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Solving Fuzzy Fractional Differential Equation with Fuzzy Laplace Transform Involving Coshx

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Abstract. In this paper we obtain the solution of fuzzy fractional differential equation (FFDEs) under Riemann Liouville H-differentiability using fuzzy Laplace transform. To solve fractional differential equation involving hyperbolic cosine function, we use Riemann Liouville to obtain the unknown solution at initial point or the solution with increasing length of their Hukuhara support using the fuzzy Laplace transform. Research work on analytical method to solve the FFDEs under Riemann Liouville H-differentiability is limited in the literature. To confirm the capability of the proposed method we present a problem and its analytical solution.

Keywords: Fuzzy fractional differential equation, Fuzzy Laplace transforms, Hyperbolic function, hyper geometric function

AMS Mathematics Subject Classification (2010): 34A07, 34A08

1. Introduction

Fractional calculus has been applied in modeling physical and chemical processes and in engineering [4,6,9]. We are reviewing the properties of derivatives and integrals of non-integer orders. Podlubnyandkilbas [10,12,] gave the idea of fractional calculus and consider Riemann Liouville differentiability to solve Fuzzy Fractional Differential Equations (FFDEs).

We use fuzzy Laplace transforms to solve FFDEs .The merits of fuzzy Laplacetrans form is that it solves the problem directly with out determining a general solution.

In section 2, we define basic definitions and Riemann Liouville H-differentiability. Fuzzy Laplace transforms are introduced and we discuss the properties in section 3. The solutions of FFDEs are determined by Fuzzy Laplace transform under Riemann Liouville H-differentiability and solve the example involving hyperbolic cosine function in section 4. In section 5, a conclusion is drawn.

2. Preliminaries

Definition 2.1. [8] Fuzzy number is a mapping $u: \mathbb{R} \rightarrow [0,1]$ with the following properties:

1. *u* is upper semi continuous.

2. *u* is fuzzy convex. i.e., $u(\lambda x + (1 - \lambda)y) \ge \min \{u(x), u(y)\}$ for all $x, y \in \mathbb{R}$, $\lambda \in [0,1]$.

3. *u* is normal. i.e., $\exists x_0 \in \mathbb{R}$ for which $u(x_0) = 1$.

4. Supp $u = \{x \in R/u(x) > 0\}$ is the support of the *u*, and its closure cl(supp *u*) is compact.

Definition 2.2. [13,14]

A fuzzy number u in parametric form is a pair $(\underline{u}, \overline{u})$ of functions $\underline{u}(r), \overline{u}(r), 0 \le r \le 1$, which satisfy the following requirements:

- 1. $\underline{u}(r)$ is a bounded non-decreasing left continuous function in (0,1], and right continuous at 0,
- 2. $\overline{u}(r)$ is a bounded non-increasing left continuous function in (0,1], and right continuous at 0,
- 3. $3.\underline{u}(r) \le \overline{u}(r), 0 \le r \le 1.$

Theorem 2.1. [15] Let f be fuzzy valued function on $[a,\infty)$ represented by $(\underline{f}(x;r), \overline{f}(x;r))$. For any fixed $r \in [0,1]$, assume $\underline{f}(x;r)$ and $\overline{f}(x;r)$ are Riemannintegrable on [a,b] for every $b \ge a$, and assume there are two positive functions $\underline{M}(r), \overline{M}(r)$ such that $\int_a^b |\underline{f}(x;r)| dx \le \underline{M}(r)$ and $\int_a^b |\overline{f}(x;r)| dx \le \overline{M}(r)$ for every $b \ge a$. Then f(x) is improper fuzzy Riemann integrable on $[a,\infty)$ and the improper fuzzy Riemann integral is a fuzzy number. Further more, we have $\int_a^\infty f(x;r) dx = \left[\int_a^\infty \underline{f}(x;r) dx, \int_a^\infty \overline{f}(x;r) dx\right]$.

Definition 2.4. Let $x, y \in E$. If there exists $z \in E$ such that x = y + z, then z is called the H- difference of x and y and it is denoted by $x \ominus y$.

Riemann Liouville H-differentiability[7]

 $C^{F}[a,b]$ is the space of all continuous fuzzy valued functions on [a,b]. Also we denote the space of all Lebesgue integrable fuzzy valued functions on [a,b] by $L^{F}[a,b]$.

Definition 2.5. Let $f \in C^F[a,b] \cap L^F[a,b], x_0$ in (a,b) and $\Phi(x) = \frac{1}{\Gamma(1-\beta)} \int_a^x \frac{f(t)dt}{(x-t)^{\beta}}$. We say that f is Riemann Liouville H- differentiable about order $0 < \beta < 1$ at x_0 , if there exists an element $\binom{RL}{a+f} \binom{\beta}{a+f} (x_0) \in E$ such that for h>0 sufficiently small

(i)
$$\binom{RL}{D_{a}^{\beta}} f(x_{0}) = \lim_{h \to 0^{+}} \frac{\phi(x_{0}+h) \ominus \phi(x_{0})}{h} = \lim_{h \to 0^{+}} \frac{\phi(x_{0}) \ominus \phi(x_{0}-h)}{h}$$
 (or)
(ii) $\binom{RL}{D_{a}^{\beta}} f(x_{0}) = \lim_{h \to 0^{+}} \frac{\phi(x_{0}) \ominus \phi(x_{0}+h)}{-h} = \lim_{h \to 0^{+}} \frac{\phi(x_{0}-h) \ominus \phi(x_{0})}{-h}$ (or)

 $(\mathrm{iii})(^{RL}D_{a^+}^{\beta}f)(x_0) =_{h \to 0^+}^{\lim \Phi(x_0+h) \ominus \Phi(x_0)} =_{h \to 0^+}^{\lim \Phi(x_0-h) \ominus \Phi(x_0)} =_{h \to 0^+}^{\lim \Phi(x_0-h) \ominus \Phi(x_0)}$

(or)

$$(iv)(^{RL}D_{a^{+}}^{\beta}f)(x_{0}) = \lim_{h \to 0^{+}} \frac{\phi(x_{0}) \ominus \phi(x_{0}+h)}{-h} = \lim_{h \to 0^{+}} \frac{\phi(x_{0}) \ominus \phi(x_{0}-h)}{h}$$

We say that the fuzzy valued function f is $({}^{RL}(i) - \beta)$ differentiable if it is differentiable as in the definition 2.5 case(i), and f is $({}^{RL}(ii) - \beta)$ differentiable if it is differentiable as in the definition 2.5 of case (ii) and so on for other cases.

Theorem 2.2. [17]

Let $f \in C^F[a,b] \cap L^F[a,b], x_0$ in (a,b) and $0 < \beta < 1$. Then

(i) Let us consider f is $\binom{RL(i) - \beta}{a^+ f}$ differentiable fuzzy valued function, then $\binom{RLD_{a^+}^{\beta}f}{a^+ f}(x_0; r) = \left[\binom{RLD_{a^+}^{\beta}f}{a^+ f}(x_0; r), \binom{RLD_{a^+}^{\beta}f}{a^+ f}(x_0; r)\right], 0 \le r \le 1$

(ii) Let us consider f is
$$\binom{RL(ii) - \beta}{a^+ f}$$
 differentiable fuzzy valued function, then $\binom{RLD_{a^+}^{\beta}f}{a^+ f}(x_0; r) = \left[\binom{RLD_{a^+}^{\beta}f}{a^+ f}(x_0; r), \binom{RLD_{a^+}^{\beta}f}{a^+ f}(x_0; r)\right], 0 \le r \le 1$

where
$$\binom{RL}{a+f} D_{a+f}^{\beta}(x_0;r) = \left[\frac{1}{\Gamma(1-\beta)}\frac{d}{dx}\int_a^x \frac{f(t;r)dt}{(x-t)^{\beta}}\right]_{x=x_0}$$
 (1)

$$\binom{RL}{a^{+}} \overline{f}(x_{0};r) = \left[\frac{1}{\Gamma(1-\beta)} \frac{d}{dx} \int_{a}^{x} \frac{\overline{f}(t;r)dt}{(x-t)^{\beta}}\right]_{x=x_{0}}$$
(2)

4. Fuzzy Laplace transforms

Definition 3.1. [16]

Let f be continuous fuzzy valued function. Suppose that $f(x) \odot e^{-px}$ is improper fuzzy Riemann integrable on $[0,\infty)$, then $\int_0^\infty f(x) \odot e^{-px} dx$ is called fuzzy Laplace transforms and denoted by

$$L[f(x)] = \int_0^\infty f(x) \Theta e^{-px} dx \quad \text{(p>0 and integer)}$$
(3)

Using Theorem 2.1 we have $0 \le r \le 1$;

$$\int_0^\infty f(x;r) \odot e^{-px} \, dx = \left[\int_0^\infty \underline{f}(x;r) \odot e^{-px} \, dx \, \int_0^\infty \overline{f}(x;r) \odot e^{-px} \, dx \right]$$

Using the classical Laplace transform,

$$l\left[\underline{f}(x;r)\right] = \int_0^\infty \underline{f}(x;r)e^{-px} \, dx \quad and \ l\left[\overline{f}(x;r)\right] = \int_0^\infty \overline{f}(x;r)e^{-px} \, dx$$

Then we get

$$L[f(x;r)] = \left[l\left[\underline{f}(x;r)\right], l[\overline{f}(x;r)\right]$$

Definition 3.2. Hypergeom (n, d, z) is the generalized hypergeometric function F(n, d, z), also known as Barnes extended hyper geometric function. For scalars a, b and c, hypergeom ([a,b],c,z) is a Gauss hyper geometric function $2F_1(a, b; c; z)$. The Gauss hypergeometric function $2F_1(a, b; c; z)$ is defined in the unit disc as the sum of the hypergeometric series

$$2F_1(a,b;c;z) = \sum_{k=0}^{\infty} \frac{(a)_k(b)_k}{(c)_k} \frac{z^k}{k!}, |z| < 1$$

Definition 3.3. The pochhammer symbol $(a)_k$ is defined by

$$(a)_0 = 1$$
,
 $(a)_n = a(a+1) \dots (a+n-1), n \in N$

Definition 3.4 A two parameters function of Mittag -Leffler type is defined by the series expansion

$$E_{\alpha,\beta}(z) = \sum_{r=0}^{\infty} \frac{z^r}{\Gamma(\alpha r + \beta)} \qquad (\alpha,\beta > 0)$$

An error function is defined by $erfc(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} e^{-t^2} dt$.

Theorem 3.1. [16]

Let f and g are continuous fuzzy valued functions. Suppose that c_1 and c_2 are constants.

$$L[(c_1 \odot f(x)) \oplus (c_2 \odot g(x))] = (c_1 \odot L[f(x)]) \oplus (c_2 \odot L[g(x)])$$

Lemma 3.2. [16] Let *f* be continuous fuzzy valued function on $[0,\infty)$ and $\lambda \in \mathbb{R}$ then $L[\lambda \odot f(x)] = \lambda \odot L[f(x)]$

Theorem 3.2. Suppose that
$$f \in C^{F}[0,\infty) \cap L^{F}[0,\infty)$$
. Then

$$L\left[\binom{RL}{a^{+}}f(x)\right] = s^{\beta}L[f(t)] \ominus \binom{RL}{a^{+}}f(0), \qquad (4)$$
if f is $\binom{RL}{i} - \beta$ differentiable, and

$$L\left[\binom{RL}{a^{+}}f(x)\right] = -\binom{RL}{a^{+}}f(0) \ominus - \binom{s^{\beta}L[f(t)]}{i} \qquad (5)$$
if f is $\binom{RL}{i} - \beta$ differentiable.

5. Fuzzy fractional differential equations under Riemann Liouville H-differentiability

Let $f \in C^F[a,b] \cap L^F[a,b]$ and consider the fuzzy fractional differential equation of order $0 < \beta < 1$ with the initial condition and $x_0 \in (a, b)$.

$$\begin{cases} ({}^{RL}D_{a^{+}}^{\beta}y)(x) = f[x,y(x)], \\ ({}^{RL}D_{a^{+}}^{\beta-1}y)(x_{0}) = ({}^{RL}y_{0}^{(\beta-1)}) \in E \end{cases}$$
(6)

Determining the solutions:

Here we use fuzzy Laplace transform and its inverse to derive the solution. By taking Laplace transform on both sides, we get

$$L[({}^{RL}D^{\beta}_{a^{+}}y)(x)] = L[f(x,y(x))],$$
(7)

Based on the Riemann Liouville H- differentiability, we have the following cases:

Case (i) Let us consider y(x) is a $\binom{RL}{i} - \beta$ differentiable function then the equation (7) is extended based on the its lower and upper functions as follows

$$s^{\beta}l\left[\underline{y}(x;r)\right] - \binom{RL}{a^{+}} \frac{D_{a^{+}}^{\beta-1}}{\underline{y}}(0;r) = l\left[\underline{f}(x,y(x);r)\right] \quad 0 \le r \le 1$$

$$s^{\beta}l[\overline{y}(x;r)] - \binom{RL}{a^{+}} \frac{D_{a^{+}}^{\beta-1}}{\overline{y}}(0;r) = l[\overline{f}(x,y(x);r)] \quad 0 \le r \le 1$$

$$\text{where } \underline{f}(x,y(x);r) = \min\left\{f(x,u)/u \in \left[\underline{y}(x;r),\overline{y}(x;r)\right]\right\}$$

$$\overline{f}(x,y(x);r) = \max\left\{f(x,u)/u \in \left[\underline{y}(x;r),\overline{y}(x;r)\right]\right\}$$

$$(8)$$

To solve the linear system (8), we assume that $H_1(p;r)$, $k_1(p;r)$ are the solutions

$$l\left[\underline{y}(x;r)\right] = H_1(p;r)$$

$$l[\overline{y}(x;r)] = k_1(p;r)$$
By using inverse Laplace transform $\underline{y}(x;r)$ and $\overline{y}(x;r)$ are computed as follows,
$$\underline{y}(x;r) = l^{-1}[H_1(p;r)]$$

$$\overline{y}(x;r) = l^{-1}[k_1(p;r)] \tag{9}$$

Case (ii)

Let us consider y(x) is a $\binom{RL(ii) - \beta}{RL(ii) - \beta}$ differentiable function then the equation (7) can be written as follows

$$\begin{cases} -\binom{RL}{a^{+}} \underbrace{y}{y}(0;r) - (-s^{\beta}l[\underbrace{y}(x;r)]) = l\left[\underline{f}(x,y(x);r)\right] \\ -\binom{RL}{a^{+}} \underbrace{\overline{y}}{y}(0;r) - (-s^{\beta}l[\underbrace{\overline{y}}(x;r)]) = l[\overline{f}(x,y(x);r)] \end{cases} \quad 0 \le r \le 1 \quad (10)$$
where $\underline{f}(x,y(x);r) = \min\left\{f(x,u)/u \in \left[\underline{y}(x;r), \overline{y}(x;r)\right]\right\}$

$$\overline{f}(x,y(x);r) = \max\left\{f(x,u)/u \in \left[y(x;r), \overline{y}(x;r)\right]\right\}$$

To solve the linear system (10), we assume that $H_2(p;r)$, $k_2(p;r)$ are the solutions

$$l[\underline{y}(x;r)] = H_2(p;r)$$

$$l[\overline{y}(x;r)] = k_2(p;r)$$
By using inverse Laplace transform $\underline{y}(x;r)$ and $\overline{y}(x;r)$ are computed as follows,
$$\underline{y}(x;r) = l^{-1}[H_2(p;r)]$$

$$\overline{y}(x;r) = l^{-1}[k_2(p;r)]$$
(11)

Example 1. Let us consider the following fuzzy fractional differential equation
$$\begin{cases} \binom{RL}{0}^{\beta}_{0^{+}} y (x) = \lambda \odot y(x) + \cosh x, \quad 0 < \beta, \ x < 1 \\ \binom{RL}{0}^{\beta-1}_{0^{+}} y (0) = \binom{RL}{y_{0}}^{(\beta-1)} \in E \end{cases}$$
(12)

Solution:

Case(i): Suppose $\lambda \in R^+ = (0, +\infty)$, then applying Laplace transform on both sides $L[({}^{RL}D_{0^+}^{\beta}y)(x)] = L[\lambda \odot y(x) + coshx],$ (13)

$$L[({}^{RL}D^{\beta}_{0^+}y)(x)] = L[\lambda \odot y(x)] + L[coshx],$$

 $\operatorname{Using}(^{RL}(i) - \beta)$ differentiability, we get

$$\begin{cases} s^{\beta}l\left[\underline{y}(x;r)\right] - \binom{RL}{a^{+}} \frac{y}{y}(0;r) = \lambda l\left[\underline{y}(x;r)\right] + \frac{s}{s^{2}-1} \\ s^{\beta}l[\overline{y}(x;r)] - \binom{RL}{a^{+}} \frac{\beta^{-1}}{\overline{y}}(0;r) = \lambda l[\overline{y}(x;r)] + \frac{s}{s^{2}-1} \\ \Rightarrow (s^{\beta} - \lambda) l\left[\underline{y}(x;r)\right] = \binom{RL}{a^{+}} \frac{p}{y}(0;r) + \frac{s}{s^{2}-1} \\ (s^{\beta} - \lambda) l\left[\overline{y}(x;r)\right] = \binom{RL}{a^{+}} \frac{p^{\beta-1}}{\overline{y}}(0;r) + \frac{s}{s^{2}-1} \\ l\left[\underline{y}(x;r)\right] = \binom{RL}{a^{+}} \frac{p^{\beta-1}}{\overline{y}}(0;r) \frac{1}{(s^{\beta}-\lambda)} + \frac{s}{(s^{2}-1)} \frac{s}{(s^{\beta}-\lambda)} \\ l[\overline{y}(x;r)] = \binom{RL}{a^{+}} \frac{p^{\beta-1}}{\overline{y}}(0;r) \frac{1}{(s^{\beta}-\lambda)} + \frac{s}{(s^{2}-1)} \frac{s}{(s^{\beta}-\lambda)} \end{cases}$$
(15)

$$\underline{y}(x;r) = \binom{RL}{a^{+}} \underbrace{y}_{a^{+}}(0;r) l^{-1} [\frac{1}{(s^{\beta}-\lambda)}] + l^{-1} [\frac{s}{(s^{2}-1)(s^{\beta}-\lambda)}]$$

$$\overline{y}(x;r) = \binom{RL}{a^{+}} \underbrace{y}_{a^{+}}(0;r) l^{-1} [\frac{1}{(s^{\beta}-\lambda)}] + l^{-1} [\frac{s}{(s^{2}-1)(s^{\beta}-\lambda)}]$$

$$\underline{y}(x;r) = \binom{RL}{2} \underbrace{y}_{0}^{(\beta-1)} (0;r) l^{-1} E_{\beta,\beta}(\lambda x^{\beta}) + \int_{0}^{x} (x-t)^{(\beta-1)} E_{\beta,\beta}(\lambda (x-t)^{\beta}) \cosh t dt$$

$$\overline{y}(x;r) = \binom{RL}{2} \underbrace{y}_{0}^{(\beta-1)} (0;r) l^{-1} E_{\beta,\beta}(\lambda x^{\beta}) + \int_{0}^{x} (x-t)^{(\beta-1)} E_{\beta,\beta}(\lambda (x-t)^{\beta}) \cosh t dt$$

$$(17)$$

Case (ii) Suppose $\lambda \in R^- = (-\infty, 0)$, then using $({}^{RL}(ii) - \beta)$ differentiability the solution will obtain similar to equ(17).

For the special case, let us consider
$$\beta = 0.5$$
, $\lambda = 1$ and
 $\binom{RL}{0^{+5}} y(0;r) = [1+r,3-r]$ in case (i).
 $\underline{y}(x;r) = [1+r,3-r] \odot x^{-\frac{1}{2}} E_{\frac{1}{2'2}}(x^{\frac{1}{2}}) + \int_{0}^{x} (x-t)^{(-\frac{1}{2})} E_{\frac{1}{2'2}}(x-t)^{\frac{1}{2}}) \cosh t dt$
 $\overline{y}(x;r) = [1+r,3-r] \odot x^{-\frac{1}{2}} E_{\frac{1}{2'2}}(x^{\frac{1}{2}}) + \int_{0}^{x} (x-t)^{(-\frac{1}{2})} E_{\frac{1}{2'2}}(x-t)^{\frac{1}{2}}) \cosh t dt$
(18)

Now consider 1^{st} term in eq (18)

$$\begin{aligned} x^{-\frac{1}{2}} E_{\frac{1}{2}\frac{1}{2}}(x^{\frac{1}{2}}) &= x^{-\frac{1}{2}} \left[\sum_{k=0}^{\infty} \frac{(x^{\frac{1}{2}})^{k}}{r(\frac{k+1}{2})} \right] &= \frac{1}{\sqrt{x}} \left[\frac{1}{r(\frac{1}{2})} + \frac{x^{\frac{1}{2}}}{r(1)} + \frac{x^{1}}{r(\frac{3}{2})} + \cdots \right] \\ &= \left[\frac{1}{\sqrt{\pi x}} + \left(\frac{x^{0}}{\Gamma(1)} + \frac{x^{\frac{1}{2}}}{r(\frac{3}{2})} + \cdots \right) \right] \\ &= \frac{1}{\sqrt{\pi x}} + \sum_{k=0}^{\infty} \frac{x^{\frac{k}{2}}}{r(\frac{k}{2}+1)} &= \frac{1}{\sqrt{\pi x}} + E_{\frac{1}{2},1}(x^{\frac{1}{2}}) \\ &= \frac{1}{\sqrt{\pi x}} + e^{(x^{\frac{1}{2}})^{2}} \text{ erfc } (-x^{\frac{1}{2}}) = \frac{1}{\sqrt{\pi x}} + e^{x} \text{ erfc } (-\sqrt{x}) \\ 2^{nd} \text{ term in eq. (18)} \\ \int_{0}^{x} (x-t)^{\left(-\frac{1}{2}\right)} E_{\frac{1}{2}\frac{1}{2^{\frac{1}{2}}}}(x-t)^{\frac{1}{2}}) \cosh t \, dt \\ &= \int_{0}^{x} \sum_{k=0}^{\infty} \frac{(x-t)^{\frac{k-1}{2}}}{r(\frac{k+1}{2})} \cosh t \, dt \\ &= \int_{0}^{x} \sum_{k=0}^{\infty} \frac{(x-t)^{\frac{k-1}{2}}}{r(\frac{k+1}{2})} \cosh t \, dt \\ &= \int_{0}^{x} \sum_{k=0}^{\infty} \frac{(x-t)^{\frac{k-1}{2}}}{r(\frac{k}{2})} \cosh t \, dt + \int_{0}^{x} \frac{(x-t)^{0}}{r(1)} \cosh t \, dt + \int_{0}^{x} \frac{(x-t)^{\frac{1}{2}}}{r(\frac{3}{2})} \cosh t \, dt + \int_{0}^{x} \frac{(x-t)^{1}}{r(2)} \cosh t \, dt + \int_{0}^{x} \frac{(x-t)^{\frac{1}{2}}}{r(\frac{3}{2})} \cosh t \, dt + \int_{0}^{x} \frac{(x-t)^{1}}{r(2)} \cosh t \, dt + \int_{0}^{x} \frac{(x-t)^{\frac{1}{2}}}{r(\frac{3}{2})} \cosh t \, dt + \dots - (*) \end{aligned}$$

Rearrange the terms split series

$$\begin{split} &= \frac{x^{\frac{1}{2}}}{(\frac{1}{2})!} \left[1 + \frac{2}{3} \cdot \frac{2}{5} x^{2} + \frac{2}{3} \cdot \frac{2}{5} \cdot \frac{2}{7} \cdot \frac{2}{9} x^{4} + \cdots \right] + \frac{x^{\frac{3}{2}}}{(\frac{2}{3})!} \left[1 + \frac{2}{5} \cdot \frac{2}{7} x^{2} + \frac{2}{5} \cdot \frac{2}{7} \cdot \frac{2}{9} \cdot \frac{2}{11} x^{4} + \cdots \right] + \frac{x^{\frac{5}{2}}}{(\frac{5}{2})!} \left[1 + \frac{2}{7} \cdot \frac{2}{9} \cdot \frac{2}{7} \cdot \frac{2}{9} \cdot \frac{2}{11} x^{4} + \cdots \right] + \frac{x^{\frac{5}{2}}}{(\frac{5}{2})!} \left[1 + \frac{2}{7} \cdot \frac{2}{9} \cdot \frac{2}{7} \cdot \frac{2}{9} \cdot \frac{2}{11} x^{4} + \cdots \right] + \frac{x^{\frac{5}{2}}}{(\frac{5}{2})!} \left[1 + \frac{2}{7} \cdot \frac{2}{9} \cdot \frac{2}{7} \cdot \frac{2}{9} \cdot \frac{2}{11} x^{4} + \cdots \right] + \frac{x^{\frac{5}{2}}}{(\frac{5}{2})!} \left[1 + \frac{2}{7} \cdot \frac{2}{9} \cdot \frac{2}{7} \cdot \frac{2}{9} \cdot \frac{2}{11} x^{4} + \cdots \right] + \frac{x^{\frac{5}{2}}}{(\frac{5}{2})!} \left[1 + \frac{2}{7} \cdot \frac{2}{9} \cdot \frac{2}{11} x^{4} + \cdots \right] + \frac{x^{\frac{5}{2}}}{(\frac{5}{2})!} \left[1 + \frac{2}{7} \cdot \frac{2}{9} \cdot \frac{2}{11} x^{4} + \cdots \right] + \frac{x^{\frac{5}{2}}}{(\frac{5}{2})!} \left[1 + \frac{2}{7} \cdot \frac{2}{9} \cdot \frac{2}{11} x^{4} + \cdots \right] + \frac{x^{\frac{5}{2}}}{(\frac{5}{2})!} \left[1 + \frac{2}{7} \cdot \frac{2}{9} \cdot \frac{2}{11} x^{4} + \cdots \right] + \frac{x^{\frac{5}{2}}}{(\frac{5}{2})!} \left[1 + \frac{2}{7} \cdot \frac{2}{9} \cdot \frac{2}{11} x^{4} + \cdots \right] + \frac{x^{\frac{5}{2}}}{(\frac{5}{2})!} \left[1 + \frac{2}{7} \cdot \frac{2}{9} \cdot \frac{2}{11} x^{4} + \cdots \right] + \frac{x^{\frac{5}{2}}}{(\frac{5}{2})!} \left[1 + \frac{2}{7} \cdot \frac{2}{9} \cdot \frac{2}{11} x^{4} + \cdots \right] + \frac{x^{\frac{5}{2}}}{(\frac{5}{2})!} \left[1 + \frac{2}{7} \cdot \frac{2}{9} \cdot \frac{2}{11} x^{4} + \cdots \right] + \frac{x^{\frac{5}{2}}}{(\frac{5}{2})!} \left[1 + \frac{2}{7} \cdot \frac{2}{9} \cdot \frac{2}{11} x^{4} + \frac{x^{6}}{6!} + \cdots \right] + \frac{x^{\frac{5}{2}}}{(\frac{5}{2}!} + \frac{x^{7}}{7!} + \frac{x^{7}}{9!} + \cdots \right] + \frac{x^{\frac{5}{2}}}{(\frac{5}{9!} \cdot \frac{2}{7!} + \frac{x^{7}}{9!} + \frac{x^{7}}{9!} + \cdots \right] + \frac{x^{\frac{5}{2}}}{(\frac{5}{9!} \cdot \frac{2}{7!} + \frac{x^{7}}{9!} + \frac{x^{7}}{9!} + \cdots \right] + \frac{x^{\frac{5}{2}}}{(\frac{5}{9!} \cdot \frac{2}{7!} + \frac{x^{7}}{9!} + \frac{x^{7}}{9!} + \frac{x^{7}}{9!} + \frac{x^{7}}{9!} + \frac{x^{7}}{9!} + \frac{x^{2}}{9!} +$$

$$\underline{y}(x;r) = [1+r] \frac{1}{\sqrt{\pi x}} + e^x \operatorname{erfc} (-\sqrt{x}) + \sum_{n=0}^{\infty} \frac{x^{n+\frac{1}{2}}}{(n+\frac{1}{2})!} \left\{ \frac{hypergeom}{even} (1, n+\frac{3}{2}, x) \right\} + \sum_{n=0}^{\infty} (n+1) \left[\frac{x^{2n+2}}{(2n+2)!} + \frac{x^{2n+1}}{(2n+1)!} \right]$$

$$\overline{y}(x;r) = [3-r] \frac{1}{\sqrt{\pi x}} + e^x \operatorname{erfc} (-\sqrt{x}) + \sum_{n=0}^{\infty} \frac{x^{n+\frac{1}{2}}}{(n+\frac{1}{2})!} \left\{ \frac{hypergeom}{even} (1, n+\frac{3}{2}, x) \right\} + \sum_{n=0}^{\infty} (n+1) \left[\frac{x^{2n+2}}{(2n+2)!} + \frac{x^{2n+1}}{(2n+1)!} \right]$$

5. Conclusion

In this paper, solving FFDEs of order $0 < \beta < 1$ using fuzzy Laplace transforms under Riemann Liouville –H differentiability was discussed. As an example, we solved a problem involving hyperbolic cosine function.

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