Atom Bond Connectivity Reverse and Product Connectivity Reverse Indices of Oxide and Honeycomb Networks

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Abstract. The connectivity indices are applied to measure the chemical characteristics of compounds in Chemical Graph Theory. In this paper, we propose a new index known as the atom bond connectivity reverse index of a molecular graph. Furthermore, we determine the atom bond connectivity reverse index and product connectivity reverse index for oxide and honeycomb networks.

Keywords: atom bond connectivity reverse index, product connectivity reverse index, oxide network, honeycomb network.

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1. Introduction
Let $G = (V(G), E(G))$ be a simple, finite, connected graph. The degree $d_G(v)$ of a vertex $v$ is the number of vertices adjacent to $v$. Let $\Delta(G)$ denote the maximum degree among the vertices of $G$. The reverse vertex degree of a vertex $v$ in a graph $G$ is defined as $c_v = \Delta(G) - d_G(v) + 1$. The reverse edge connecting the reverse vertices $u$ and $v$ will be denoted by $uv$. Any undefined term in this paper may be found in Kulli [1].

We propose the atom bond connectivity reverse index of a graph $G$ as

$$ABCC(G) = \sum_{uv \in E(G)} \frac{c_u + c_v - 2}{c_u c_v}.$$ 

Recently some reverse indices were studied, for example, in [2, 3, 4, 5, 6].

The product connectivity reverse index was introduced by Kulli in [4]. The product connectivity reverse index of a graph $G$ is defined as

$$PC(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{c_u c_v}}.$$ 

Recently several topological indices were studied, for example, in [7, 8, 9, 10, 11, 12, 13, 14, 15, 16].
In this paper, the atom bond connectivity reverse index and product connectivity reverse index of oxide networks and honeycomb networks are determined. For networks see [17].

2. Results for Oxide networks
We consider oxide networks. These networks are vital importance in the study of silicate networks. An oxide network of dimension \( n \) is denoted by \( OX_n \). A 5-dimensional oxide network is shown in Figure 1.

![Figure 1: An oxide network of dimension five](image)

Let \( G \) be the graph of oxide network \( OX_n \). From Figure 1, it is easy to see that the vertices of \( OX_n \) are either of degree 2 or 4. By calculation, we obtain that \( G \) has \( 9n^2 + 3n \) vertices and \( 18n^2 \) edges. Clearly we have \( c_u = \Delta(G) - d_G(u) + 1 = d_G(u) \). In \( OX_n \), by algebraic method, there are two types of edges based on the degree of the end vertices of each edge as follows:

\[
E_{24} = \{ uv \in E(G) \mid d_G(u) = 2, d_G(v) = 4 \}, \quad |E_{24}| = 12n.
\]

\[
E_{44} = \{ uv \in E(G) \mid d_G(u) = d_G(v) = 4 \}, \quad |E_{44}| = 18n^2 - 12n.
\]

Thus there are two types of reverse edges based on the degree of the reverse end vertices of each reverse edge as follows:

\[
CE_{31} = \{ uv \in E(G) \mid c_u = 3, c_v = 1 \}, \quad |CE_{31}| = 12n.
\]

\[
CE_{11} = \{ uv \in E(G) \mid c_u = c_v = 1 \}, \quad |CE_{11}| = 18n^2 - 12n.
\]

We compute the atom bond connectivity reverse index of oxide networks.

**Theorem 1.** The atom bond connectivity reverse index of an oxide network is given by

\[
ABCC(OX_n) = 4\sqrt{6n}.
\]

**Proof:** By definition, we have

\[
ABCC(G) = \sum_{uv \in E(G)} \sqrt{\frac{c_u + c_v - 2}{c_u c_v}}
\]

\[
ABCC(OX_n) = \sum_{CE_{31}} \sqrt{\frac{c_u + c_v - 2}{c_u c_v}} + \sum_{CE_{11}} \sqrt{\frac{c_u + c_v - 2}{c_u c_v}}
\]
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\[
= \left( \frac{3+1-2}{3\times1} \right) 12n + \left( \frac{1+1-2}{1\times1} \right) (18n^2 - 12n)
= 4\sqrt{6}n.
\]

In the following theorem, we compute the product connectivity reverse index of oxide networks.

**Theorem 2.** The product connectivity reverse index of oxide networks is given by

\[
PC(OX_n) = 18n^2 + (4\sqrt{3} - 12)n.
\]

**Proof:** By definition, we have

\[
PC(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{c_u c_v}}.
\]

Thus

\[
PC(OX_n) = \sum_{c_{E_u}} \frac{1}{\sqrt{c_u c_v}} + \sum_{c_{E_v}} \frac{1}{\sqrt{c_u c_v}}
= \left( \frac{1}{\sqrt{3\times1}} \right) 12n + \left( \frac{1}{\sqrt{1\times1}} \right) (18n^2 - 12n)
= 18n^2 + (4\sqrt{3} - 12)n.
\]

3. Results for Honeycomb networks

Honeycomb networks are very useful in computer graphics and chemistry. A honeycomb network of dimension \(n\) is denoted by \(HC_n\) where \(n\) is the number of hexagons between central and boundary hexagon. A 4-dimensional honeycomb network is shown in Figure 2.

![Figure 2: A honeycomb network of dimension four](image)

Let \(H\) be the graph of honeycomb network \(HC_n\). From Figure 2, we see that the vertices of \(HC_n\) are either of degree 2 or 3. By algebraic method, we obtain that \(|V(HC_n)| = 6n^2\) and \(|E(HC_n)| = 9n^2 - 3n\). Clearly we have \(c_u = \Delta(H) - d_H(u) + 1 = 4 - d_H(u)\). By
algebraic method, in $HC_n$, there are three types of edges based on the degree of the end vertices of each edge as follows:

$E_{22} = \{uv \in E(H) \mid d_H(u) = d_H(v) = 2\}$, \quad |E_{22}| = 6.

$E_{23} = \{uv \in E(H) \mid d_H(u) = 2, \; d_H(v) = 3\}$, \quad |E_{23}| = 12n - 12.

$E_{33} = \{uv \in E(H) \mid d_H(u) = d_H(v) = 3\}$, \quad |E_{33}| = 9n^2 - 15n + 6.

Thus there are three types of reverse edges based on the degree of the reverse end vertices of each reverse edge as follows:

$CE_{22} = \{uv \in E(H) \mid \bar{c}_u = \bar{c}_v = 2\}$, \quad |CE_{22}| = 6.

$CE_{21} = \{uv \in E(H) \mid \bar{c}_u = 2, \; \bar{c}_v = 1\}$, \quad |CE_{21}| = 12n - 12.

$CE_{11} = \{uv \in E(H) \mid \bar{c}_u = \bar{c}_v = 1\}$, \quad |CE_{11}| = 9n^2 - 15n + 6.

In the following theorem, we compute the atom bond connectivity reverse index of honeycomb networks.

**Theorem 3.** The atom bond connectivity reverse index of honeycomb networks is given by

$$ABCC(HC_n) = 6\sqrt{2n} - 3\sqrt{2}.$$  

**Proof:** By definition, we have

$$ABCC(H) = \sum_{uv \in E(H)} \sqrt{\frac{c_u + c_v - 2}{c_u c_v}}.$$  

Thus,

$$ABCC(HC_n) = \sum_{CE_{22}} \sqrt{\frac{c_u + c_v - 2}{c_u c_v}} + \sum_{CE_{21}} \sqrt{\frac{c_u + c_v - 2}{c_u c_v}} + \sum_{CE_{11}} \sqrt{\frac{c_u + c_v - 2}{c_u c_v}}$$

$$= \left(\frac{2 + 2 - 2}{2 \times 2}\right)6 + \left(\frac{2 + 1 - 2}{2 \times 1}\right)(12n - 12) + \left(\frac{1 + 1 - 2}{1 \times 1}\right)(9n^2 - 15n + 6)$$

$$= 6\sqrt{2n} - 3\sqrt{2}.$$  

In the following theorem, we compute the product connectivity reverse index of honeycomb networks.

**Theorem 4.** The product connectivity reverse index of honeycomb networks is given by

$$PC(HC_n) = 9n^2 + \left(6\sqrt{2} - 15\right)n + \left(9 - 6\sqrt{2}\right).$$  

**Proof:** By definition, we have

$$PC(H) = \sum_{uv \in E(H)} \frac{1}{c_u c_v}.$$  

Thus,

$$PC(HC_n) = \sum_{CE_{22}} \frac{1}{c_u c_v} + \sum_{CE_{21}} \frac{1}{c_u c_v} + \sum_{CE_{11}} \frac{1}{c_u c_v}$$

$$= \left(\frac{1}{\sqrt{2 \times 2}}\right)6 + \left(\frac{1}{\sqrt{2 \times 1}}\right)(12n - 12) + \left(\frac{1}{\sqrt{1 \times 1}}\right)(9n^2 - 15n + 6)$$

$$= 9n^2 + \left(6\sqrt{2} - 15\right)n + \left(9 - 6\sqrt{2}\right).$$

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REFERENCES