

**Proof:** For the only if direction suppose that A is a strongly $\lambda$-robust FNSM, i.e. for each $x \in \mathcal{N}_{(n)} / M(A)$ there is number $k \in N^+$ such that $A^k \otimes \langle x^T, x^I, x^F\rangle = \langle x^T, x^I, x^F\rangle^*(A,\lambda)$ and $x^*(A,\lambda) = c^*(A)$ by Lemma 5.8. In particular, for each FNSV $\langle x^T, x^I, x^F\rangle = e_i \in \mathcal{N}_{(n)} / M(A)$ there exists $k_i \in N^+$ such that equalities $A^{ki} \otimes e_i = e^{(ki)} = c^*(A)$ and $c^*(A) = A^s \otimes c^*(A)$ hold for all $s \geq k_i$, i.e. the strong $\lambda$-robustness of A implies that conditions(i) of Lemma 5.5 holds and the assertion follows. Conversely, let us suppose that $\langle x^T, x^I, x^F\rangle^*(A,\lambda) = c^*(A)$ and $G(A,c(A))$ is a strongly connected digraph with period equal to 1. We shall consider two cases.

case-1: $\langle x^T, x^I, x^F\rangle \in \mathcal{N}_{(n)} / M(A)$ and $\langle x^T, x^I, x^F\rangle \geq c^*(A)$. By Theorem 5.4 the condition $\langle x^T, x^I, x^F\rangle^*(A,\lambda) = c^*(A)$ implies that $G(A,c^+(A))$ is acyclic digraph.

Notice that a threshold digraph $G(A,c^{+}(A))=(N,E)$ is acyclic if and only if its nodes can be relabeled such that if $(i,j)\in E$ then $i<j$. That means that without loss of generality it can be assumed that $c(A)=\bigoplus_{j\in N}\langle a_{nj}^{T},a_{nj}^{I},a_{ij}^{F}\rangle$ and if $\langle a_{ij}^{T},a_{ij}^{I},a_{ij}^{F}\rangle>c(A)$ then $i<j$. For arbitrary FNSV $\langle x^{T},x^{I},x^{F}\rangle\in\mathcal{N}_{(n)}/M(A)$ and $\langle x^{T},x^{I},x^{F}\rangle\geq c^{*}(A)$ we have to prove that $A^{n}\otimes\langle x^{T},x^{I},x^{F}\rangle=c^{*}(A)$. For this purpose we shall construct the sequence of FNSV $\langle x^{T},x^{I},x^{F}\rangle^{(1)},\ldots,\langle x^{T},x^{I},x^{F}\rangle^{(n)}$ in the following way. Put $\langle x^{T},x^{I},x^{F}\rangle^{(1)}=(\langle x^{T},x^{I},x^{F}\rangle_{1}^{(1)},\ldots,\langle x^{T},x^{I},x^{F}\rangle_{n}^{(1)})^{T}$ and define the FNSVs $\langle x^{T},x^{I},x^{F}\rangle^{(2)},\ldots,\langle x^{T},x^{I},x^{F}\rangle^{(n)}$ as follows:

$A\otimes\langle x^{T},x^{I},x^{F}\rangle^{(i)}=\langle x^{T},x^{I},x^{F}\rangle^{(i+1)}$ for all $i\in\{1,\ldots,n-1\}$. We aim to obtain the equality $A^{n}\otimes\langle x^{T},x^{I},x^{F}\rangle=c^{*}(A)$.

Let us define $M_{n}=\{n\},M_{0}=\phi$ and recursively define sets $M_{n-k}$ as follows:

$$M_{n-k}=i\in N/\bigcup_{j=0}^{k-1}M_{n-j};\ \bigoplus_{j\in N\setminus\cup_{j=0}^{k-1}M_{n-j}}\langle a_{ij}^{T},a_{ij}^{I},a_{ij}^{F}\rangle<c(A)$$
,

for $k=1,\ldots,n-1$. Let s be the first index from the set $\{1,\ldots,n-1\}$ such that $M_{n-k}\neq\phi$ for $k=1,\ldots,s$ and $M_{n-(s+1)}=\phi$. Then $A\otimes\langle x^{T},x^{I},x^{F}\rangle^{(1)}=(\langle x^{T},x^{I},x^{F}\rangle_{1}^{(2)},\ldots,\langle x^{T},x^{I},x^{F}\rangle_{n}^{(2)})^{t}$, where $\langle x^{T},x^{I},x^{F}\rangle_{i}^{(2)}=c(A)$ for $i\in M_{n}$

$A^{2}\otimes\langle x^{T},x^{I},x^{F}\rangle^{(1)}=A\otimes(A\otimes\langle x^{T},x^{I},x^{F}\rangle^{(1)})=A\otimes(\langle x^{T},x^{I},x^{F}\rangle_{1}^{(2)},\ldots,\langle x^{T},x^{I},x^{F}\rangle_{n}^{(2)})^{t})$
$=\langle x^{T},x^{I},x^{F}\rangle_{1}^{(3)},\ldots,\langle x^{T},x^{I},x^{F}\rangle_{n}^{(3)})^{t}$.

where $\langle x^{T},x^{I},x^{F}\rangle_{i}^{(3)}=c(A)$ for $i\in M_{n-1}$, a.s.o., and for $s$ we get

$A^{s+1}\otimes\langle x^{T},x^{I},x^{F}\rangle^{(1)}=A\otimes(A^{s}\otimes\langle x^{T},x^{I},x^{F}\rangle^{(1)})=A\otimes(\langle x^{T},x^{I},x^{F}\rangle_{1}^{(s+1)},\ldots,$
$\langle x^{T},x^{I},x^{F}\rangle_{n}^{(s+1)})^{t})=(\langle x^{T},x^{I},x^{F}\rangle_{1}^{(s+2)},\ldots,\langle x^{T},x^{I},x^{F}\rangle_{n}^{(s+2)})^{t}$,

where $\langle x^{T},x^{I},x^{F}\rangle_{i}^{(s+2)}=c(A)$ for $i\in M_{n-s}$,

Let D denotes the FNSM which arises from the FNSM A on omitting the $i^{th}$ row and the corresponding column for all $i\in\bigcup_{j=0}^{s}M_{n-j}$. If $N/\bigcup_{j=0}^{s}M_{n-j}\neq\phi$ then $c(A)=c(D)$ and the digraph $G(D,c^{+}(A))$ is again acyclic. Using the above procedure at most n times we obtain $A^{n}\otimes\langle x^{T},x^{I},x^{F}\rangle=c^{*}(A)=\langle x^{T},x^{I},x^{F}\rangle^{*}(A,\lambda)$.

Case 2: $\langle x^{T},x^{I},x^{F}\rangle\in\mathcal{N}_{(n)}/M(A)$ and $\langle x^{T},x^{I},x^{F}\rangle\ngeq c^{*}(A)$, i.e. there are indices $i,j\in N$ such that $x_{i}\geq c(A)$ and $\langle x^{T},x^{I},x^{F}\rangle_{j}<c(A)$. It is easily seen that an arbitrary

FNSV $\langle x^T, x^I, x^F \rangle = (\langle x^T, x^I, x^F \rangle_1, ..., \langle x^T, x^I, x^F \rangle_n)^T \in \mathcal{N}_{(n)}$ can be written as a linear (with respect to $\oplus, \otimes$) combination of the FNSV $e_i$,

i.e. $\langle x^T, x^I, x^F \rangle = \overset{n}{\oplus}_{i=1} \langle x^T, x^I, x^F \rangle_i \otimes e_i$. Let us note that the digraph $G(A, c(A))$ is under our assumption strongly connected with period equal to 1. Then by Lemma 5.6 we get $(\forall i \in N)(\exists r_i \in N^+)(\forall k \geq r_i) \quad (A^k \otimes e_i = e_i^{(k)} \geq c^*(A))$
and hence

$$\langle x^T, x^I, x^F \rangle^r = A^r \otimes \langle x^T, x^I, x^F \rangle = A^r \otimes \bigoplus_{i=1}^{n} \langle x^T, x^I, x^F \rangle_i \otimes e_i = \bigoplus_{i=1}^{n} \langle x^T, x^I, x^F \rangle_i \otimes (A^r \otimes e_i)$$

$$\geq \bigoplus_{i=1}^{n} \langle x^T, x^I, x^F \rangle_i \otimes c^*(A) = c^*(A)$$

for $r = \max_{i \in N} r_i$. Now , case 1 can be used for the FNSV
$\langle x^T, x^I, x^F \rangle^{(r)} = A^r \otimes \langle x^T, x^I, x^F \rangle \in \mathcal{N}_{(n)} / M(A)$ and the assertion follows.

We can us the results to derive an algorithm for checking that a given FNSM $A \in \mathcal{N}_{(n,n)}$ is strongly $\lambda$-robust.

**Algorithm Strong Robust**

Input. $A = (\langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle) \in \mathcal{N}_{(n,n)}, \lambda \in B.$

Output. yes in variable sr if A is strong $\lambda$-robust; no sr otherwise.

begin

1. If $\lambda \leq c(A)$ then $sr :=$ no; go to end;
2. Compute $\langle x^T, x^I, x^F \rangle^*(A, \lambda), c^*(A)$;
3. Check the strongly connectivity of $G(A, c(A))$;
4. Compute the period of $G(A, c(A))$;
5. If $\langle x^T, x^I, x^F \rangle^*(A, \lambda), c^*(A)$ and $G(A, c(A))$ is strongly connected with period equal to 1 then $sr := yes$, else sr:=no:

**Theorem 3.3.9.** Let $A \in \mathcal{N}_{(n,n)}$ and $\lambda \in \mathcal{N}_{(n)}$. The algorithm strong robust correctly decides whether a FNSM A is strongly $\lambda$-robust in $0(n^3)$ arithmetic operations.

**Proof:** To determine a complexity of the algorithm, recall first that to compute $\langle x^T, x^I, x^F \rangle^*(A, \lambda)$ and $c^*(A)$ we need $O(n^2 log\, n)$ operations. The number of operations for checking of strongly connectivity and computing the period of $G(A, c(A))$ is $O(n^2) + O(n^3) = O(n^3)$. Thus, the complexity of the whole algorithm is $O(n^3)$.

**Corollary 3.3.10.** Let $A\in \mathcal{N}_{(n,n)}$ and $\lambda\in \mathcal{N}_{(n)}, \lambda > c(A) = 0$. Then A is a strongly $\lambda$-robust FNSM if and only if $\langle x^T, x^I, x^F\rangle^*(A,\lambda) = (\langle 0,0,1\rangle,...,\langle 0,0,1\rangle)^t$. Let us suppose that $A\in \mathcal{N}_{(n,n)}, \lambda\in \mathcal{N}, \lambda > c(A) = 0$. It is easily seen that $\langle x^T, x^I, x^F\rangle^*(A,\lambda) = (\langle 0,0,1\rangle,...,\langle 0,0,1\rangle)^t$ holds if and only if the FNSM A is equivalent to an upper triangular FNSM. Then the fact that a power of an upper triangular FNSM is again an upper triangular one allows us to present a result which is similar to that of theorem 3.6 of [4].

**Corollary 3.3.11.** Let $A\in \mathcal{N}_{(n,n)}$ and $\lambda\in \mathcal{N}, \lambda > c(A) = 0$. Then A is a strongly $\lambda$-robus if and only if $V(A,\lambda) = V(A^2,\lambda) =,... = V(A^n,\lambda) = \{(\langle 0,0,1\rangle,...,\langle 0,0,1\rangle)^t\}$. }

**3.4. A possible application of robust FNSMs-a realization of fuzzy discrete dynamic systems**

Suppose that the fuzzy discrete-event dynamic system starts at time 0 in state

$$\langle x^T, x^I, x^F\rangle^{(0)} = \langle b^T, b^I, b^F\rangle$$

and its states at the following time points are

$$\langle x^T, x^I, x^F\rangle^1 = A\otimes\langle x^T, x^I, x^F\rangle^0, \langle x^T, x^I, x^F\rangle^2 = A\otimes\langle x^T, x^I, x^F\rangle^1, ..., \langle x^T, x^I, x^F\rangle^{t+1}$$
$$= A\otimes\langle x^T, x^I, x^F\rangle^t, ....$$

We know only the scalar output of the system in the form $g_t = c^T \otimes \langle x^T, x^I, x^F\rangle_t$, where $c^T = (c_1,...,c_n)$ is the observation FNSV. The sequence $\{g_t\}^\infty{}_{t=0}$ is called the sequence of Markov parameters of the system with the observation FNSV $\langle c^T, c^I, c^F\rangle$ and staring FNSV $\langle b^T, b^I, b^F\rangle$. The triple $(A, \langle b^T, b^I, b^F\rangle, \langle c^T, c^I, c^F\rangle)$ is called a realization of the sequence of Markov parameters. The task is: to find a FNSM A and the FNSV $\langle c^T, c^I, c^F\rangle, \langle b^T, b^I, b^F\rangle$ such that triple $(A, \langle b^T, b^I, b^F\rangle, \langle c^T, c^I, c^F\rangle)$ realizes the sequence $\{g_t\}^\infty{}_{t=0}$. The sequence $\{g_t\}^\infty{}_{t=0}$ is called stabilized if there exists a natural number $m$ such that $g_{k+1} = g_k$ for all $k \geq m$. The next lemma provides a method for finding a simple realization of any stabilized sequence of Markov parameters

**Lemma 3.4.1.** Any stabilized sequence $\{g_t\}^\infty{}_{t=0}$ has an $(A, \langle b^T, b^I, b^F\rangle, \langle c^T, c^I, c^F\rangle)$ realization with an I-robust FNSM A.

**Proof:** Suppose that the sequence $\{g_t\}^\infty{}_{t=0}$ is stabilized, i.e. $g_{k+1} = g_k$ for all $k \geq m$ for some natural m, put $\langle c^T, c^I, c^F\rangle = (\langle 1,1,0\rangle, \langle 0,0,1\rangle, ..., \langle 0,0,1\rangle)^t \in \mathcal{N}_{(m+1)}$;

$$A = \begin{pmatrix} \langle 0,0,1\rangle & \langle 1,1,0\rangle & \langle 0,0,1\rangle & \dots & \langle 0,0,1\rangle & \langle 0,0,1\rangle \\ \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 1,1,0\rangle & \dots & \langle 0,0,1\rangle & \langle 0,0,1\rangle \\ \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle & \dots & \langle 0,0,1\rangle & \langle 0,0,1\rangle \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle & \dots & \langle 0,0,1\rangle & \langle 1,1,0\rangle \\ \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle & \dots & \langle 0,0,1\rangle & \langle 1,1,0\rangle \end{pmatrix} \in \mathcal{N}_{(m+1,m+1)},$$

$$b = \begin{pmatrix} \langle g_0^T \; g_0^I \; g_0^F \rangle \\ \langle g_1^T \; g_1^I \; g_1^F \rangle \\ . \\ . \\ . \\ \langle g_m^T \; g_m^I \; g_m^F \rangle \end{pmatrix} \in \mathcal{N}_{(m+1)}.$$

It is easily seen that $g_k = c^T \otimes A^k \otimes b$ for every $k$ and

$$A^m = A^{m+s} = \begin{pmatrix} \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle & \dots & \langle 0,0,1\rangle & \langle 1,1,0\rangle \\ \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 1,1,0\rangle & \dots & \langle 0,0,1\rangle & \langle 1,1,0\rangle \\ \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle & \dots & \langle 0,0,1\rangle & \langle 1,1,0\rangle \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle & \dots & \langle 0,0,1\rangle & \langle 0,0,1\rangle \\ \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle & \dots & \langle 0,0,1\rangle & \langle 1,1,0\rangle \end{pmatrix} \in \mathcal{N}_{(m+1,m+1)}$$

for every natural s. Hence $per(A,I)=1$ and $A$ is I-robus.

## 4. Conclusion

In this paper, the authors presented background of the problem, $\lambda$-robust FNSMs, Strongly $\lambda$-robust FNSMs, and a possible application of robust FNSMs-realization of fuzzy discrete dynamic systems.

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