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Modal Type Operators over Interval Valued Intuitionistic Fuzzy Sets of Second Type

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Abstract. In this paper, we introduce modal type operators over interval valued intuitionistic fuzzy sets of second type and establish some of their properties.

Keywords: Intuitionistic fuzzy sets, intuitionistic fuzzy sets of second type, interval valued fuzzy sets, interval valued intuitionistic fuzzy sets, interval valued intuitionistic fuzzy sets of second type.

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1. Introduction

An intuitionistic fuzzy set for a given underlying set *x* where introduced by Atanassov [2] which is the generalization of ordinary fuzzy sets. Atanassov and Gargov [3] further introduced the concepts of interval valued intuitionistic fuzzy set. The present authors further introduced the new extension of IVIFS namely interval valued intuitionistic fuzzy sets of second type (IVIFSST) and established some of their properties [4]. The rest of the paper is designed as follows: In Section 2, we give some basic definitions. In Section 3, we introduce modal type operators over interval valued intuitionistic fuzzy sets of second type and establish some of their properties. This paper is concluded in section 4.

2. Preliminaries

In this section, we give some basic definitions.

Definition 2.1. [2] Let X be a non-empty set. An Intuitionistic Fuzzy Set (IFS) A in X is defined as an object of the following form.

$$\mathrm{A} = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in \mathrm{X}\}$$

where the functions $\mu_A: X \to [0,1]$ and $\nu_A: X \to [0,1]$ denote the degree of membership and the degree of non-membership of the element $x \in X$, respectively, and for every $x \in X$.

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1$$

Definition 2.2. [2] Let a set X be fixed. An intuitionistic fuzzy sets of second type (IFSST) A in X is defined as an object of the following form.

$$\mathrm{A} = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in \mathrm{X}\}$$

where the functions $\mu_A: X \to [0,1]$ and $\nu_A: X \to [0,1]$ denote the degree of membership and the degree of non-membership of the element $x \in X$, respectively, and for every $x \in X$.

$$0 \leq \mu_A{}^2(x) + \nu_A{}^2(x) \leq 1$$

Definition 2.3. [3] An interval valued intuitionistic fuzzy sets (IVIFS) A in X is given by

$$A = \{\langle x, M_A(x), N_A(x)\rangle \mid x \in X\}$$

where $M_A : X \to [0, 1]$, $N_A : X \to [0, 1]$. The intervals $M_A(x)$ and $N_A(x)$ denote the degree of membership and the degree of non-membership of the element x in X, where $M_A(x) = [M_{AL}(x), M_{AU}(x)]$ and $N_A(x) = [N_{AL}(x), N_{AU}(x)]$ with the condition that

$$M_{AU}(x) + N_{AU}(x) \leq 1 \ \forall x \in X$$

Definition 2.4. [4] An interval valued intuitionistic fuzzy sets of second type A in X is given by

$$A = \{\langle x, M_A(x), N_A(x)\rangle \mid x \in X\}$$

where $M_A: X \to [0, 1]$, $N_A: X \to [0, 1]$. The intervals $M_A(x)$ and $N_A(x)$ denote the degree of membership and the degree of non-membership of the element x in X, where $M_A(x) = [M_{AL}(x), M_{AU}(x)]$ and $N_A(x) = [N_{AL}(x), N_{AU}(x)]$ with the condition that

$$\boldsymbol{M^2}_{AU}(x) + \boldsymbol{N^2}_{AU}(x) \leq 1 \ \forall \ x \in X.$$

Definition 2.5. [4] For every two IVIFSST A and B, we have the following relations and operations

1. $A \subset B$ iff $M_{AU}(x) \leq M_{BU}(x) \& M_{AL}(x) \leq M_{BL}(x) \&$
   $N_{AU}(x) \geq N_{BU}(x) \& N_{AL}(x) \geq N_{BL}(x)$
2. $A = B$ iff $A \subset B$ & $B \subset A$
3. $\bar{A} = \{\langle x, [N_{AL}(x), N_{AU}(x)], [M_{AL}(x), M_{AU}(x)]\rangle \mid x \in X\}$
4. $A \cup B = \{\langle x, [\max(M_{AL}(x), M_{BL}(x)), \max(M_{AU}(x), M_{BU}(x))],$
   $[\min(N_{AL}(x), N_{BL}(x)), \min(N_{AU}(x), N_{BU}(x))]\rangle \mid x \in X\}$
5. $A \cap B = \{\langle x, [\min(M_{AL}(x), M_{BL}(x)), \min(M_{AU}(x), M_{BU}(x))],$
   $[\max(N_{AL}(x), N_{BL}(x)), \max(N_{AU}(x), N_{BU}(x))]\rangle \mid x \in X\}$

Definition 2.6. [5] For every IVIFSST, we define the following operators

Necessity operator

$$\square A = \left\{\langle x, [M_{AL}(x), M_{AU}(x)], \left[N_{AL}(x), \sqrt{1 - M^2{}_{AU}(x)}\right]\rangle \mid x \in X\right\}$$

Possibility operator

$$\lozenge A = \left\{\langle x, \left[M_{AL}(x), \sqrt{1 - N^2{}_{AU}(x)}\right], [N_{AL}(x), N_{AU}(x)]\rangle \mid x \in X\right\}$$

Definition 2.7. [5] Given an IVIFSST A and for every $\alpha, \beta \in [0, 1]$, we define the operators D_α and $F_{\alpha,\beta}$

$$D_\alpha(A) = \left\{\left\langle x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + \alpha^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right], \right.\right.$$
$$\left.\left.\left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + (1-\alpha)^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right]\right\rangle \mid x \in X\right\}$$

$$F_{\alpha,\beta}(A) = \left\{x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + \alpha^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right],\right.$$

$$\left.\left[\boldsymbol{N_{AL}(x)}, \sqrt{\boldsymbol{N^2{}_{AU}(x) + \beta^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}}\right]\right\rangle \mid \boldsymbol{x} \in \mathbf{X}\right\}$$

3. Modal type operators on IVIFSST

In this section, we define the new modal type operators over IVIFSST and also we establish some of their properties.

Definition 3.1. Given an IVIFSST A and for every $\alpha, \beta \in [0,1]$, we define the operators $G_{\alpha,\beta}$, $H_{\alpha,\beta}$ and $J_{\alpha,\beta}$

(i). $G_{\alpha,\beta}(A) = \{\langle x, [\alpha M_{AL}(x), \alpha M_{AU}(x)], [\beta N_{AL}(x), \beta N_{AU}(x)]\rangle \mid x \in X\}$

(ii). $H_{\alpha,\beta}(A) = \{\langle x, [\alpha\, M_{AL}(x), \alpha\, M_{AU}(x)],$

$$\left.\left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + \beta^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right]\right\rangle \mid x \in X\right\}$$

(iii). $$J_{\alpha,\beta}(A) = \left\{\left\langle x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + \alpha^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right],\right.\right.$$

$$[\beta N_{AL}(x), \beta N_{AU}(x)]\rangle \mid x \in X\}$$

Definition 3.2. Given an IVIFSST A and for every $\alpha, \beta \in [0,1]$, we define the operators $H^*{}_{\alpha,\beta}$ and $J^*{}_{\alpha,\beta}$

(i). $H^*{}_{\alpha,\beta}(A) = \{\langle x, [\alpha\, M_{AL}(x), \alpha\, M_{AU}(x)],$

$$\left.\left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + \beta^2\left(1 - \alpha^2 M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right]\right\rangle \mid x \in X\right\}$$

(ii). $$J^*{}_{\alpha,\beta}(A) = \left\{\left\langle x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + \alpha^2\left(1 - M^2{}_{AU}(x) - \beta^2 N^2{}_{AU}(x)\right)}\right],\right.\right.$$

$$[\beta N_{AL}(x), \beta N_{AU}(x)]\rangle \mid x \in X\}$$

Proposition 3.1. *Let X be a non-empty set. For every IVIFSST A and for every $\alpha, \beta \in [0,1]$ we have the following*

(i). $\overline{H_{\alpha,\beta}(\bar{A})} = J_{\beta,\alpha}(A)$

(ii). $\overline{J_{\alpha,\beta}(\bar{A})} = H_{\beta,\alpha}(A)$

(iii). $\overline{H_{1,0}(\bar{A})} = \overline{J_{0,1}(\bar{A})} = G_{1,1}(A)$

Proof: Let $A = \{\langle x, [M_{AL}(x), M_{AU}(x)], [N_{AL}(x), N_{AU}(x)]\rangle \mid x \in X\}$,
Then,

$$\bar{A} = \{\langle x, [N_{AL}(x), N_{AU}(x)], [M_{AL}(x), M_{AU}(x)]\rangle \mid x \in X\},$$

$G_{\alpha,\beta}(A) = \{\langle x, [\alpha M_{AL}(x), \alpha M_{AU}(x)], [\beta N_{AL}(x), \beta N_{AU}(x)]\rangle \mid x \in X\}$,

$$H_{\alpha,\beta}(A) = \{\langle x, [\alpha\, M_{AL}(x), \alpha\, M_{AU}(x)], \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + \beta^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right]\right\rangle |x \in \mathrm{X}\right\}$$

$$J_{\alpha,\beta}(A) = \left\{\left\langle x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + \alpha^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right], [\beta N_{AL}(x), \beta N_{AU}(x)]\rangle |\, x \in \mathrm{X}\}$$

Now

$$H_{\alpha,\beta}(\bar{A}) = \{\langle x, [\alpha\, N_{AL}(x), \alpha\, N_{AU}(x)], \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + \beta^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right]\right\rangle |x \in \mathrm{X}\right\}$$

$$\overline{H_{\alpha,\beta}(\bar{A})} = \left\{\left\langle x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + \beta^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right], [\alpha\, N_{AL}(x), \alpha\, N_{AU}(x)]\rangle |\, x \in \mathrm{X}\}$$

$$= J_{\beta,\alpha}(A)$$

This is (i)

(ii) $$J_{\alpha,\beta}(\bar{A}) = \left\{\left\langle x, \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + \alpha^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right], [\beta M_{AL}(x), \beta M_{AU}(x)]\rangle |\, x \in \mathrm{X}\}$$

$$\overline{J_{\alpha,\beta}(\bar{A})} = \{\langle x, [\beta M_{AL}(x), \beta M_{AU}(x)], \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + \alpha^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right]\right\rangle |\, x \in \mathrm{X}\right\}$$

$$= H_{\beta,\alpha}(A)$$

Therefore,

$$\overline{J_{\alpha,\beta}(\bar{A})} = H_{\beta,\alpha}(A)$$

(iii) Let $A = \{\langle x, [M_{AL}(x), M_{AU}(x)], [N_{AL}(x), N_{AU}(x)]\rangle | x \in \mathrm{X}\}$,

Then,

$$\bar{A} = \{\langle x, [N_{AL}(x), N_{AU}(x)], [M_{AL}(x), M_{AU}(x)]\rangle | x \in \mathrm{X}\}$$

$$H_{\alpha,\beta}(\bar{A}) = \{\langle x, [\alpha\, N_{AL}(x), \alpha\, N_{AU}(x)], \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + \beta^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right]\right\rangle |x \in \mathrm{X}\right\}$$

$$\overline{H_{\alpha,\beta}(\bar{A})} = \left\{\left\langle x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + \beta^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right],\right.\right.$$

$[\alpha\, N_{AL}(x), \alpha\, N_{AU}(x)]\rangle |\, x \in X\}$

Put $\alpha = 1$ and $\beta = 0$ in $\overline{H_{\alpha,\beta}(\bar{A})}$

$$\overline{H_{1,0}(\bar{A})} = \left\{\left\langle x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + 0^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right],\right.\right.$$
$$[1N_{AL}(x), 1N_{AU}(x)]\rangle |\, x \in X\}$$

$$= \{\langle x, [M_{AL}(x), M_{AU}(x)], [N_{AL}(x), N_{AU}(x)]\rangle |\, x \in X\} \quad (1)$$

$$J_{\alpha,\beta}(\bar{A}) = \left\{\left\langle x, \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + \alpha^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right],\right.\right.$$
$$[\beta M_{AL}(x), \beta M_{AU}(x)]\rangle |\, x \in X\}$$

$$\overline{J_{\alpha,\beta}(\bar{A})} = \{\langle x, [\beta M_{AL}(x), \beta M_{AU}(x)],$$
$$\left.\left.\left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + \alpha^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right]\right\rangle |\, x \in X\right\}$$

Put $\alpha = 0$ and $\beta = 1$ in $\overline{J_{\alpha,\beta}(\bar{A})}$

$$\overline{J_{0,1}(\bar{A})} = \{\langle x, [1M_{AL}(x), 1M_{AU}(x)],$$

$$\left.\left.\left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + 0^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right]\right\rangle |\, x \in X\right\}$$

$$= \{\langle x, [M_{AL}(x), M_{AU}(x)], [N_{AL}(x), N_{AU}(x)]\rangle |\, x \in X\} \quad (2)$$

$G_{\alpha,\beta}(A) = \{\langle x, [\alpha M_{AL}(x), \alpha M_{AU}(x)], [\beta N_{AL}(x), \beta N_{AU}(x)]\rangle |\, x \in X\}$

Put $\alpha = 1$ and $\beta = 1 G_{\alpha,\beta}(A)$ we have

$$G_{1,1}(A) = \{\langle x, [M_{AL}(x), M_{AU}(x)], [N_{AL}(x), N_{AU}(x)]\rangle |\, x \in X\} \quad (3)$$

From (1), (2) and (3) we have

$$\overline{H_{1,0}(\bar{A})} = \overline{J_{0,1}(\bar{A})} = G_{1,1}(A)$$

Proposition 3.2. *Let X be a non-empty set. For every IVIFSST A and for every* $\alpha, \beta \in [0,1]$ *we have* the following

(i). $\overline{H^*{}_{\alpha,\beta}(\bar{A})} = J^*{}_{\beta,\alpha}(A)$

(ii). $\overline{J^*{}_{\alpha,\beta}(\bar{A})} = H^*{}_{\beta,\alpha}(A)$

Proof: Let $A = \{\langle x, [M_{AL}(x), M_{AU}(x)], [N_{AL}(x), N_{AU}(x)]\rangle | x \in X\}$

Then,

$$\bar{A} = \{\langle x, [N_{AL}(x), N_{AU}(x)], [M_{AL}(x), M_{AU}(x)]\rangle | x \in \mathrm{X}\},$$

$$\mathrm{H}^*_{\alpha,\beta}(A) = \{\langle x, [\alpha\, M_{AL}(x), \alpha\, M_{AU}(x)], \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + \beta^2\left(1 - \alpha^2 M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right]\rangle | x \in \mathrm{X}\right\},$$

$$\mathrm{J}^*_{\alpha,\beta}(A) = \left\{\left\langle x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + \alpha^2\left(1 - M^2{}_{AU}(x) - \beta^2 N^2{}_{AU}(x)\right)}\right], [\beta N_{AL}(x), \beta N_{AU}(x)]\rangle | x \in \mathrm{X}\},$$

$$\mathrm{H}^*_{\alpha,\beta}(\bar{A}) = \{\langle x, [\alpha\, N_{AL}(x), \alpha\, N_{AU}(x)], \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + \beta^2\left(1 - \alpha^2 N^2{}_{AU}(x) - M^2{}_{AU}(x)\right)}\right]\right\rangle | x \in \mathrm{X}\},$$

and

$$\mathrm{J}^*_{\alpha,\beta}(\bar{A}) = \left\{\left\langle x, \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + \alpha^2\left(1 - \beta^2 M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right], [\beta M_{AL}(x), \beta M_{AU}(x)]\rangle | x \in \mathrm{X}\}$$

Now,

(i) $$\overline{\mathrm{H}^*_{\alpha,\beta}(\bar{A})} = \left\{\left\langle x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + \beta^2\left(1 - M^2{}_{AU}(x) - \alpha^2 N^2{}_{AU}(x)\right)}\right], [\alpha\, N_{AL}(x), \alpha\, N_{AU}(x)]\rangle | x \in \mathrm{X}\}$$

$$= \mathrm{J}^*_{\beta,\alpha}(A)$$

Therefore,

$$\overline{\mathrm{H}^*_{\alpha,\beta}(\bar{A})} = \mathrm{J}^*_{\beta,\alpha}(A)$$

(ii) $$\mathrm{J}^*_{\alpha,\beta}(\bar{A}) = \left\{\left\langle x, \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + \alpha^2\left(1 - \beta^2 M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right], [\beta M_{AL}(x), \beta M_{AU}(x)]\rangle | x \in \mathrm{X}\}$$

$$\overline{\mathrm{J}^*_{\alpha,\beta}(\bar{A})} = \{\langle x, [\beta M_{AL}(x), \beta M_{AU}(x)], \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + \alpha^2\left(1 - \beta^2 M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right]\right\rangle | x \in \mathrm{X}\right\}$$

$$= \mathrm{H}^*_{\beta,\alpha}(A)$$

Hence,

$$\overline{\mathrm{J}^*_{\alpha,\beta}(\bar{A})} = \mathrm{H}^*_{\beta,\alpha}(A)$$

Proposition 3.3. *Let X be a non-empty set. For every IVIFSST A and for every* $\alpha, \beta \in [0,1]$ *we have the following*

i. $\mathrm{H}^*{}_{\alpha,\beta}(\mathrm{A}) = \mathrm{F}_{0,\beta}\left(\mathrm{G}_{\alpha,1}(\mathrm{A})\right)$

ii. $\mathrm{J}^*{}_{\beta,\alpha}(\mathrm{A}) = \mathrm{F}_{\beta,0}\left(\mathrm{G}_{1,\alpha}(\mathrm{A})\right)$

Proof: Let $A = \{\langle x, [M_{AL}(x), M_{AU}(x)], [N_{AL}(x), N_{AU}(x)]\rangle | x \in \mathrm{X}\}$,
Then,

$$\bar{A} = \{\langle x, [N_{AL}(x), N_{AU}(x)], [M_{AL}(x), M_{AU}(x)]\rangle | x \in \mathrm{X}\},$$

$$F_{\alpha,\beta}(A) = \left\{\left\langle x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + \alpha^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right], \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + \beta^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right]\right\rangle | \; x \in \mathrm{X}\right\},$$

$\mathrm{G}_{\alpha,\beta}(\mathrm{A}) = \{\langle x, [\alpha M_{AL}(x), \alpha M_{AU}(x)], [\beta N_{AL}(x), \beta N_{AU}(x)]\rangle | \; x \in \mathrm{X}\}$,

$$\mathrm{H}^*{}_{\alpha,\beta}(A) = \left\{\left\langle x, [\alpha\, M_{AL}(x), \alpha\, M_{AU}(x)], \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + \beta^2\left(1 - \alpha^2 M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right]\right\rangle | \; x \in \mathrm{X}\right\}$$

$$\mathrm{J}^*{}_{\alpha,\beta}(A) = \left\{\left\langle x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + \alpha^2\left(1 - M^2{}_{AU}(x) - \beta^2 N^2{}_{AU}(x)\right)}\right], [\beta N_{AL}(x), \beta N_{AU}(x)]\right\rangle | \; x \in \mathrm{X}\right\}$$

(i). Put $\beta = 1$ in $\mathrm{G}_{\alpha,\beta}(A)$ we have
$\mathrm{G}_{\alpha,1}(\mathrm{A}) = \{\langle x, [\alpha M_{AL}(x), \alpha M_{AU}(x)], [N_{AL}(x), N_{AU}(x)]\rangle | \; x \in \mathrm{X}\}$

Again put $\alpha = 0$ in $\mathrm{F}_{\alpha,\beta}(A)$ we have

$$F_{0,\beta}(A) = \left\{\left\langle x, [M_{AL}(x), M_{AU}(x)], \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + \beta^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right]\right\rangle | \; x \in \mathrm{X}\right\}$$

$$F_{0,\beta}\left(\mathrm{G}_{\alpha,1}(\mathrm{A})\right) = \left\{\left\langle x, [\alpha M_{AL}(x), \alpha M_{AU}(x)], \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + \beta^2\left(1 - \alpha^2 M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right]\right\rangle | \; x \in \mathrm{X}\right\}$$

From $\mathrm{H}^*{}_{\alpha,\beta}(A)$ and $F_{0,\beta}\left(\mathrm{G}_{\alpha,1}(\mathrm{A})\right)$ we have

$$\mathrm{H}^*{}_{\alpha,\beta}(A) = F_{0,\beta}\left(\mathrm{G}_{\alpha,1}(\mathrm{A})\right)$$

(ii). Put $\alpha = \beta$ and $\beta = \alpha$ in $J^*_{\alpha,\beta}(A)$ we have

$$\mathrm{J}^*{}_{\beta,\alpha}(A) = \left\{\left\langle x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + \beta^2\left(1 - M^2{}_{AU}(x) - \alpha^2 N^2{}_{AU}(x)\right)}\right], [\alpha N_{AL}(x), \alpha N_{AU}(x)]\right\rangle | \; x \in \mathrm{X}\right\}$$

put $\alpha = 1$ and $\beta = \alpha$ in $\mathrm{G}_{1,\alpha}(\mathrm{A})$ we have

$G_{1,\alpha}(A) = \{\langle x, [M_{AL}(x), M_{AU}(x)], [\alpha N_{AL}(x), \alpha N_{AU}(x)]\rangle | \; x \in X\}$

Put $\alpha = \beta$ and $\beta = 0$ in $F_{\alpha,\beta}(A)$ we have

$$F_{\beta,0}(A) = \left\{\left\langle x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + \beta^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right],\right.\right.$$
$$\left.\left.[N_{AL}(x), N_{AU}(x)]\right\rangle | \; x \in X\right\}$$

Now

$$F_{\beta,0}\left(G_{1,\alpha}(A)\right) = \left\{\left\langle x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + \beta^2\left(1 - M^2{}_{AU}(x) - \alpha^2 N^2{}_{AU}(x)\right)}\right],\right.\right.$$
$$\left.\left.\left[\alpha N_{AL}(x), \sqrt{\alpha^2 N^2{}_{AU}(x) + 0\left(1 - \alpha^2 M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right]\right\rangle | \; x \in X\right\}$$
$$= \left\{\left\langle x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + \beta^2\left(1 - M^2{}_{AU}(x) - \alpha^2 N^2{}_{AU}(x)\right)}\right],\right.\right.$$
$$\left.\left.[\alpha N_{AL}(x), \alpha N_{AU}(x)]\right\rangle | \; x \in X\right\}$$

From $J^*{}_{\beta,\alpha}(A)$and $F_{\beta,0}\left(G_{1,\alpha}(A)\right)$ we have

$$F_{\beta,0}\left(G_{1,\alpha}(A)\right) = J^*{}_{\beta,\alpha}(A)$$

4. Conclusion

We have introduced modal type operators$\boldsymbol{G_{\alpha,\beta}}$, $\boldsymbol{H_{\alpha,\beta}}$, $\boldsymbol{J_{\alpha,\beta}}$, $\boldsymbol{H^*{}_{\alpha,\beta}}$ and $\boldsymbol{J^*{}_{\alpha,\beta}}$ over IVIFSST also we established some of their properties. It is still open to define some more operators over IVIFSST.

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