S-\(\alpha\) Anti Fuzzy Normal Subsemigroups in S-\(\alpha\) Anti Fuzzy Semigroups

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Abstract. The notion of S-\(\alpha\) anti fuzzy cosets with representative \(x^t\) and S-\(\alpha\) anti fuzzy normal subsemigroups are introduced and their characterizations are obtained. It is also proved that the set of all S-\(\alpha\) anti fuzzy cosets will form a semigroup under a suitable binary operation and its structural properties are determined. A necessary condition for an S-\(\alpha\) anti fuzzy semigroup to be S-\(\alpha\) anti fuzzy normal is also proved.

Keywords: S-semigroup, anti-fuzzy group, \(\alpha\)-anti fuzzy set, \(\alpha\)-anti fuzzy group, S- anti fuzzy semigroup, S-\(\alpha\) anti fuzzy semigroup.

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1. Introduction

The concept of fuzzy set was introduced by Zadeh in 1965 [9]. A fuzzy subgroup of a group was defined by Rosenfeld in 1971 [5]. Fuzzy groups were redefined by Anthony and H. Sherwood in 1979[1]. Malik, Mordeson and Nair introduced the notion of fuzzy cosets of \(B\) in \(A\) with representative \(x_t\) and fuzzy normal, where \(A\) and \(B\) are fuzzy subgroups of a group \(G\) such that \(B \subset A\) [3]. Rajeshkumar analyzed Fuzzy Algebra in 1993[4]. VasanthaKandasamy studied Smarandache fuzzy semigroups in [7]. Sharma introduced the concept of \(\alpha\)-anti fuzzy set, \(\alpha\)-anti fuzzy group, \(\alpha\)-anti fuzzy coset, \(\alpha\)-anti fuzzy normal subsemigroups and obtained their properties in [6]. Gowri and Rajeswari introduced the idea of S-\(\alpha\) anti fuzzy semigroup, S-\(\alpha\) anti fuzzy left, right cosets and S-\(\alpha\) anti fuzzy normal subsemigroup and analyzed their properties [2]. Vijayakumar and T. Rajeswari introduced the concept of anti fuzzy cosets of \(B\) in \(A\) with representative \(x^t\) and anti fuzzy normal[8]. In this paper, the concepts of S-\(\alpha\) anti fuzzy cosets with representative \(x^t\) and S-\(\alpha\) anti fuzzy normal subsemigroups are introduced. These ideas differ from those in [2]. It is also proved that the set of all S-\(\alpha\) anti fuzzy cosets will form a semigroup under a suitable binary operation and its structural properties are determined.

Throughout this paper, \(\alpha\) will always denote a member of \([0, 1]\).
2. Preliminaries

Definition 2.1. Let \( X \) be a non-empty set. A fuzzy subset \( A \) of \( X \) is a function
\( A : X \rightarrow [0,1] \)

Definition 2.2. A fuzzy subset \( A \) of a group \( G \) is called an anti fuzzy group of \( G \) if
(i) \( A(xy) \leq \max\{A(x), A(y)\} \)
(ii) \( A(x^{-1}) = A(x) \), for all \( x, y \in G \)

Definition 2.3. Let \( A \) be a fuzzy subset of a group \( G \). Let \( \alpha \in [0,1] \). Then an \( \alpha \)-anti fuzzy subset of \( G \) (with respect to a fuzzy set \( A \)), denoted by \( A_\alpha \), is defined as
\( A_\alpha(x) = \max\{A(x), 1-\alpha\} \), for all \( x \in G \).

Definition 2.4. Let \( A \) be a fuzzy subset of a group \( G \) and \( \alpha \in [0,1] \). Then \( A \) is called an \( \alpha \)-anti fuzzy subgroup of \( G \) if \( A_\alpha \) is an anti fuzzy group.

Definition 2.5. A semigroup \( S \) is said to be a Smarandache semigroup (S-semigroup) if there exists a proper subset \( P \) of \( S \) which is a group under the same binary operation in \( S \).

Definition 2.6. Let \( G \) be an S-semigroup. Let \( A \) be a fuzzy subset of \( G \) and \( \alpha \in [0,1] \). \( A \) is called a Smarandache \( \alpha \) anti fuzzy semigroup (S-\( \alpha \) anti fuzzy semigroup) if there exists a proper subset \( P \) of \( G \) which is a group and the restriction of \( A \) to \( P \) is such that \( A_\alpha \) is an anti fuzzy group. That is,
(i) \( A_\alpha(xy) \leq \max\{A_\alpha(x), A_\alpha(y)\} \)
(ii) \( A_\alpha(x^{-1}) = A_\alpha(x) \), for all \( x, y \in P \)

Result 2.7. [2] If \( A : G \rightarrow [0,1] \) is an S-\( \alpha \) anti fuzzy semigroup of an S-semigroup \( G \) relative to a group \( P \) which is a proper subset of \( G \), then
(i) \( A_\alpha(x) \geq A_\alpha(e) \), where \( e \) is the identity element of \( P \)
(ii) \( A_\alpha(xy^{-1}) = A_\alpha(e) \Rightarrow A_\alpha(x) = A_\alpha(y) \), for all \( x, y \in P \)

Result 2.8. [2] Let \( G \) be an S-semigroup and \( P \) be a proper subset of \( G \) which is a group. Then \( A : G \rightarrow [0,1] \) is an S-\( \alpha \) anti fuzzy semigroup of \( G \) relative to \( P \) iff
\( A_\alpha(xy^{-1}) \leq \max\{A_\alpha(x), A_\alpha(y)\} \), for all \( x, y \in P \)
3. S-α anti fuzzy cosets and S-α anti fuzzy normal subsemigroups

In this section, we define S-α anti fuzzy cosets with representative \( x^t \) and S-α anti fuzzy normal subsemigroups and obtain their characterizations.

**Definition 3.1.** [8] Let \( \mathbb{X} \) be a non empty set. For any \( x \in \mathbb{X} \) and \( t \in [0,1] \), a fuzzy singleton, denoted by \( x^t \), is defined as \( x^t : \mathbb{X} \rightarrow [0,1] \) is a mapping.

**Definition 3.2.** [8] Let \( \mathbb{X} \) be a non empty set and let \( \cdot \) be a binary operation on \( \mathbb{X} \). Let \( A \) and \( B \) be fuzzy subsets of \( \mathbb{X} \). Define the fuzzy subset \( A \odot B \) of \( \mathbb{X} \) by

\[
A \odot B = \left\{ \begin{array}{ll}
\inf \{ \sup \{ A(y), B(z) \} / x = yz \} & \text{if } x = yz \\
1 & \text{if } x \neq yz
\end{array} \right.
\]

**Remark 3.3.** [8] If the operation \( \cdot \) in \( \mathbb{X} \) is associative, commutative respectively, then so is \( \odot \).

**Definition 3.4.** Let \( \mathbb{X} \) be an \( \mathbb{S}\)-semigroup. Let \( A \) and \( B \) be \( \mathbb{S}\)-anti fuzzy semigroups of \( \mathbb{X} \) relative to a group \( \mathbb{G} \) in \( \mathbb{X} \) such that \( B \subseteq A \). Let \( x \in \mathbb{P} \) and let \( x^t \subseteq A \). Then the fuzzy subset \( x^t \odot B_{P_a} \) is called the Smarandache-\( \mathbb{S}\)-anti fuzzy left coset(S-\( \mathbb{S}\)-anti fuzzy left coset) of \( B \) in \( A \) with representative \( x^t \).

That is, \( (x^t \odot B_{P_a}) (z) = \inf \{ \sup \{ x^t (u), B_{P_a} (v) \} / z = uv \} \) for all \( z \in \mathbb{P} \)

**Example 3.5.** Let \( G = \{ e, a, b, c, d, e, f, g \} \) which is a semigroup by the following table.

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Let \( P = \{ e, a, b, c \} \) which is the klein four group. \( P \subseteq G \). Two fuzzy subsets \( A \) and \( B \) of \( G \) are defined as

\[
A(x) = \begin{cases} 
1, & \text{if } x = e, a \\
\frac{3}{4}, & \text{if } x = b, c \\
0, & \text{otherwise}
\end{cases} \quad \text{and} \quad B(x) = \begin{cases} 
1, & \text{if } x = e, a \\
\frac{1}{2}, & \text{if } x = b, c \\
0, & \text{otherwise}
\end{cases}
\]

If take \( \alpha = 0.85 \), then \( A \) and \( B \) be \( \mathbb{S}\)-anti fuzzy semigroups. Also \( B \subseteq A \) and let \( t = 0.6 \). For \( x = a \), \( x^t (y) = \begin{cases} 
0.6, & \text{if } y = x \\
1, & \text{if } y \neq x
\end{cases} \) for all \( y \in P \).
Thus $\alpha^t \subset A$ and an $S$-\(\alpha\) anti fuzzy left coset of $B$ in $A$ with representatives $x^t$ is given by $(\alpha^t B_{p_a})(z) = \begin{cases} 0.75, & \text{if } z = e, a \\ 1, & \text{if } z = b, c \end{cases}$

**Definition 3.6.** Let $G$ be an $S$-semigroup. Let $A$ and $B$ be $S$-\(\alpha\) anti fuzzy semigroups of $G$ relative to a group $P$ in $G$ such that $B \subset A$. Let $x \in P$ and let $x^t \subset A$. Then the fuzzy subset $B_{p_a} \alpha^t$ is called the Smarandache-\(\alpha\) anti fuzzy right coset of $B$ in $A$ with representatives $x^t$.

That is, $(B_{p_a} \alpha^t)(z) = \sup \{\inf \{B_{p_a}(u), x^t(v)\}/z = uv \} \text{ for all } z \in P$

**Example 3.7.** In example 3.4, for $x = a$, an $S$-\(\alpha\) anti fuzzy right coset of $B$ in $A$ with representatives $x^t$ is given by $(B_{p_a} \alpha^t)(z) = \begin{cases} 0.75, & \text{if } z = e, a \\ 1, & \text{if } z = b, c \end{cases}$

**Theorem 3.8.** Let $G$ be an $S$-semigroup. Let $A$ and $B$ be $S$-\(\alpha\) anti fuzzy semigroups of $G$ relative to a group $P$ in $G$ such that $B \subset A$. Let $x, y \in P$ and let $x^t, y^t \subset A$. Then $x^t, y^t \subset A$.

(i) $x^t \cdot B_{p_a} = y^t \cdot B_{p_a}$ iff $\sup \{t, B_{p_a}(e)\} = \sup \{s, B_{p_a}(y^t x)\}$ and $\sup \{s, B_{p_a}(y^t x)\} = \sup \{t, B_{p_a}(x^t y)\}$.

(ii) $B_{p_a} \alpha^t = B_{p_a} \alpha^t$ iff $\sup \{t, B_{p_a}(e)\} = \sup \{s, B_{p_a}(x^t y)\}$

and $\sup \{t, B_{p_a}(e)\} = \sup \{s, B_{p_a}(y^t x)\}$

**Proof:**
(i) Suppose that $x^t \cdot B_{p_a} = y^t \cdot B_{p_a}$. Then $(x^t \cdot B_{p_a})(z) = (y^t \cdot B_{p_a})(z)$ for all $z \in P$. If we take $z = x$ and then $z = y$, then by theorem 3.8, we have $\sup \{t, B_{p_a}(e)\} = \sup \{s, B_{p_a}(y^t x)\}$ and $\sup \{s, B_{p_a}(y^t x)\} = \sup \{t, B_{p_a}(x^t y)\}$. Conversely, suppose that the conditions concerning the supremum hold. Let $z \in P$. Then $(x^t \cdot B_{p_a})(z) = \sup \{t, B_{p_a}(x^t y)\} = \sup \{t, B_{p_a}(x^t y)\}$

(ii) This can be proved similarly.

**Corollary 3.10.** Let $G$ be an $S$-semigroup. Let $A$ and $B$ be $S$-\(\alpha\) anti fuzzy semigroups of $G$ relative to a group $P$ in $G$ such that $B \subset A$. Let $x^t, y^t \subset A$, where
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where \(x, y \in P\). If \(B_{P_e}(y^{-1}x) = B_{P_e}(e)\), then \(x' \ B_{P_e} = y' \ B_{P_e}\).

**Proof:** Since \(B_{P_e}(x^{-1}y) = B_{P_e}(y^{-1}x)\), \(\sup\{t, B_{P_e}(e)\} = \sup\{t, B_{P_e}(x^{-1}y)\} = \sup\{t, B_{P_e}(y^{-1}x)\}\) which leads to \(x' \ B_{P_e} = y' \ B_{P_e}\), by theorem 3.9(i).

**Theorem 3.11.** Let \(G\) be an S-semigroup. Let \(A\) and \(B\) be S-\(\alpha\) anti fuzzy semigroups of \(G\) relative to a group \(P\) in \(G\) such that \(B \subset A\). Let \(x, y \in P\), where \(x, y \in P\). Then the following conditions are equivalent

(i) \(x' \ B_{P_e} = y' \ B_{P_e}\)
(ii) \((y^{-1})' \ B_{P_e} = e' \ B_{P_e}\)
(iii) \((x' y')' \ B_{P_e} = e' \ B_{P_e}\)

**Proof:** \(x' \ B_{P_e} = y' \ B_{P_e}\) iff \(\sup\{t, B_{P_e}(e)\} = \sup\{t, B_{P_e}(y^{-1}x)\}\) and \(\sup\{t, B_{P_e}(e)\} = \sup\{t, B_{P_e}(x^{-1}y)\}\) iff \((y^{-1})' \ B_{P_e} = e' \ B_{P_e}\), by theorem 3.9(i) and hence (i) \(\iff\) (ii). Similarly it is easy to see that (i) \(\iff\) (iii).

**Theorem 3.12.** Let \(G\) be an S-semigroup. Let \(A\) and \(B\) be S-\(\alpha\) anti fuzzy semigroups of \(G\) relative to a group \(P\) in \(G\) such that \(B \subset A\). Let \(x, y \in P\) and \(s, t \in [A_{P_e}(e), 1]\).

Let \(x, y \subset A\). Suppose that \(B_{P_e}(e) = A_{P_e}(e)\). Then

(i) \(x' \ B_{P_e} = y' \ B_{P_e} \iff t = \sup\{s, B_{P_e}(y^{-1}x)\} \text{ and } s = \sup\{t, B_{P_e}(x^{-1}y)\}\)
(ii) \(x' \ B_{P_e} = y' \ B_{P_e} \iff (y^{-1})' \ B_{P_e} \supseteq B_{P_e}\)
(iii) \(x' \ B_{P_e} = y' \ B_{P_e} \iff t = s \geq B_{P_e}(x^{-1}y)\)
(iv) \(x' \ B_{P_e} = y' \ B_{P_e} \iff t = s\)

**Proof:** (i) By theorem 3.9, \(x' \ B_{P_e} = y' \ B_{P_e} \iff \sup\{t, A_{P_e}(e)\} = \sup\{s, B_{P_e}(y^{-1}x)\}\) and \(\sup\{s, A_{P_e}(e)\} = \sup\{t, B_{P_e}(x^{-1}y)\}\) \(\iff t = \sup\{s, B_{P_e}(y^{-1}x)\}\) and \(s = \sup\{t, B_{P_e}(x^{-1}y)\}\).

(ii) By (i), \(x' \ B_{P_e} = y' \ B_{P_e} \iff t = \sup\{t, B_{P_e}(y^{-1}x)\}\) and \(t = \sup\{t, B_{P_e}(x^{-1}y)\}\) iff \((y^{-1})' \supseteq B_{P_e}\).

(iii) If \(x' \ B_{P_e} = y' \ B_{P_e}\), then by (i), \(t = \sup\{s, B_{P_e}(x^{-1}y)\}\) and \(s = \sup\{t, B_{P_e}(x^{-1}y)\}\) which implies that \(t = s \geq B_{P_e}(x^{-1}y)\). Conversely, assume that \(t = s \geq B_{P_e}(x^{-1}y)\). This implies that \(s, B_{P_e}(y^{-1}x)\) \(\subset\) \(t\) and \(s, B_{P_e}(x^{-1}y)\) \(=\) \(t\) and hence \(x' \ B_{P_e} = y' \ B_{P_e}\).

(iv) From (iii) the result is obvious.

**Corollary 3.13.** Let \(G\) be an S-semigroup. Let \(A\) and \(B\) be S-\(\alpha\) anti fuzzy
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semigroups of $G$ relative to a group $P$ in $G$ such that $B \subset A$. Let $x, y \in P$ and $s, t \in [A_p(e), 1]$. Let $x', y' \in A$. Suppose that $B_{p_s}(e) = A_{p_s}(e)$. If $t \neq s$, then

\[
\{x' \cdot B_{p_s} / x' \subset A\} \cap \{y' \cdot B_{p_t} / y' \subset A\} = \emptyset.
\]

**Proof:** By theorem 3.12(iii), the result is obvious.

**Definition 3.14.** Let $G$ be an $S$-semigroup. Let $A$ and $B$ be $S$-$\alpha$ anti fuzzy semigroups of $G$ relative to a group $P$ in $G$ such that $B \subset A$. $B$ is said to be Smarandache-$\alpha$ anti fuzzy normal subsemigroup (S-$\alpha$ fuzzy anti normal subsemigroup) in $A$ if

\[
(x' \cdot B_{p_s} = B_{p_s} \cdot \phi', \forall x' \subset A), where x \in P.
\]

**Example 3.15.** From example 3.5 and 3.7, it can be easily seen that

\[
(x' \cdot B_{p_s} = B_{p_s} \cdot \phi', \forall x' \subset A).
\]

**Theorem 3.16.** Let $G$ be an $S$-semigroup. Let $A$ and $B$ be $S$-$\alpha$ anti fuzzy semigroups of $G$ relative to a group $P$ in $G$ such that $B \subset A$. Let $x, y \in P$ and $x', y' \in A$. If $B$ is an $S$-$\alpha$ anti fuzzy normal subsemigroup in $A$, then

\[
(x' \cdot B_{p_s}) \cup (y' \cdot B_{p_t}) = (xy)' \cdot B_{p_r}, where r = \sup\{t, s\}.
\]

**Proof:** By remark[3.3],

\[
(x' \cdot B_{p_s} \cup y' \cdot B_{p_t}) = (x' \cdot \phi') \cdot B_{p_r}.
\]

By the definition of fuzzy singleton set $x' \cdot \phi' = (xy)'$ which leads to the result.

**Theorem 3.17.** Let $G$ be an $S$-semigroup. Let $A$ and $B$ be $S$-$\alpha$ anti fuzzy semigroups of $G$ relative to a group $P$ in $G$ such that $B \subset A$. Let

\[
A/B = \{x' \cdot B_{p_s} / x' \subset A \text{ and } x \in P\}.
\]

Assume that $B$ is $S$-$\alpha$ anti fuzzy normal in $A$. Then $(A/B, \phi)$ is a semigroup with identity. If $B_{p_s}(e) = A_{p_s}(e)$, then $A/B$ is completely regular. That is, $A/B$ is a union of disjoint groups.

**Proof:** If $x' \cdot B_{p_s}, y' \cdot B_{p_t} \in A/B$, where $x', y' \subset A$, then by theorem 3.16,

\[
(xy)' \cdot B_{p_r} \in A/B, \text{ where } r = \sup\{t, s\}.\]

It can be easily seen that $e^{A_p(e)}$ is the identity of $A/B$. By remark[3.3], $\phi$ is associative and hence $(A/B, \phi)$ is a semigroup with identity $e^{A_p(e)}$. For fixed $t \in [A_{p_s}(e), 1]$, define $(A/B)^{(t)} = \{x' \cdot B_{p_t} / x' \subset A \text{ and } x \in P\}$. Then $(A/B)^{(t)}$ is closed and the identity of $(A/B)^{(t)}$ is $e^{t} \cdot B_{p_t}$. It is also easy to see that $(x^{-1})' \cdot B_{p_t}$ is the inverse of $x' \cdot B_{p_t}$. Thus $(A/B)^{(t)}$ is a group. Moreover $A/B = \bigcup_{t \in [A_{p_s}(e), 1]} (A/B)^{(t)}$ and hence $A/B$ is completely regular.
Remark 3.18. Let $G$ be an $S$-semigroup. Let $A$ and $B$ be $S\alpha$ anti fuzzy semigroups of $G$ relative to a group $P$ in $G$ such that $B \subseteq A$. For $t \in [0,1]$, we define $B_p^\prime = \{ x \in P / B_p^\prime (x) \leq t \}. Then it can be proved that if $t \in \text{Im} B_p^\prime$, then $B_p^\prime$ is a subgroup of $P$. Since $B \subseteq A$, $A_p^\prime$ is also a subgroup of $P$.

Theorem 3.19. Let $G$ be an $S$-semigroup. Let $A$ and $B$ be $S\alpha$ anti fuzzy semigroups of $G$ relative to a group $P$ in $G$ such that $B \subseteq A$. If $B$ is $S\alpha$ anti fuzzy normal in $A$, then for all $t \in [0,1]$, $B_p^\prime$ is normal in $A_p^\prime$.

Proof: Assume that $B$ is $S\alpha$ anti fuzzy normal in $A$. Let $t \in [B_p^\prime (e),1]$. Then $x^t B_p^\prime = B_p^\prime x^\alpha$ and $B_p^\prime$ and $A_p^\prime$ are subgroups of $P$. Let $x \in A_p^\prime$ and $b \in B_p^\prime$. Therefore $(x^t B_p^\prime)(b) = (B_p^\prime x^\alpha)(b)$ which implies that $\sup \{ t, B_p^\prime (x^{-1}bx) \} = \sup \{ t, B_p^\prime (b) \} = t$. Thus $B_p^\prime (x^{-1}bx) \leq t$. Therefore $x^{-1}bx \in B_p^\prime$, which implies that $B_p^\prime$ is normal in $A_p^\prime$.

Theorem 3.20. Let $G$ be an $S$-semigroup. Let $A$ and $B$ be $S\alpha$ anti fuzzy semigroups of $G$ relative to a group $P$ in $G$ such that $B \subseteq A$. Assume that $B_p^\prime (e) \leq t \leq 1$ and $x^t \subseteq A$. Let $1 - \alpha \leq t$ and $t \geq s$. Then $(x^t B_p^\prime)_p^\prime = xB_p^\prime$ and $(B_p^\prime x^\alpha)_p^\prime = B_p^\prime x$.

Proof: $B_p^\prime = \{ x \in P / B_p^\prime (x) \leq t \}$. Then $(x^t B_p^\prime)_p^\prime = \{ y \in P / (x^t B_p^\prime)_p^\prime (y) \leq t \}$. If $y \in (x^t B_p^\prime)_p^\prime$, then $(x^t B_p^\prime)_p^\prime (y) \leq t$. This implies that $(x^t B_p^\prime)(y) \leq t \Rightarrow \sup \{ s, B_p^\prime (x^{-1}y) \} \leq t \Rightarrow B_p^\prime (x^{-1}y) \leq t \Rightarrow y \in xB_p^\prime$. Thus $(x^t B_p^\prime)_p^\prime \subseteq xB_p^\prime$. Now let $y \in xB_p^\prime$. Then $B_p^\prime (x^{-1}y) \leq t \Rightarrow (x^t B_p^\prime)_p^\prime (y) \leq t$ (since $s \leq t$) $\Rightarrow \max \{ (x^t B_p^\prime)(y), 1 - \alpha \} \leq t$ (since $\alpha \leq t$) $\Rightarrow y \in (x^t B_p^\prime)_p^\prime$. Thus $(x^t B_p^\prime)_p^\prime = xB_p^\prime$. Similarly, $(B_p^\prime x^\alpha)_p^\prime = B_p^\prime x$ can be proved.

4. Conclusion
In this paper, $S\alpha$ anti fuzzy normal subsemigroups, which are some special types of fuzzy normal subgroups, are introduced by defining $S\alpha$ anti fuzzy left and right cosets. Also their characterizations have been developed. The characterizations of $S\alpha$ anti fuzzy normal subsemigroups may be extended to fuzzy semirings, fuzzy semi vectorspaces and fuzzy bigroups.

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