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S-α Anti Fuzzy Normal Subsemigroups in S-α Anti Fuzzy Semigroups

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Abstract. The notion of S-α anti fuzzy cosets with representative x^t and S-α anti fuzzy normal subsemi groups are introduced and their characterizations are obtained. It is also proved that the set of all S-α anti fuzzy cosets will form a semigroup under a suitable binary operation and its structural properties are determined. A necessary condition for an S-α anti fuzzy semigroup to be S-α anti fuzzy normal is also proved.

Keywords: S-semigroup, anti-fuzzy group, α-anti fuzzy set, α-anti fuzzy group, S- anti fuzzy semigroup, S-α anti fuzzy semigroup.

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1. Introduction

The concept of fuzzy set was introduced by Zadeh in 1965 [9]. A fuzzy subgroup of a group was defined by Rosenfeld in 1971 [5]. Fuzzy groups was redefined by Anthony and H. Sherwood in 1979[1]. Malik, Mordeson and Nair introduced the notion of fuzzy cosets of B in A with representative x_t and fuzzy normal, where A and B are fuzzy subgroups of a group G such that B $\subset$ A [3]. Rajeshkumar analyzed Fuzzy Algebra in 1993[4]. VasanthaKandasamy studied Smarandache fuzzy semigroups in [7]. Sharma introduced the concept of α-anti fuzzy set, α- anti fuzzy group, α-anti fuzzy coset, α-anti fuzzy normal subsemigroups and obtained their properties in [6]. Gowri and Rajeswari introduced the idea of S-α anti fuzzy semigroup, S-α anti fuzzy left, right cosets and S-α anti fuzzy normal subsemigroup and analyzed their properties [2]. Vijayakumar and T. Rajeswari introduced the concept of anti fuzzy cosets of B in A with representative x^t and anti fuzzy normal[8]. In this paper, the concepts of S-α anti fuzzy cosets with representative x^t and S-α anti fuzzy normal subsemigroups are introduced. These ideas differ from those in [2]. It is also proved that the set of all S-α anti fuzzy cosets will form a semigroup under a suitable binary operation and its structural properties are determined.

Throughout this paper, α will always denote a member of [0, 1].

2. Preliminaries

Definition 2.1. Let X be a non empty set. A fuzzy subset A of X is a function $A : X \to [0,1]$

Definition 2.2. A fuzzy subset A of a group G is called an anti fuzzy group of G if
$(i)\ A(xy) \le \max\{A(x), A(y)\}$
$(ii)\ A(x^{-1}) = A(x),$ for all $x, y \in G$

Definition 2.3. Let A be a fuzzy subset of a group G. Let $\alpha \in [0,1]$.Then an α - anti fuzzy subset of G (with respect to a fuzzy set A), denoted by A_α, is defined as $A_\alpha(x) = \max\{A(x), 1-\alpha\}$, for all $x \in G$.

Definition 2.4. Let A be a fuzzy subset of a group G and $\alpha \in [0,1]$.Then A is called an α - anti fuzzy subgroup of G if A_α is an anti fuzzy group.

Definition 2.5. A semigroup S is said to be a Smarandache semigroup (S-semigroup) if there exists a proper subset P of S which is a group under the same binary operation in S.

Definition 2.6. Let G be an semigroup. Let A be a fuzzy subset of G and $\alpha \in [0,1]$. A is called a Smarandache α anti fuzzy semigroup (S-α anti fuzzy semigroup) if there exists a proper subset P of G which is a group and the restriction of A to P is such that A_{P_α} is an anti fuzzy group. That is,
$(i)\ A_{P_\alpha}(xy) \le \max\{A_{P_\alpha}(x), A_{P_\alpha}(y)\}$
$(ii)\ A_{P_\alpha}(x^{-1}) = A_{P_\alpha}(x),$ for all $x, y \in P$

Result 2.7. [2] If $A : G \to [0,1]$ is an S-α anti fuzzy semigroup of an S-semigroup G relative to a group P which is a proper subset of G, then
$(i) A_{P_\alpha}(x) \ge A_{P_\alpha}(e)$, where e is the identity element of P
$(ii) A_{P_\alpha}(xy^{-1}) = A_{P_\alpha}(e) \Rightarrow A_{P_\alpha}(x) = A_{P_\alpha}(y)$, for all $x, y \in P$

Result 2.8. [2] Let G be an S-semigroup and P be a proper subset of G which is a group. Then $A : G \to [0,1]$ is an S-α anti fuzzy semigroup of G relative to P iff $A_{P_\alpha}(xy^{-1}) \le \max\{A_{P_\alpha}(x), A_{P_\alpha}(y)\}$, for all $x, y \in P$

3. S-α anti fuzzy cosets and S-α anti fuzzy normal subsemigroups

In this section, we define S-α anti fuzzy cosets with representative x^t and S-α anti fuzzy normal subsemigroups and obtain their characterizations.

Definition 3.1. [8] Let X be a non empty set. For any $x \in X$ and $t \in [0,1]$, a fuzzy singleton, denoted by x^t, is defined as $x^t(y) = \begin{cases} t, if\ y = x \\ 1, if\ y \neq x \end{cases}$ for all $y \in X$. That is $x^t : X \to [0,1]$ is a mapping.

Definition 3.2. [8] Let G be a non empty set and let $\cdot$ be a binary operation on G. Let A and B be fuzzy subsets of G. Define the fuzzy subset $A \diamond B$ of G by $(A \diamond B)(x) = \inf\{\sup\{A(y), B(z)\}/x = yz\}$ for all $x \in G$, That is

$$A \diamond B = \begin{cases} \inf\{sup\{A(y),\ B(z)\}\},\ if\ x = yz \\ 1 \qquad\qquad\qquad\qquad ,\ if\ x \neq yz \end{cases}$$

Remark 3.3. [8] If the operation $\cdot$ in G is associative, commutative respectively, then so is $\diamond$

Definition 3.4. Let G be an S-semigroup. Let A and B be S-α anti fuzzy semigroups of G relative to a group P in Gsuch that $B \subset A$. Let $x \in P$ and let $x^t \subset A$. Then the fuzzy subset $x^t \diamond B_{P_\alpha}$ is called the Smarandache- α anti fuzzy left coset(S-α **anti fuzzy left coset)** of B in A with respresentative x^t.
That is, $(x^t \diamond B_{P_\alpha})(z) = \inf\{\sup\{x^t(u),\ B_{P_\alpha}(v)\}/z = uv\}$ for all $z \in P$

Example 3.5. Let $G = \{e, a, b, c, d, e, f, g\}$ which is a semigroup by the following table.

*	e	a	b	c	d	f	g
e	e	a	b	c	d	f	g
a	a	e	c	b	a	a	a
b	b	c	e	a	b	b	b
c	c	b	a	e	c	c	c
d	d	a	a	a	d	f	g
f	f	b	b	b	d	f	g
g	g	c	c	c	d	f	g

Let $P = \{e, a, b, c\}$ which is the klein four group. $P \subset G$. Two fuzzy subsets A and B of G are defined as

$$A(x) = \begin{cases} 1,\ if\ x = e, a \\ \frac{3}{4},\ if\ x = b, c \\ 0,\ otherwise \end{cases} \quad \text{and } B(x) = \begin{cases} 1,\ if\ x = e, a \\ \frac{1}{2},\ if\ x = b, c \\ 0,\ otherwise \end{cases}$$

If take $\alpha = 0.85$, then A and B be S-α anti fuzzy semigroups. Also $B \subset A$ and let $t = 0.6$. For $x = a$, $x^t(y) = \begin{cases} 0.6, if\ y = x \\ 1, if\ y \neq x \end{cases}$ for all $y \in P$.

Thus $x^t \subset A$ and an S-α anti fuzzy left coset of B in A with respresentative x^t is given by $(x^t \diamond B_{P_\alpha})(z) = \begin{cases} 0.75, & if\ z = e, a \\ 1 \quad, & if\ z = b, c \end{cases}$

Definition 3.6. Let G be an S-semigroup. Let A and B be S-α anti fuzzy semigroups of G relative to a group P in G such that $B \subset A$. Let $x \in P$ and let $x^t \subset A$. Then the fuzzy subset $B_{P_\alpha} \diamond x^t$ is called the Smarandache- α anti fuzzy right coset(S-α **anti fuzzy right coset)** of B in A with respresentative x^t.
That is, $(B_{P_\alpha} \diamond x^t)(z) = \sup\{\inf\{B_{P_\alpha}(u),\ x^t(v)\}/z = uv\}$ for all $z \in P$

Example 3.7. In example 3.4, for $x = a$, an S-α anti fuzzy right coset of B in A with respresentative x^t is given by $(B_{P_\alpha} \diamond x^t)(z) = \begin{cases} 0.75, & if\ z = e, a \\ 1 \quad, & if\ z = b, c \end{cases}$

Theorem 3.8. Let G be an S-semigroup. Let A and B be S-α anti fuzzy semigroups of G relative to a group P in Gsuch that $B \subset A$. Let $x \in P$ and let $x^t \subset A$. Then for all $z \in P$, $(x^t \diamond B_{P_\alpha})(z) = \sup\{t, B_{P_\alpha}(x^{-1}z)\}$ and $(B_{P_\alpha} \diamond x^t)(z) = \sup\{t, B_{P_\alpha}(zx^{-1})\}$.
Proof: For $z \epsilon P$, $(x^t \diamond B_{P_\alpha})(z) = \inf\{\sup\{x^t(u),\ B_{P_\alpha}(v)\}/z = uv, u, v \in P\}$. If $z = uv$, then $v = u^{-1}z$. Also $z = x(x^{-1}z)$. Thus $(x^t \diamond B_{P_\alpha})(z) = \inf\{\sup\{t, B_{P_\alpha}(x^{-1}z)\}, 1\} = \sup\{t, B_{P_\alpha}(x^{-1}z)\}$. Similarly $(B_{P_\alpha} \diamond x^t)(z) = \sup\{t, B_{P_\alpha}(zx^{-1})\}$.

Theorem 3.9. Let G be an S-semigroup. Let A and B be S-α anti fuzzy semigroups of G relative to a group P in G such that $B \subset A$. Let $x, y \in P$ and let $t, s \in [0,1]$. Let x^t, $y^s \subset A$. Then
(i) $x^t \diamond B_{P_\alpha} = y^s \diamond B_{P_\alpha}$ iff $\sup\{t, B_{P_\alpha}(e)\} = \sup\{s, B_{P_\alpha}(y^{-1}x)\}$ and $\sup\{s, B_{P_\alpha}(e)\} = \sup\{t, B_{P_\alpha}(x^{-1}y)\}$.
(ii) $B_{P_\alpha} \diamond x^t = B_{P_\alpha} \diamond y^s$ *iff* $\sup\{t, B_{P_\alpha}(e)\} = \sup\{s, B_{P_\alpha}(xy^{-1})\}$
and $\sup\{s, B_{P_\alpha}(e)\} = \sup\{t, B_{P_\alpha}(yx^{-1})\}$
Proof: (i) Suppose that $x^t \diamond B_{P_\alpha} = y^s \diamond B_{P_\alpha}$. Then $(x^t \diamond B_{P_\alpha})(z) = (y^s \diamond B_{P_\alpha}(z)$ for all $z \in P$. If we take $z = x$ and then $z = y$, then by theorem 3.8, we have $\sup\{t, B_{P_\alpha}(e)\} = \sup\{s, B_{P_\alpha}(y^{-1}x)\}$ and $\sup\{s, B_{P_\alpha}(e)\} = \sup\{t, B_{P_\alpha}(x^{-1}y)$.Conversely, suppose that the conditions concerning the supremum hold. Let $z \in P$.Then $(x^t \diamond B_{P_\alpha})(z) = \sup\{t, B_{P_\alpha}(x^{-1}z)\} = \sup\{t, B_{P_\alpha}(x^{-1}y)(y^{-1}z)\}$
$\leq \sup\{t, \sup\{B_{P_\alpha}(x^{-1}y), B_{P_\alpha}(y^{-1}z)\}\} = \sup\{\sup\{s, B_{P_\alpha}(e)\}, B_{P_\alpha}(y^{-1}z)\}$ (by assumption)
$= \sup\{s, B_{P_\alpha}(y^{-1}z)\} = (y^s \diamond B_{P_\alpha})(z)$ (by theorem 3.8).Thus $x^t \diamond B_{P_\alpha} \subset y^s \diamond B_{P_\alpha}$. Similarly it can be proved that $x^t \diamond B_{P_\alpha} \supset y^s \diamond B_{P_\alpha}$ and hence $x^t \diamond B_{P_\alpha} = y^s \diamond B_{P_\alpha}$
(ii) This can be proved similarly.

Corollary 3.10. Let G be an S-semigroup. Let A and B be S-α anti fuzzy semigroups of G relative to a group P in G such that $B \subset A$. Let $x^t, y^t \subset A$, where

where $x, y \in P$. If $B_{P_\alpha}(y^{-1}x) = B_{P_\alpha}(e)$, then $x^t \diamond B_{P_\alpha} = y^t \diamond B_{P_\alpha}$.

Proof: Since $B_{P_\alpha}(x^{-1}y) = B_{P_\alpha}(y^{-1}x)$, $\sup\{t, B_{P_\alpha}(e)\} = \sup\{t, B_{P_\alpha}(x^{-1}y)\}$
$= \sup\{t, B_{P_\alpha}(y^{-1}x)\}$ which leads to $x^t \diamond B_{P_\alpha} = y^t \diamond B_{P_\alpha}$, by theorem 3.9(i).

Theorem 3.11. Let G be an S-semigroup. Let A and B be S-α anti fuzzy semigroups of G relative to a group P in G such that $B \subset A$. Let $x^t, y^t \subset A$, where $x, y \in P$ .Then the following conditions are equivalent

(i) $x^t \diamond B_{P_\alpha} = y^t \diamond B_{P_\alpha}$

(ii) $(y^{-1}x)^t \diamond B_{P_\alpha} = e^t \diamond B_{P_\alpha}$

(iii) $(x^{-1}y)^t \diamond B_{P_\alpha} = e^t \diamond B_{P_\alpha}$

Proof: $x^t \diamond B_{P_\alpha} = y^t \diamond B_{P_\alpha}$ iff $\sup\{t, B_{P_\alpha}(e)\} = \sup\{t, B_{P_\alpha}(y^{-1}x)\}$ and $\sup\{t, B_{P_\alpha}(e)\} = \sup\{t, B_{P_\alpha}(x^{-1}y)\} \Leftrightarrow (y^{-1}x)^t \diamond B_{P_\alpha} = e^t \diamond B_{P_\alpha}$, by theorem 3.9(i) and hence (i) $\Leftrightarrow$ (ii). Similarly it is easy to see that (i) $\Leftrightarrow$ (iii).

Theorem 3.12. Let G be an S-semigroup. Let A and B be S-α anti fuzzy semigroups of G relative to a group P in G such that $B \subset A$. Let $x, y \in P$ and $s, t \in [A_{P_\alpha}(e), 1]$. Let $x^t, y^s \subset A$ .Suppose that $B_{P_\alpha}(e) = A_{P_\alpha}(e)$ .Then

(i) $x^t \diamond B_{P_\alpha} = y^s \diamond B_{P_\alpha} \Leftrightarrow t = \sup\{s, B_{P_\alpha}(y^{-1}x\}$ and $\mathrm{s} = \sup\{t, B_{P_\alpha}(x^{-1}y)\}$

(ii) $x^t \diamond B_{P_\alpha} = y^t \diamond B_{P_\alpha} \Leftrightarrow (y^{-1}x)^t \supseteq B_{P_\alpha}$

(iii) $x^t \diamond B_{P_\alpha} = y^s \diamond B_{P_\alpha} \Leftrightarrow t = s \geq B_{P_\alpha}(x^{-1}y\}$

(iv) $x^t \diamond B_{P_\alpha} = y^s \diamond B_{P_\alpha} \Leftrightarrow t = \mathrm{s}$

Proof: (i) By theorem 3.9, $x^t \diamond B_{P_\alpha} = y^s \diamond B_{P_\alpha} \Leftrightarrow \sup\{t, A_{P_\alpha}(e\} = \sup\{s, B_{P_\alpha}(y^{-1}x\}$ and $\sup\{s, A_{P_\alpha}(e\} = \sup\{t, B_{P_\alpha}(x^{-1}y\} \Leftrightarrow t = \sup\{s, B_{P_\alpha}(y^{-1}x)\}$ and $s = \sup\{t, B_{P_\alpha}(x^{-1}y)\}$.

(ii) By (i), $x^t \diamond B_{P_\alpha} = y^t \diamond B_{P_\alpha} \Leftrightarrow t = \sup\{t, B_{P_\alpha}(y^{-1}x)\}$ and $t = \sup\{t, B_{P_\alpha}(x^{-1}y)\}$ iff $t \geq B_{P_\alpha}(y^{-1}x)$ and $t \geq B_{P_\alpha}(x^{-1}y)$ iff $(y^{-1}x)^t \supset B_{P_\alpha}$.

(iii) If $x^t \diamond B_{P_\alpha} = y^s \diamond B_{P_\alpha}$, then by (i), $t = \sup\{s, B_{P_\alpha}(x^{-1}y)\}$ and $s = \sup\{t, B_{P_\alpha}(y^{-1}x)\}$ which implies that $t = s \geq B_{P_\alpha}(x^{-1}y\}$. Conversely, assume that $t = s \geq B_{P_\alpha}(x^{-1}y\}$. This implies that $\sup\{s, B_{P_\alpha}(y^{-1}x)\} = t$ and $\sup\{t, B_{P_\alpha}(x^{-1}y)\} = s$ and hence $x^t \diamond B_{P_\alpha} = y^s \diamond B_{P_\alpha}$.

(iv) From (iii) the result is obvious.

Corollary 3.13. Let G be an S-semigroup. Let A and B be S-α anti fuzzy semigroups of G relative to a group P in G such that $B \subset A$. Let $x, y \in P$ and $s, t \in [A_{P_\alpha}(e),1]$. Let $x^t, y^s \subset A$. Suppose that $B_{P_\alpha}(e) = A_{P_\alpha}(e)$. If $t \neq s$, then $\{x^t \circ B_{P_\alpha} / x^t \subset A\} \cap \{y^s \circ B_{P_\alpha} / y^s \subset A\} = \Phi$.

Proof: By theorem 3.12(iii), the result is obvious.

Definition 3.14. Let G be an S-semigroup. Let A and B be S-α anti fuzzy semigroups of G relative to a group P in G such that $B \subset A$. B is said to be Smarandache-α anti fuzzy normal subsemigroup (S-α fuzzy anti normal subsemigroup) in A if $x^t \circ B_{P_\alpha} = B_{P_\alpha} \circ x^t$, for all $x^t \subset A$, where $x \in P$.

Example 3.15. From example 3.5 and 3.7, it can be easily seen that $x^t \circ B_{P_\alpha} = B_{P_\alpha} \circ x^t$, for all $x^t \subset A$.

Theorem 3.16. Let G be an S-semigroup. Let A and B be S-α anti fuzzy semigroups of G relative to a group P in G such that $B \subset A$. Let $x, y \in P$ and $x^t, y^s \subset A$. If B is an S-α anti fuzzy normal subsemigroup in A, then

$$(x^t \circ B_{P_\alpha}) \circ (y^s \circ B_{P_\alpha}) = (xy)^r \circ B_{P_\alpha}, \text{ where } r = \sup\{t, s\}.$$

Proof: By remark[3.3], $(x^t \circ B_{P_\alpha}) \circ (y^s \circ B_{P_\alpha}) = (x^t \circ y^s) \circ B_{P_\alpha}$.

By the definition of fuzzy singleton set $x^t \circ y^s = (xy)^r$ which leads to the result.

Theorem 3.17. Let G be an S-semigroup. Let A and B be S-α anti fuzzy semigroups of G relative to a group P in G such that $B \subset A$. Let

$$A/B = \{x^t \circ B_{P_\alpha} / x^t \subset A \text{ and } x \in P\}.$$

Assume that B is S-α anti fuzzy normal in A. Then $(A/B, \circ)$ is a semigroup with identity. If $B_{P_\alpha}(e) = A_{P_\alpha}(e)$, then A/B is completely regular. That is, A/B is a union of disjoint groups.

Proof: If $x^t \circ B_{P_\alpha}, y^s \circ B_{P_\alpha} \in A/B$, where $x^t, y^s \subset A$, then by theorem 3.16, $(xy)^r \circ B_{P_\alpha} \in A/B$, where $r = \sup\{t, s\}$. It can be easily seen that $e^{A_{P_\alpha}(e)}$ is the identity of A/B. By remark[3.3], $\circ$ is associative and hence $(A/B, \circ)$ is a semigroup with identity $e^{A_{P_\alpha}(e)}$. For fixed $t \in [A_{P_\alpha}(e),1]$,

define $(A/B)^{(t)} = \{x^t \circ B_{P_\alpha} / x^t \subset A \text{ and } x \in P\}$. Then $(A/B)^{(t)}$ is closed and the identity of $(A/B)^{(t)}$ is $e^t \circ B_{P_\alpha}$. It is also easy to see that $(x^{-1})^t \circ B_{P_\alpha}$ is the inverse of $x^t \circ B_{P_\alpha}$. Thus $(A/B)^{(t)}$ is a group. Moreover $A/B = \bigcup_{t \in [A_{P_\alpha}(e),1]} (A/B)^{(t)}$ and hence A/B is completely regular.

Remark 3.18. Let G be an S-semigroup. Let A and B be S-α anti fuzzy semigroups of G relative to a group P in G such that $B \subset A$. For $t \in [0,1]$, we define ${B_{P_\alpha}}^t = \{x \in P / B_{P_\alpha}(x) \leq t$ .Then it can be proved that if $t \in \operatorname{Im} B_{P_\alpha}$, then ${B_{P_\alpha}}^t$ is a subgroup of P . Since $B \subset A$, ${A_{P_\alpha}}^t$ is also a subgroup of P .

Theorem 3.19. Let G be an S-semigroup. Let A and B be S-α anti fuzzy semigroups of G relative to a group P in G such that $B \subset A$. If B is S-α anti fuzzy normal in A, then for all $t \in [B_{P_\alpha}(e),1]$, ${B_{P_\alpha}}^t$ is normal in ${A_{P_\alpha}}^t$.

Proof: Assume that B is S-α anti fuzzy normal in A. Let $t \in [B_{P_\alpha}(e),1]$**.** Then $x^t \diamond B_{P_\alpha} = B_{P_\alpha} \diamond x^t$ and ${B_{P_\alpha}}^t$ and ${A_{P_\alpha}}^t$ are subgroups of P. Let $x \in {A_{P_\alpha}}^t$ and $b \in {B_{P_\alpha}}^t$. Therefore $(x^t \diamond B_{P_\alpha})(bx) = (B_{P_\alpha} \diamond x^t)(bx)$ which implies that $\sup\{t, B_{P_\alpha}(x^{-1}bx)\} = \sup\{t, B_{P_\alpha}(bxx^{-1})\} = \sup\{t, B_{P_\alpha}(b)\} = t$ .Thus $B_{P_\alpha}(x^{-1}bx) \leq t$ .Therefore $x^{-1}bx \in {B_{P_\alpha}}^t$ which implies that ${B_{P_\alpha}}^t$ is normal in ${A_{P_\alpha}}^t$.

Theorem 3.20. Let G be an S-semigroup. Let A and B be S-α anti fuzzy semigroups of G relative to a group P in G such that $B \subset A$. Assume that $B_{P_\alpha}(e) \leq t \leq 1$ and $x^s \subset A$. Let $1-\alpha \leq t$ and $t \geq s$. Then ${(x^s \diamond B_{P_\alpha})_{P_\alpha}}^t = x{B_{P_\alpha}}^t$ and ${(B_{P_\alpha} \diamond x^s)_{P_\alpha}}^t = {B_{P_\alpha}}^t x.$

Proof: ${B_{P_\alpha}}^t = \{x \in P / B_{P_\alpha}(x) \leq t$. ${(x^s \diamond B_{P_\alpha})_{P_\alpha}}^t = \{y \in P / (x^s \diamond B_{P_\alpha})_{P_\alpha}(y) \leq t\}$.

If $y \in {(x^s \diamond B_{P_\alpha})_{P_\alpha}}^t$, then $(x^s \diamond B_{P_\alpha})_{P_\alpha}(y) \leq t$ .This implies that $(x^s \diamond B_{P_\alpha})(y) \leq t \Rightarrow \sup\{s, B_{P_\alpha}(x^{-1}y)\} \leq t \leq B_{P_\alpha}(x^{-1}y) \leq t \Rightarrow y \in x{B_{P_\alpha}}^t$. Thus ${(x^s \diamond B_{P_\alpha})_{P_\alpha}}^t \subset x{B_{P_\alpha}}^t$. Now let $y \in x{B_{P_\alpha}}^t$. Then $B_{P_\alpha}(x^{-1}y) \leq t \Rightarrow (x^s \diamond B_{P_\alpha})_{P_\alpha}(y) \leq t$ (since $s \leq t$)

$\Rightarrow \max\{(x^s \diamond B_{P_\alpha})(y), 1-\alpha\} \leq t$ (since $\alpha \leq t$) $\Rightarrow y \in {(x^s \diamond B_{P_\alpha})_{P_\alpha}}^t$.

Thus ${(x^s \diamond B_{P_\alpha})_{P_\alpha}}^t = x{B_{P_\alpha}}^t$. Similarly , ${(B_{P_\alpha} \diamond x^s)_{P_\alpha}}^t = {B_{P_\alpha}}^t x$ can be proved.

4. Conclusion

In this paper, S-α anti fuzzy normal subsemigroups, which are some special types of fuzzy normalsubgroups, are introduced by defining S-α anti fuzzy left and right cosets. Also their characterizations have been developed. The characterizations of S-α anti fuzzy normal subsemigroups may be extended to fuzzy semirings, fuzzy semi vectorspaces and fuzzy bigroups.

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