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Optimal Solutions of the Fuzzy Multi Index Bi-criteria Fixed Charge Bottleneck Transportation Problem

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Abstract. In this paper, fuzzy multi index bi-criteria fixed charge bottleneck transportation problem (FMIBCFCBTP) is considered and for the first time all parameters are taken as trapezoidal fuzzy numbers. An algorithm is developed to find fuzzy time-cost trade-off pairs of FMIBCFCBTP. A numerical example is given to explain the proposed algorithm.

Keywords: Multi Index Transportation Problem, efficient time-cost pairs, fixed charge, Trapezoidal fuzzy numbers, Bottleneck transportation Problem.

AMS Mathematics Subject Classification (2010): 90B06

1. Introduction

Cost Minimizing Transportation Problem (CTP) is widely studied in literature which has transportation cost as criteria. But if transportation time is taken as the criteria instead of cost then the problem becomes Time Minimization Transportation Problem which is also called Bottleneck Transportion Problem (BTP). BTP's have been considered by several researchers [6,11,9,19,20,21]. By including additional indices as commodity, modes of transport etc. to the BTP, the problem becomes Multi Index Transportation Problem (MITP). In 1975, Bhatia, Swarup and Puri [6] have given the solution procedure to solve the BTP. Further the authors [7,10] have extended the algorithm to solve the MITP by including commodity as an the additional index. Bhatia, Swarup and Puri [8] have further extended the algorithm to solve multi index bi-criteria transportation problem (MIBCTP). In their problem, cost and time are taken as the two criteria in which time is minimized first and then cost. In literature, most of the authors [3,4] have solved the problem by minimizing cost first and then time. When fixed charges like toll charges, warehouse rent etc. are included in MIBCTP, the problem becomes Multi Index Bicriteria Fixed Charge Transportation Problem (MIBCFCTP). Ahuja and Arora [1], Khurana and Adlakha [12], Arora and Khurana [2] are names of a few authors who have solved the MIBCFCTP to find cost-time trade-off pairs. Uncertainties due to weather changes, accidents, traffic jams etc. led to the consideration of fuzzy parameters in the transportation problem. The fuzzy concept was introduced by Zadeh [22]. The Multi Index Fixed Charge Transportation Problem (MIFCTP) was solved by Ritha and Vinotha [16] with symmetrical trapezoidal fuzzy numbers while Kumar, Gupta and Sharma [14] proposed algorithm for bi-criteria fixed charge transportation problem (BCFCTP) using trapezoidal fuzzy number.

In this paper, FMIBCFCBTP is considerd. The problem is solved by extending the method given by Singh, Tuli and Sarode [17] by incorportating fuzzy costs, durations, supply, demand, capacity and fixed charges. The rest of the paper is organised as follows: In section 2, some preliminaries and formulation of FMIBCFCBTP are given. The algorithm developed for finding optimal solutions of the FMIBCFCBTP is given in section 3. Section 4 describes the solution procedure by a numerical example. Section 5 compares the proposed method with existing method and gives concluding remarks.

2. Preliminaries and Formulation of Bottleneck MIBCFCTP using trapezoidal fuzzy numbers

In real life situation, crisp MIBCFCBTP may not give the optimal solution because of uncertainty in parameters. To solve this uncertain problem, the crisp problem can be reformulated as fuzzy problem using trapezoidal fuzzy numbers $\tilde{A} = (a, b, c, d)$ [5,13, 15].

2.1. Preliminaries of trapezoidal fuzzy numbers

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers. Then

(i) Membership function for $\tilde{A} = (a_1, a_2, a_3, a_4)$:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, x < a_1, x > a_4 \\ \dfrac{x - a_1}{a_2 - a_1}, a_1 \le x < a_2 \\ 1, \quad a_2 \le x \le a_3 \\ \dfrac{a_4 - x}{a_4 - a_3}, a_3 < x \le a_4 \end{cases}$$

(ii) Ranking of $\tilde{A} = (a_1, a_2, a_3, a_4)$ is $\Re(\tilde{A}) = \frac{a_1 + a_2 + a_3 + a_4}{4}$

(iii) $(a_1, a_2, a_3, a_4) + (b_1, b_2, b_3, b_4) = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$

(iv) $(a_1, a_2, a_3, a_4) - (b_1, b_2, b_3, b_4) = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$

(v) $\lambda. (a_1, a_2, a_3, a_4) = (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4)$

(v) $(a_1, a_2, a_3, a_4) * (b_1, b_2, b_3, b_4) = \Re(\tilde{B}). (a_1, a_2, a_3, a_4)$

2.2. Formulation of FMIBCFCBTP

Let there be as m sources, n destinations and p types of commodities. Let cost, time, supply, demand, capacity and fixed charge be trapezoidal fuzzy numbers. Then the problem can be formulated as

$$\text{(P)} Minimize \left\{ \max_{\substack{1 \le i \le m \\ 1 \le j \le n \\ 1 \le k \le p}} [\tilde{t}_{ijk} | \tilde{x}_{ijk} > (0,0,0,0)], \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} (\tilde{c}_{ijk}\, \tilde{x}_{ijk} + \tilde{f}_{ijk} \tilde{y}_{ijk}) \right\}$$

subject to

$$\sum_{i=1}^{m} \Re(\tilde{x}_{ijk}) = \Re(\tilde{A}_{jk}), (j = 1,2,\dots,n; k = 1,2,\dots,p)$$

$$\sum_{j=1}^{n} \Re(\tilde{x}_{ijk}) = \Re(\tilde{B}_{ki}), (k = 1,2,\dots,p; i = 1,2,\dots,m)$$

$$\sum_{k=1}^{p} \Re(\tilde{x}_{ijk}) = \Re(\tilde{E}_{ij}), (i = 1,2,\dots,m; j = 1,2,\dots,n)$$

$$\Re(\tilde{x}_{ijk}) \geq 0, (i = 1,2,\dots,m; j = 1,2,\dots,n; k = 1,2,\dots,p)$$

$$\sum_{j=1}^{n} \Re(\tilde{A}_{jk}) = \sum_{i=1}^{m} \Re(\tilde{B}_{ki}), \sum_{k=1}^{p} \Re(\tilde{B}_{ki}) = \sum_{j=1}^{n} \Re(\tilde{E}_{ij}), \sum_{i=1}^{m} \Re(\tilde{E}_{ij}) = \sum_{k=1}^{p} \Re(\tilde{A}_{jk})$$

$$\sum_{j=1}^{n}\sum_{k=1}^{p} \Re(A_{jk}) = \sum_{k=1}^{p}\sum_{i=1}^{m} \Re(B_{ki}) = \sum_{i=1}^{m}\sum_{j=1}^{n} \Re(E_{ij})$$

$$\tilde{y}_{ijk} = \begin{cases} (1,1,1,1) \; if \; \Re(\tilde{x}_{ijk}) > 0 \\ (0,0,0,0) \qquad otherwise \end{cases}$$

where

$\tilde{t}_{ijk}$ is fuzzy time required for transporting k^{th} commodity from i^{th} source to j^{th} destination.

$\tilde{c}_{ijk}$ is unit fuzzy transportation cost for k^{th} commodity transported from i^{th} source to j^{th} destination.

$\tilde{f}_{ijk}$ is fuzzy fixed charge incurred when k^{th} commodity is transported from i^{th} source to j^{th} destination.

$\tilde{x}_{ijk}$ is fuzzy units of k^{th} commodity transported from i^{th} source to j^{th} destination .

$\Re(\tilde{A}_{jk})$ is the rank of total fuzzy requirement of k^{th} commodity at j^{th} destination.

$\Re(\tilde{B}_{ki})$ is the rank of total fuzzy availability of k^{th} commodity at i^{th} source.

$\Re(\tilde{E}_{ij})$ is the rank of maximum fuzzy unit that can be sent from i^{th} source to j^{th} destination.

The problem (P) can be divided into two sub problems - fuzzy time minimization multi index transportation problem (P_1) and total fuzzy cost minimization multi index transportation problem (P_2).

The first sub problem (P_1) is

$$(P_1)\text{: } Minimize\{\max_{\substack{1\leq i\leq m\\1\leq j\leq n\\1\leq k\leq p}}[\tilde{t}_{ijk} | \tilde{x}_{ijk} > (0,0,0,0)\}$$

The second sub problem (P_2) is

$$(P_2)\text{: } Minimize \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} (\tilde{c}_{ijk}\,\tilde{x}_{ijk} + \tilde{f}_{ijk}\tilde{y}_{ijk})$$

3. Algorithm for solving FMIBCFCBTP

The steps of the proposed algorithm are as follows:

Step 1. Fuzzy transportation time $\tilde{t}_{ijk}$ and fuzzy transportation cost $\tilde{c}_{ijk}$ for the FMIBCFCBTP with m sources, n destinations and p type of commodities are shown in Table 1(a) and Table 1(b) respectively.

Step 2. The corresponding crisp Time Table (Table (2a)) and crisp Cost Table (Table (2b)) are obtained by applying the ranking approach on the trapezoidal fuzzy numbers.

Step 3. In Table (2a) a modified VAM method is applied to get the initial solution. In this method, penalties of all rows, columns and cell diagonals are calculated. The row/column/cell diagonal with largest penalty is selected and allocation is made in the cell in that row/column/cell diagonal having minimum time. In case of tie in largest penalty, the row/column/cell diagonal having cell with minimum time is selected. In case of tie in minimum time also, the row/column/cell diagonal in which maximum allocation can be made in the cell with minimum time is selected.

Step 4. Step 3 is repeated till the total demand, supply and capacity is allocated.
Table 3 shows the allocation of the initial solution obtained by applying the modified VAM method on Table (2a).

Step 5. From Table 3 the corresponding fuzzy time table is obtained as shown in Table 4.

Step 6. In Table 4, if there is cell with negative allocation, to make the solution feasible an e-loop is introduced to remove that allocation. Forming an e-loop can be seen in figure 1.

Figure 1:

	j=1		j=2	
i=1	1 (-1) +		3 8-	
		2 8-		4 5+
i=2	5 9-		7 +	
		6 2+		8 6-

It can be seen from figure 1 that cell (1,1,1) has negative allocation of -1 units. The e-loop is formed consisting of 2^3 cells with two adjacent cells lying in the same row/column/cell diagonal.

The cell with negative allocation is given a '+' sign and all other cells alternately '+' and '-' sign. All cells are allocated except one non-allocated cell with '+' sign. After shifting in the e-loop, the allocations of feasible solution are shown in figure 2.

Figure 2:

	j=1		j=2	
i=1	1		3 7	
		2 7		4 6
i=2	5 8		7 1	
		6 3		8 5

Step 7. The feasible solution is made basic feasible by counting the number of allocated cells.
If number of allocated cells=mnp-(m-1)(n-1)(p-1) then solution is feasible.
If number of allocated cells< mnp-(m-1)(n-1)(p-1) then $\tilde{\mathcal{E}} = (\mathcal{E},\mathcal{E},\mathcal{E},\mathcal{E})$ is added to an independent non-allocated cell.
The allocations of fuzzy Initial Basic Feasible solution are shown in Table 5.

Step 8. From Table (5), the fuzzy time $\tilde{t}_1$ and $\sum \Re(x_{ijk})$ which is the sum of ranks of total number of units transported to cells corresponding to $\tilde{t}_1$ is obtained.

Step 9. Table (6) is obtained by converting each $\tilde{t}_{ijk}$ in Table (5) as

$$\tilde{t}_{ijk} = \begin{cases} (1,1,1,1) & ,\Re(\tilde{t}_{ijk}) < R(\tilde{t}_1) \\ (0,0,0,0) & , \quad otherwise \end{cases}$$

Step 10. From Table (6), fuzzy shadow costs $\tilde{u}_{jk}, \tilde{v}_{ki}, \tilde{w}_{ij}$ are calculated by putting (m-1)(n-1)(p-1) shadow costs equal to zero and find the remaining shadow costs for mnp-(m-1)(n-1)(p-1) allocations. The shadow costs satisfy $\tilde{u}_{jk} + \tilde{v}_{ki} + \tilde{w}_{ij} = 0$.

Step 11. The non-basic cell for which $\tilde{t}_{ijk} = (1,1,1,1)\ and\ (\tilde{u}_{jk} + \tilde{v}_{ki} + \tilde{w}_{ij}) \leq 0$ is now made to enter the basis by forming e-loop as explained below:
The e-loop is similar to the e-loop explained in step (6) with the difference being that it starts from non-allocated cell and has only allocated cells as its vertices. The modified allocations are shown in Table 7.

Step 12. From Table 7, the fuzzy time $\tilde{t}_2$ is calculated.

Step 13. Steps (9-12) are repeated until for all non-basic cells for which $\tilde{t}_{ijk} = (1,1,1,1)$, the sum $(\tilde{u}_{jk} + \tilde{v}_{ki} + \tilde{w}_{ij}) > 0$.The allocations obtained give the first fuzzy Efficient Time $\tilde{T}_1$ is shown in Table 8.

Step 14. The corresponding fuzzy cost $\tilde{Z}_1$ is obtained from Table (1b) as follows:-

(i) Modify fuzzy transportation cost of sub problem (P_2) as

$$\tilde{c}'_{ijk} = \begin{cases} \tilde{c}_{ijk}\ if\ \Re(\tilde{t}_{ijk}) \leq \Re(\tilde{T}_1) \\ (\infty,\infty,\infty,\infty)\ if\ \Re(\tilde{t}_{ijk}) > R(\tilde{T}_1) \end{cases}$$

(ii) The fuzzy shadow costs, $\tilde{u}_{jk}, \tilde{v}_{ki}, \tilde{w}_{ij}$ are calculated by putting (m-1)(n-1)(p-1) shadow costs equal to zero and find the remaining shadow costs for mnp-(m-1)(n-1)(p-1) allocations. The shadow costs satisfy $\tilde{c}'_{ijk}(for\ allocated\ cells) = \tilde{u}_{jk} + \tilde{v}_{ki} + \tilde{w}_{ij}$.

(iii) Calculate $\tilde{\delta}_{ijk} = \tilde{c}'_{ijk} - (\tilde{u}_{jk} + \tilde{v}_{ki} + \tilde{w}_{ij})$ for non-allocated cells.

(iv) Select the cell with minimum $\Re(\tilde{\delta}_{ijk})$ and make it to enter the basis by e-loop explained in step (11)

(v) Let the allocation of the selected cell be $\tilde{I}_{ijk}$.

(vi) Calculate $\tilde{A}_{ijk} = \tilde{\delta}_{ijk} \times \tilde{I}_{ijk}$.

(vii) Calculate the fixed charge difference as
$\tilde{F}_{ijk}(difference) = \tilde{F}_{ijk}(for\ newly\ enter\ cell) - \tilde{F}_{ijk}(for\ leaving\ cell)$

(viii) Calculate total cost difference as $\tilde{\Delta}_{ijk} = \tilde{A}_{ijk} + \tilde{F}_{ijk}(difference)$

(ix) If $\Re(\tilde{\Delta}_{ijk}) < 0$ then enter the cell with e-loop as explained in step (11) and repeat the steps 14(ii)-(vii) till all $\Re(\tilde{\Delta}_{ijk}) \geq 0$.

(x) Calculate the Total transportation cost as

$$\tilde{Z}_1 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p}(\tilde{c}_{ijk}\,\tilde{x}_{ijk} + \tilde{f}_{ijk}(for\ allocated\ cells))$$

Step 15. The allocation for the first fuzzy efficient Optimal Cost obtained is shown in Table 9.
Table 9 is modified to Table 10 by retaining the original fuzzy costs as given in Table 1(b) and the final allocation of the first efficient solution by the modified MODI method.

(i) Calculate the shadow costs as explained in step 14(ii) and apply steps 14(iii-viii).
(ii) The cell where $\Re(\tilde{\Delta}_{ijk}) < 0$ enters the basis by e-loop as explained in step (11).
(iii) Find the second optimal time $\tilde{T}_2$ from Table 11 obtained after applying Steps 15 (i), (ii) on Table 10.
(iv) Apply step (14) to find the second fuzzy total transportation cost $\tilde{Z}_2$ corresponding to $\tilde{T}_2$ which is shown in Table 12.

Step 16. Repeat step (15) till all $\tilde{c}_{ijk}^{'} = \tilde{c}_{ijk}$ to obtain third and subsequent efficient time-cost trade-off pairs.

4. Numerical example

Let there be *m* = 3sources, *n* = 3 destinations and *p* = 3 types of commodities. Table 1(a) shows fuzzy time together with fuzzy supply, demand and capacity, Table 1(b) shows fuzzy cost together with fuzzy supply, demand and capacity. The fuzzy fixed charges $\tilde{f}_{ijk}$ are given below.

$\tilde{f}_{111}$=(4,6,10,20) $\tilde{f}_{121}$=(14,16,30,60) $\tilde{f}_{131}$=(9,11,20,40)
$\tilde{f}_{112}$=(9,11,20,40) $\tilde{f}_{122}$=(9,11,20,40) $\tilde{f}_{132}$=(9,11,20,40)
$\tilde{f}_{113}$=(14,16,30,60) $\tilde{f}_{123}$=(9,11,20,40) $\tilde{f}_{133}$=(4,6,10,20)
$\tilde{f}_{211}$=(4,6,10,20) $\tilde{f}_{221}$=(9,11,20,40) $\tilde{f}_{231}$=(9,11,20,40)
$\tilde{f}_{212}$=(4,6,10,20) $\tilde{f}_{222}$=(4,6,10,20) $\tilde{f}_{232}$=(14,16,30,60)
$\tilde{f}_{213}$=(19,21,40,80) $\tilde{f}_{223}$=(4,6,10,20) $\tilde{f}_{233}$=(4,6,10,20)
$\tilde{f}_{311}$=(4,6,10,20) $\tilde{f}_{321}$=(19,21,40,80) $\tilde{f}_{331}$=(9,11,20,40)
$\tilde{f}_{312}$=(9,11,20,40) $\tilde{f}_{322}$=(4,6,10,20) $\tilde{f}_{332}$=(14,16,30,60)
$\tilde{f}_{313}$=(9,11,20,40) $\tilde{f}_{323}$=(4,6,10,20) $\tilde{f}_{333}$=(4,6,10,20)

Table 1(a): (Fuzzy time table)

Destinations → Sources ↓	j=1			j=2			j=3			$\tilde{B}_{ki}$		
i=1	(1,2,3,6)			(3,5,8,16)			(3,4,7,14)			(2,4,6,12)		
		(2,3,5,10)			(2,4,6,12)			(1,3,4,8)			(4,5,9,18)	
	$\tilde{E}_{11}$=(4,6,10,20)		(1,3,4,8)	$\tilde{E}_{12}$=(2,4,6,12)		(3,5,8,16)	$\tilde{E}_{13}$=(4,5,9,18)		(0,0.5,1.5,2)			(4,6,10,20)

i=2	(0,1,2,5)			(1,3,4,8)			(2,4,6,12)			(6,7,13,26)		
		(0,0.5,1.5,2)			(0,1,2,5)			(0,0.5,1.5,2)			(6,8,14,28)	
	$\tilde{E}_{21}$=(10,11,21,42)		(2,4,6,12)	$\tilde{E}_{22}$=(4,5,9,18)		(0,1,2,5)	$\tilde{E}_{23}$=(6,8,14,28)		(0,0.5,1.5,2)			(8,9,17,34)
i=3	(1,3,4,8)			(1,2,3,6)			(1,3,4,8)			(7,8,15,30)		
		(3,5,8,16)			(0,1,2,5)			(0,1,2,5)			(6,7,13,26)	
	$\tilde{E}_{31}$=(10,11,21,42)		(0,0.5,1.5,2)	$\tilde{E}_{32}$=(6,7,13,26)		(0,0.5,1.5,2)	$\tilde{E}_{33}$=(5,7,12,24)		(3,5,8,16)			(8,10,18,36)
$\tilde{A}_{jk}$	(7,8,15,30)			(3,5,8,16)			(5,6,11,22)					
		(8,9,17,34)			(5,6,11,22)			(3,5,8,16)				
			(9,11,20,40)			(4,5,9,18)			(7,9,16,32)			

Table 1(b): (Fuzzy cost table)

Destinations → Sources ↓	j=1			j=2			j=3			$\tilde{B}_{ki}$		
i=1	(3,5,8,16)			(1,5,6,8)			(3,4,7,14)			(2,4,6,12)		
		(3,4,7,14)			(2,4,6,12)			(1,2,3,6)			(4,5,9,18)	
	$\tilde{E}_{11}$=(4,6,10,20)		(2,4,6,12)	$\tilde{E}_{12}$=(2,4,6,12)		(4,6,10,20)	$\tilde{E}_{13}$=(4,5,9,18)		(5,6,11,22)			(4,6,10,20)
i=2	(5,6,11,22)			(4,5,9,18)			(6,7,13,26)			(6,7,13,26)		
		(3,5,8,16)			(7,8,15,30)			(3,4,7,14)			(6,8,14,28)	
	$\tilde{E}_{21}$=(10,11,21,42)		(6,7,13,26)	$\tilde{E}_{22}$=(4,5,9,18)		(5,7,12,24)	$\tilde{E}_{23}$=(6,8,14,28)		(3,5,8,16)			(8,9,17,34)
i=3	(1,5,6,8)			(3,5,8,16)			(4,6,10,20)			(7,8,15,30)		
		(2,4,6,12)			(4,5,9,18)			(2,4,6,12)			(6,7,13,26)	
	$\tilde{E}_{31}$=(10,11,21,42)		(3,4,7,14)	$\tilde{E}_{32}$=(6,7,13,26)		(3,4,7,14)	$\tilde{E}_{33}$=(5,7,12,24)		(5,7,12,24)			(8,10,18,36)
$\tilde{A}_{jk}$	(7,8,15,30)			(3,5,8,16)			(5,6,11,22)					
		(8,9,17,34)			(5,6,11,22)			(3,5,8,16)				
			(9,11,20,40)			(4,5,9,18)			(7,9,16,32)			

After applying the ranking approach to the trapezoidal fuzzy numbers in Table 1(a) and Table 1(b), the crisp time table and crisp cost table are obtained which are shown in Table 2(a) and Table 2(b) respectively.

Table 2(a): (Crisp time table)

Destinations → Sources ↓	j=1	j=2	j=3	B_{ki}
i=1	3 5 4 E_{11}=10	8 6 8 E_{12}= 6	7 4 1 E_{13}=9	6 0 9 10
i=2	2 1 6 E_{21}=21	4 2 2 E_{22}=9	6 1 1 E_{23}=14	13 14 17
i=3	4 8 1 E_{31}=21	3 2 1 E_{32}=13	4 2 8 E_{33}=12	15 13 18
A_{jk}	15 17 20	8 11 9	11 8 16	34 36 45

Table 2(b): (Crisp cost table)

Destinations → Sources ↓	j=1	j=2	j=3	B_{ki}
i=1	8 7 6 E_{11}=10	5 6 10 E_{12}= 6	7 3 11 E_{13}=9	6 9 10
i=2	11 8 13 E_{21}=21	9 15 12 E_{22}=9	13 7 8 E_{23}=14	13 14 17
i=3	5 6 7 E_{31}=21	8 9 7 E_{32}=13	10 6 12 E_{33}=12	15 13 18
A_{jk}	15 17 20	8 11 9	11 8 16	34 36 45

The initial solution to the crisp transportation time is obtained by applying modified VAM method (steps (3-4)) as shown below in Table 3.

Table 3: (Initial solution for crisp transportation time)

Destinations → Sources ↓	j=1	j=2	j=3
i=1	3 ⑤	8	7 ①
	5 ③	6	4 ⑥
	4 ②	8 ⑥	1 ②
i=2	2 ⑦	4 ⑥	6
	1 ⑭	2	1
	6	2 ③	1 ⑭
i=3	4 ③	3 ②	4 ⑩
	8	2 ⑪	2 ②
	1 ⑱	1	8

From Table 3, corresponding fuzzy allocations obtained are shown in bold in Table 4.

Table 4: (Initial solution for fuzzy transportation time)

Destinations → Sources ↓	j=1	j=2	j=3
i=1	(1,2,3,6) **(-55,-12,21,66)**	(3,5,8,16)	(3,4,7,14) **(-54,-15,16,57)**
	(2,3,5,10) **(-20,-5,9,28)**	(2,4,6,12)	(1,3,4,8) **(-18,-2,12,32)**
	(1,3,4,8) **(-34,-8,11,39)**	(3,5,8,16) **(2,4,6,12)**	(0.0.5,1,2) **(-21,-5,8,26)**
i=2	(0,1,2,5) **(-18,-3,13,36)**	(1,3,4,8) **(-12,0,10,26)**	(2,4,6,12)
	(0.0.5,1,2) **(6,8,14,28)**	(0,1,2,5)	(0.0.5,1,2)
	(2,4,6,12)	(0,1,2,5) **(-8,-1,5,16)**	(0.0.5,1,2) **(6,8,14,28)**
i=3	(1,3,4,8) **(-26,-7,11,34)**	(1,2,3,6) **(-16,-4,7,21)**	(1,3,4,8) **(-16,0,16,40)**
	(3,5,8,16)	(0,1,2,5) **(5,6,11,22)**	(0,1,2,5) **(-16,-4,7,21)**
	(0.0.5,1,2) **(8,10,18,36)**	(0.0.5,1,2)	(3,5,8,16)

It is seen from Table 4 that there is no negative allocation. Also from Table 4, number of allocated cells=18 which is less than (mnp-(m-1)(n-1)(p-1)), so $\tilde{\varepsilon} = (\varepsilon, \varepsilon, \varepsilon, \varepsilon)$ is added to the non-allocated cell (3,2,3). The allocations of Initial Basic Feasible solution (IBFS) obtained are shown in bold in Table 5.

Table 5: (IBFS for fuzzy time table)

Destinations → Sources ↓	j=1			j=2			j=3		
i=1	(1,2,3,6) **(-55,-12,21,66)**	(2,3,5,10) **(-20,-5,9,28)**	(1,3,4,8) **(-34,-8,11,39)**	(3,5,8,16)	(2,4,6,12)	(3,5,8,16) **(2,4,6,12)**	(3,4,7,14) **(-54,-15,16,57)**	(1,3,4,8) **(-18,-2,12,32)**	(0.0.5,1,2) **(-21,-5,8,26)**
i=2	(0,1,2,5) **(-18,-3,13,36)**	(0.0.5,1,2) **(6,8,14,28)**	(2,4,6,12)	(1,3,4,8) **(-12,0,10,26)**	(0,1,2,5)	(0,1,2,5) **(-8,-1,5,16)**	(2,4,6,12)	(0.0.5,1,2)	(0.0.5,1,2) **(6,8,14,28)**
i=3	(1,3,4,8) **(-26,-7,11,34)**	(3,5,8,16)	(0.0.5,1,2) **(8,10,18,36)**	(1,2,3,6) **(-16,-4,7,21)**	(0,1,2,5) **(5,6,11,22)**	(0.0.5,1,2) $(\boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}, \boldsymbol{\varepsilon})$	(1,3,4,8) **(-16,0,16,40)**	(0,1,2,5) **(-16,-4,7,21)**	(3,5,8,16)

From Table 5, fuzzy transportation time is calculated as $\tilde{t}_1 = (3,5,8,16)$and $\sum \Re(x_{ijk})$=6 for cell corresponding to $\tilde{t}_1$. After applying steps (8-9) to Table 5, modified transportation time and fuzzy shadow time is calculated which is shown in Table 6.

Table 6: (Modified fuzzy time with IBFS)

Destinations → Sources ↓	j=1			j=2			j=3			$\tilde{u}_{jk}$		
i=1	(1,1,1,1) **(-55,-12,21,66)**-	(1,1,1,1) **(-20,-5,9,28)**	(1,1,1,1) **(-34,-8,11,39)**+	(0,0,0,0)	(1,1,1,1) +	(0,0,0,0) **(2,4,6,12)**-	(1,1,1,1) **(-54,-15,16,57)** +	(1,1,1,1) **(-18,-2,12,32)**-	(1,1,1,1) **(-21,-5,8,26)**	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)
	$\tilde{E}_{11}$ = (0,0,0,0)			$\tilde{E}_{12}$=(-1,-1,-1,-1)			$\tilde{E}_{13}$ = (0,0,0,0)					

i=2	(1,1,1,1) **(-18,-3,13,36)**			(1,1,1,1) **(-12,0,10,26)**			(1,1,1,1)			(0,0,0,0)		
		(1,1,1,1) **(6,8,14,28)**			(1,1,1,1)			(1,1,1,1)			(0,0,0,0)	
	$\tilde{E}_{21} = (0,0,0,0)$		(1,1,1,1)	$\tilde{E}_{22} = (0,0,0,0)$		(1,1,1,1) **(-8,-1,5,16)**	$\tilde{E}_{23} = (0,0,0,0)$		(1,1,1,1) **(6,8,14,28)**			(0,0,0,0)
i=3	(1,1,1,1) **(-26,-7,11,34)**+			(1,1,1,1) **(-16,-4,7,21)**			(1,1,1,1) **(-16,0,16,40)**-			(0,0,0,0)		
		(0,0,0,0)			(1,1,1,1) **(5,6,11,22)**-			(1,1,1,1) **(-16,-4,7,21)**+			(0,0,0,0)	
	$\tilde{E}_{31} = (0,0,0,0)$		(1,1,1,1) **(8,10,18,36)**-	$\tilde{E}_{32} = (0,0,0,0)$		(1,1,1,1) $(\varepsilon, \varepsilon, \varepsilon,$	$\tilde{E}_{33} = (0,0,0,0)$		(0,0,0,0)			(0,0,0,0)
$\tilde{v}_{ki}$	(1,1,1,1)			(1,1,1,1)			(1,1,1,1)					
		(1,1,1,1)			(1,1(,1,1)			(1,1,1,1)				
			(1,1,1,1)			(1,1,1,1)			(1,1,1,1)			

On applying steps (10-11) on table 6, the allocations of new Basic Feasible solution obtained are shown in bold in Table 7.

Table 7: (Basic feasible solution)

Destinations → Sources ↓	j=1			j=2			j=3		
i=1	(1,1,1,1)			(0,0,0,0)			(1,1,1,1) **(-109,-27,37,123)**		
		(1,1,1,1) **(-20,-5,9,28)**			(1,1,1,1) **(-55,-12,21,66)**			(1,1,1,1) **(-84,-23,24,87)**	
			(1,1,1,1) **(-89,-20,32,105)**			(0,0,0,0) **(-64,-17,18,67)**			(1,1,1,1) **(-21,-5,8,26)**
i=2	(1,1,1,1) **(-18,-3,13,36)**			(1,1,1,1) **(-12,0,10,26)**			(1,1,1,1)		
		(1,1,1,1) **(6,8,14,28)**			(1,1,1,1)			(1,1,1,1)	
			(1,1,1,1)			(1,1,1,1) **(-8,-1,5,16)**			(1,1,1,1) **(6,8,14,28)**
i=3	(1,1,1,1) **(-81,-19,32,100)**			(1,1,1,1) **(-16,-4,7,21)**			(1,1,1,1) **(-82,-21,28,95)**		
		(0,0,0,0)			(1,1,1,1) **(-61,-**			(1,1,1,1) **(-71,-**	

					15,23,77)			**16,28,87)**	
			(1,1,1,1) **(-58,-11,30,91)**			(1,1,1,1)+ **(-55,-12,21,66)**			(0,0,0,0)

From Table 7, fuzzy time $\tilde{t}_2$ =(3,5,8,16) and $\sum \Re(x_{ijk})$=1 for cell corresponding to $\tilde{t}_2$
After applying the step (13), first fuzzy efficient time $\tilde{T}_1$ with fuzzy allocations (in bold) are shown in Table 8.

Table 8: (First fuzzy efficient time)

Destinations → Sources ↓	j=1			j=2			j=3		
i=1	(1,2,3,6) **(-230,-60,70,244)**	(2,3,5,10) **(-20,-5,9,28)**	(1,3,4,8) **(-333,-90,92,335)**	(3,5,8,16)	(2,4,6,12) **(-119,-29,39,133)**	(3,5,8,16)	(3,4,7,14)	(1,3,4,8)	(0,0.5,1.5,2) **(-315,-82,96,337)**
i=2	(0,1,2,5) **(-196,-52,61,211)**	(0,0.5,1.5,2) **(6,8,14,28)**	(2,4,6,12) **(-175,-48,49,178)**	(1,3,4,8)	(0,1,2,5)	(0,1,2,5) **(-127,-30,44,149)**	(2,4,6,12) **(-294,-77,88,311)**	(0,0.5,1.5,2) (□, □, □, □)	(0,0.5,1.5,2) **(-305,-80,91,322)**
i=3	(1,3,4,8) **(-147,-40,44,155)**	(3,5,8,16)	(0,0.5,1.5,2) **(-133,-23,51,157)**	(1,2,3,6) **(-135,-33,46,154)**	(0,1,2,5) **(-128,-33,40,141)**	(0,0.5,1.5,2) $(\boldsymbol{\epsilon},\boldsymbol{\epsilon},\boldsymbol{\epsilon},\boldsymbol{\epsilon})$	(1,3,4,8) **(-149,-39,45,159)**	(0,1,2,5) **(-135,-33,46,154)**	(3,5,8,16)

From Table 8, first fuzzy efficient time is calculated as $\tilde{T}_1 = (2,4,6,12)$
The first fuzzy efficient cost $\tilde{Z}_1$corresponding to $\tilde{T}_1$ is calculated by applying step (14) to fuzzy cost table 1(b). The allocations obtained are shown in bold in Table 9.

Table 9: (First fuzzy efficient solution)

Destinations → Sources ↓	j=1			j=2			j=3		
i=1	(3,5,8,16) **(2,4,6,12)**	(3,4,7,14) **(-8,-1,5,16)**	(2,4,6,12) **(-24,-5,7,26)**	$(\infty,\infty,\infty,\infty)$	(2,4,6,12) **(2,4,6,12)**	$(\infty,\infty,\infty,\infty)$	$(\infty,\infty,\infty,\infty)$	(1,2,3,6) $(\boldsymbol{\epsilon},\boldsymbol{\epsilon},\boldsymbol{\epsilon},\boldsymbol{\epsilon})$	(5,6,11,22) **(-20,-1,15,42)**

i=2	(5,6,11,22)	(3,5,8,16) **(-8,4,18,42)**	(6,7,13,26) **(-32,-7,17,50)**	(4,5,9,18)	(7,8,15,30) **(-63,-15,24,78)**	(5,7,12,24) **(-74,-19,24,81)**	(6,7,13,26) **(-52,-11,22,69)**	(3,4,7,14)	(3,5,8,16) **(-35,-6,17,52)**
i=3	(1,5,6,8) **(-5,2,11,28)**	(∞, ∞, ∞, ∞)	(3,4,7,14) **(-18,0,19,47)**	(3,5,8,16) **(-78,-20,24,82)**	(4,5,9,18) **(-7,0,7,20)**	(3,4,7,14) **(-39,-9,18,54)**	(4,6,10,20) **(-47,-11,17,57)**	(2,4,6,12) **(-33,-5,18,52)**	(∞, ∞, ∞, ∞)

From Table 9, total fuzzy transportation cost is calculated as sum of direct fuzzy cost and fuzzy fixed charges which is equal to

$\tilde{Z}_1 = (393,586,979,1922) + (152,188,340,680) = (545,774,1319,2602)$

The first fuzzy efficient time-cost pair is

$(\tilde{T}_1, \tilde{Z}_1) = ((2,5,6,12),((545,774,1319,2602))$

Table 9 is modified to Table 10 by retaining the original fuzzy costs.

Table 10:

Destinations → Sources ↓	j=1			j=2			j=3		
i=1		(3,4,7,14) **(-8,-1,5,16)**	(2,4,6,12) **(-24,-5,7,26)**	(1,5,6,8)	(2,4,6,12) **(2,4,6,12)**	(4,6,10,20)	(2,4,6,12)	(1,2,3,6) $(\varepsilon,\varepsilon,\varepsilon,\varepsilon)$	(5,6,11,22) **(-20,-1,15,42)**
i=2	(5,6,11,22)	(3,5,8,16) **(-8,4,18,42)**	(6,7,13,26) **(-32,-7,17,50)**	(4,5,9,18) **(-63,-15,24,78)**	(7,8,15,30)	(5,7,12,24) **(-74,-19,24,81)**	(6,7,13,26) **(-52,-11,22,69)**	(3,4,7,14)	(3,5,8,16) **(-35,-6,17,52)**
i=3	(1,5,6,8) **(-5,2,11,28)**	(2,4,6,12)	(3,4,7,14) **(-18,0,19,47)**	(3,5,8,16) **(-78,-20,24,82)**	(4,5,9,18) **(-7,0,7,20)**	(3,4,7,14) **(-39,-9,18,54)**	(4,6,10,20) **(-47,-11,17,57)**	(2,4,6,12) **(-33,-5,18,52)**	(5,7,12,24)

By applying steps 15(i-iii), Table 11 is obtained showing the allocations in bold of second efficient time $\tilde{T}_2$.

Table 11: (Second fuzzy efficient time)

Destinations → Sources ↓	j=1			j=2			j=3		
i=1	(3,5,8,16) **(-80,-20,26,90)**			(1,5,6,8)			(3,4,7,14) **(-78,-20,24,82)**		
		(3,4,7,14) **(-8,-1,5,16)**			(2,4,6,12) **(2,4,6,12)**			(1,2,3,6) **($\epsilon,\epsilon,\epsilon,\epsilon$)**	
			(2,4,6,12) **(-106,-29,27,104)**			(3,8,16,32)			(5,6,11,22) **(-102,-25,35,120)**
i=2	(5,6,11,22)			(4,5,9,18) **(-141,-35,48,160)**			(6,7,13,26) **(-134,-35,42,147)**		
		(3,5,8,16) **(-8,4,18,42)**			(7,8,15,30)			(3,4,7,14)	
			(6,7,13,26) **(-32,-7,17,50)**			(5,7,12,24) **(-156,-43,44,159)**			(3,5,8,16) **(-113,-26,41,134)**
i=3	(1,5,6,8) **(-83,-18,35,110)**			(3,5,8,16)			(4,6,10,20) **(-47,-11,17,57)**		
		(2,4,6,12)			(4,5,9,18) **(-7,0,7,20)**			(2,4,6,12) **(-33,-5,18,52)**	
			(3,4,7,14) **(-100,-24,39,125)**			(3,4,7,14) **(-117,-29,42,136)**			(5,7,12,24)

By applying step 15(iv), the corresponding allocations of fuzzy cost are shown in Table 12.

Table 12: (Second fuzzy efficient cost)

Destinations → Sources ↓	j=1			j=2			j=3		
i=1	(3,5,8,16)			($\infty,\infty,\infty,\infty$)			(3,4,7,14) **(2,4,6,12)**		
		(3,4,7,14) **(-8,-1,5,16)**			(2,4,6,12) **(2,4,6,12)**			(1,2,3,6) **($\varepsilon,\varepsilon,\varepsilon,\varepsilon$)**	
			(2,4,6,12) **(-12,1,11,28)**			($\infty,\infty,\infty,\infty$)			(5,6,11,22) **(-8,1,5,16)**
i=2	(5,6,11,22) **(-33,-9,14,44)**			(4,5,9,18) **(3,5,8,16)**			(6,7,13,26) **(-28,-7,8,31)**		
		(3,5,8,16) **(6,8,14,28)**			(7,8,15,30)			(3,4,7,14)	
			(6,7,13,26) **(-75,-20,24,83)**			(5,7,12,24) **(-29,-8,9,32)**			(3,5,8,16) **(-9,4,17,40)**

	j=1			j=2			j=3		
i=3	(1,5,6,8) **(-14,1,16,41)**	$(\infty,\infty,\infty,\infty)$)	(3,4,7,14) **(-31,-5,20,56)**	(3,5,8,16)	(4,5,9,18) **(-7,0,7,20)**	(3,4,7,14) **(-14,0,13,33)**	(4,6,10,20) **(-11,-1,7,21)**	(2,4,6,12) **(3,5,8,16)**	$(\infty,\infty,\infty,\infty$

From Tables 11 and 12, second fuzzy time-cost efficient pair is $(\tilde{T}_2, \tilde{Z}_2) = ((3,4,7,14), (489,726,1215,2386))$.
From Step 16 the allocations of third fuzzy efficient solution obtained are shown in bold in Table 13.

Table 13: (Third fuzzy efficient solution)

Destinations → Sources ↓	j=1			j=2			j=3		
i=1	(3,5,8,16)	(3,4,7,14) $(\varepsilon,\varepsilon,\varepsilon,\varepsilon)$	(2,4,6,12) **(4,6,10,20)**	(1,5,6,8) **(-78,-20,23,83)**	(2,4,6,12) **(-71,-17,24,80)**	(4,6,10,20)	(3,4,7,14) **(-71,-17,24,80)**	(1,2,3,6 **(-62,-15,22,75)**	(5,6,11,22)
i=2	(5,6,11,22) **(-88,-21,31,106)**	(3,5,8,16) **(-118,-26,48,152)**	(6,7,13,26)	(4,5,9,18) **(-80,-18,28,94)**	(7,8,15,30)	(5,7,12,24) **(-97,-26,30,105)**	(6,7,13,26)	(3,4,7,14)	(3,5,8,16) **(6,8,14,28)**
i=3	(1,5,6,8) **(-86,-20,33,105)**	(2,4,6,12) **(-144,-39,43,152)**	(3,4,7,14) **(-123,-31,46,148)**	(3,5,8,16)	(4,5,9,18) **(-75,-18,28,93)**	(3,4,7,14) **(-87,-21,31,101)**	(4,6,10,20) **(-75,-18,28,93)**	(2,4,6,12) **(-59,-14,20,65)**	(5,7,12,24) **(-21,-5,8,26)**

The third efficient time-cost pair is obtained from Table 13 which is equal to $(\tilde{T}_3, \tilde{Z}_3) = ((3,5,8,16), (476,717,1193,2346))$.
It is observed that no more efficient solutions can be obtained as all $\tilde{c}'_{ijk} = \tilde{c}_{ijk}$.

5. Comparative study between existing method and proposed method

The FMIBCFCBTP with all parameters as trapezoidal fuzzy numbers has been solved for the first time. So it cannot be compared with existing bottleneck problems. However Singh, Tuli and Sarode [18] have found the efficient cost-time pairs for fuzzy multi index bi-criteria fixed charge transportation problem (FMIBCFCTP) with cost and duration as trapezoidal fuzzy numbers. A comparison of the two methods illustrates that while the proposed method gives minimum duration of (2,4,6,12) in 13 iterations, the same has been obtained by existing method [18] in 17 iterations. Hence the proposed method is a far better alternative in real life problem with all parameters as fuzzy numbers in which time minimization is more important. Tables 14 and 15 show the fuzzy solutions, crisp solutions and number of iterations required to obtain the solutions by both methods.

Table 14: (Solution by existing method [18])

	Fuzzy Cost	Crisp Cost by Ranking	Number of Iterations		Fuzzy Time	Crisp Time by Ranking	Number of Iterations
First Efficient Cost	(476,717, 1193,2346)	1183	7	First Efficient Time	(3,5,8,16)	8	7
Second Efficient Cost	(489,726, 1215,2386)	1204	12	Second Efficient Time	(3,4,7,14)	7	12
Third Efficient Cost	(545,774, 1319,2602)	1310	17	Third Efficient Time	(2,4,6,12)	6	17

Table 15: (Solution by proposed method)

	Fuzzy Time	Crisp Time by Ranking	Number of Iterations		Fuzzy Cost	Crisp Cost by Ranking	Number of Iterations
First Efficient Time	(2,4,6,12)	6	13	First Efficient Cost	(545,774,1319, 2602)	1310	17
Second Efficient Time	(3,4,7,14)	7	18	Second Efficient Cost	(489,726,1215, 2386)	1204	22
Third Efficient Time	(3,5,8,16)	8	23	Third Efficient Cost	(476,717,1193, 2346)	1183	31

6. Concluding remarks

In this paper, FMIBCFCBTP is studied with the parameters of cost, time, supply, demand, capacity and fixed charges being taken for the first time as trapezoidal fuzzy numbers. Use of the fuzzy number makes the problem more realistic than existing crisp problems. If time minimization is more important than cost minimization then the proposed method gives the solution faster than existing methods.

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