Intern. J. Fuzzy Mathematical Archive
Vol. 15, No. 1, 2018, 73-81
ISSN: 2320 –3242 (P), 2320 –3250 (online)
Published on 28 February 2018
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/ijfma.v15n1a7

International Journal of
Fuzzy Mathematical Archive

(λ,μ)-Intuitionistic Fuzzy Submodule of a Module

***KR.Balasubramanian*[1] and *R.Rajangam*[2]**

[1]Department of Mathematics, H.H.The Rajah's College
Pudukkottai, Tamilnadu, India.
Email: [1]balamohitha@gmail.com, [2]rajangamscience@gmail.com

Received 8 February 2018; accepted 24 February 2018

Abstract. In this paper we introduce the concepts of (λ,μ)-intuitionistic fuzzy submodule and (λ,μ)-intuitionistic anti-fuzzy submodules. It can be regarded as generalization of (λ,μ)-intuitionistic fuzzy subring and (λ,μ)-intuitionistic anti-fuzzy subring.

Keywords: intuitionistic fuzzy submodules, (λ,μ)-intuitionistic fuzzy submodule and (λ,μ)-intuitionistic anti-fuzzy submodules

AMS Mathematics Subject Classification (2010): 03F55, 20N25, 08A72, 16D10

1. Introduction

After the introduction of fuzzy sets by Zadeh [17] in 1965, several researchers explored on the generalization of fuzzy sets. The concept of intuitionistic fuzzy subset was introduced by Atanassov [3] as a generalization of fuzzy set. Naegoita and Ralescue [5] introduced the concept of fuzzy submodules of a module over a ring R and Liu [15] introduced the concept of fuzzy ring and fuzzy ideal. The concept of (λ,μ)-intuitionistic fuzzy subring was introduced by Yao [16]. Now, we introduce the concept of (λ,μ)-intuitionistic fuzzy submodule and discuss some algebraic properties.

2. Preliminaries

Definition 2.1. Let X be a non-empty set. A fuzzy subset A of X is a function $A:X\rightarrow[0,1]$.

Definition 2.2. An intuitionisic fuzzy set A of a non-empty set X is an object of the form $A=\{<x,\mu_A(x),\nu_A(x)>\}$,where $\mu_A:X\rightarrow[0,1]$ and $\nu_A:X\rightarrow[0,1]$ are membership and non-membership functions such that for each $x \in X$, we have $0\leq \mu_A(x)+\nu_A(x)\leq 1$.

Remark 2.1. (i) When $\mu_A(x)+\nu_A(x)=1$, i.e. $\nu_A(x)= 1- \mu_A(x)$, then A is called fuzzy set.
(ii) We denote the intuitionistic fuzzy set $A=\{<x,\mu_A(x),\nu_A(x)> : x\in X\}$ by $A=(\mu_A ,\nu_A)$

Definition 2.3. Let M be a Modulus over a ring R. An IFS $A=(\mu_A ,\nu_A)$ of M is called intuitinistic fuzzy (left) submodule (IFSM) if (i) $\mu_A(0)=1,\nu_A(0)=0$
(ii) $\mu_A(x+y) \geq \min \{ \mu_A(x), \mu_A(y)\}$ and $\nu_A(x+y) \leq \max \{ \nu_A(x), \nu_A(y)\}$
(iii) $\mu_A(rx) \geq \mu_A(x)$ and $\nu_A(rx) \leq \nu_A(x)$, for all $x \in X ,r \in R$.

Definition 2.4. Let A and B be any two intuitionistic fuzzy subset of a set X. Then

1. $A \cap B = \{<x, \min\{\mu_A(x), \mu_B(x)\} \wedge \mu, \max\{\nu_A(x), \nu_B(x)\} \vee \lambda>\}$, for all $x \in X$
2. $A \cup B = \{<x, \max\{\mu_A(x), \mu_B(x)\} \vee \lambda, \min\{\nu_A(x), \nu_B(x)\} \wedge \mu>\}$, for all $x \in X$
3. $\Diamond A = \{<x, \mu_A(x) \vee \lambda, (1-\mu_A(x)) \wedge \mu>\}$, for all $x \in X$
4. $\Diamond A = \{<x, (1-\nu_A(x)) \vee \lambda, \nu_A(x) \wedge \mu>\}$, for all $x \in X$

3. Main results

Definition 3.1. Let M be a modulus over R.An IFS $A = (\mu A, \nu A)$ of M is called (λ,μ)-intuitionistic fuzzy submodule $(((\lambda,\mu)$-IFSM) of M if (i) $\mu_A(0) \vee \lambda = \mu$, $\nu_A(0) \wedge \mu = \lambda$
(ii) $\mu_A(x+y) \vee \lambda \geq \min\{\mu_A(x), \mu_A(y)\} \wedge \mu$ and $\nu_A(x+y) \wedge \mu \leq \max\{\nu_A(x), \nu_A(y)\} \vee \lambda$
(iii) $\mu_A(rx) \vee \lambda \geq \mu_A(x) \wedge \mu$ and $\nu_A(rx) \wedge \mu \leq \nu_A(x) \vee \lambda$, for all $x, y \in M, r \in R$, where $0 \leq \lambda+\mu \leq 1$.

Definition 3.2. Let M be a modulus over R. An IFS $A = (\mu_A, \nu_A)$ of M is called (λ,μ)-intuitionistic antifuzzy submodule $(((\lambda,\mu)$-IAFSM) of M if (i) $\mu_A(0) \wedge \mu = \lambda$, $\nu_A(0) \vee \lambda = \mu$
(ii) $\mu_A(x+y) \wedge \mu \leq \max\{\mu_A(x), \mu_A(y)\} \vee \lambda$ and $\nu_A(x+y) \vee \lambda \geq \min\{\nu_A(x), \nu_A(y)\} \wedge \mu$
(iii) $\mu_A(rx) \wedge \mu \leq \mu_A(x) \vee \lambda$ and $\nu_A(rx) \vee \lambda \geq \nu_A(x) \wedge \mu$, for all $x, y \in M$, $r \in R$, where $0 \leq \lambda+\mu \leq 1$.

Theorem 3.1. Let $A=(\mu_A,\nu_A)$ be a IFS of an R-module M. Then A is (λ,μ)-IFSM of M iff A satisfies the following conditions (i) $\mu_A(0_M) \vee \lambda = \mu$, $\nu_A(0_M) \wedge \mu = \lambda$
(ii) $\mu_A(rx+sy) \vee \lambda \geq \min\{\mu_A(x), \mu_A(y)\} \wedge \mu$ and $\nu_A(rx+sy) \wedge \mu \leq \max\{\nu_A(x), \nu_A(y)\} \vee \lambda$ for all $x, y \in M, r \in R$.

Proof: Firstly let A be (λ,μ)-IFSM of M, for all $x,y \in M$, $r,s \in R$ and $\lambda \leq \mu$. Then

$$\mu_A(rx+sy) \vee \lambda = \{\mu_A(rx+sy) \vee \lambda\} \vee \lambda \geq \{\min\{\mu_A(rx), \mu_A(ry)\} \wedge \mu\} \vee \lambda$$
$$= \{\mu_A(rx) \vee \lambda\} \wedge \{\mu_A(sy) \vee \lambda\} \wedge \{\mu \vee \lambda\}$$
$$\geq \min\{\mu_A(x), \mu_A(y)\} \wedge \mu \qquad \text{(i)}$$
$$\nu_A(rx+sy) \wedge \mu = \{\nu_A(rx+sy) \wedge \mu\} \wedge \mu \leq \{\max\{\nu_A(rx), \nu_A(ry)\} \wedge \mu\} \wedge \mu$$
$$= \{\nu_A(rx) \wedge \mu\} \vee \{\nu_A(sy) \wedge \mu\} \vee \{\mu \vee \lambda\}$$
$$\leq \max\{\mu_A(x), \mu_A(y)\} \vee \lambda \qquad \text{(ii)}$$

Therefore condition (i) and (ii) holds. Then conversely, if (i) and (ii) holds, then putting $r=s=1_R$ in (ii) we get $\mu_A(rx+sy) \vee \lambda = \{\mu_A(1_R x+1_R y) \vee \lambda\} \vee \lambda \geq \min\{\mu_A(x), \mu_A(y)\} \wedge \mu$, $\nu_A(rx+sy) \wedge \mu = \{\nu_A(1_R x+1_R y) \wedge \mu\} \wedge \mu \leq \max\{\nu_A(x), \nu_A(y)\} \vee \lambda$. Hence, A is (λ,μ)-intuitionistic fuzzy submodule of M.

Remark 3.1. If an intuitionistic fuzzy set of a ring R is said to be (λ,μ)-intuitionistic fuzzy subring, if the following should hold.
(i) $\mu_A(0) \geq \mu_A(x)$ and $\nu_A(0) \leq \nu_A(y)$, for all x in R.
(ii) For all non-zero x in R, $\max\{\mu_A(x)\} \leq \lambda$ or $\min\{\mu_A(x)\} \geq \mu$ and $\min\{\nu_A(x)\} \geq \mu$ or $\max\{\nu_A(x)\} \leq \lambda$.

Theorem 3.2. Let A and B be two (λ,μ)-IFSMs of R-Module M, then $(A \cap B)_M$ is also (λ,μ)-IFSM of M.

Proof: Now $\mu_{(A\cap B)_M}(0) \vee \lambda = \{\mu_{A_M}(0) \wedge \mu_{A_M}(0)\} \vee \lambda = \mu \wedge \mu = \mu$ and
$\nu_{(A\cap B)_M}(0) \wedge \mu = \{\nu_{A_M}(0) \wedge \nu_{A_M}(0)\} \wedge \mu = \lambda \vee \lambda = \lambda$.
Let $x,y \in M$ and $r,s \in R$, we have,

$$\mu_{(A\cap B)_M}(rx+sy) \vee \lambda = \{\mu_{A_M}(rx+sy) \wedge \mu_{B_M}(rx+sy)\} \vee \lambda$$
$$\geq \{\mu_A(x) \wedge \mu_A(y) \wedge \mu\} \wedge \{\mu_B(x) \wedge \mu_B(y) \wedge \mu\}$$
$$= \mu_{(A\cap B)_M}(x) \wedge \mu_{(A\cap B)_M}(y) \wedge \mu \qquad \text{(i)}$$

Similarly, $\nu_{(A\cap B)_M}$(rx + sy) ∧ μ ={ ν_{A_M}(rx + sy) v ν_{B_M} (rx + sy)} ∧ μ

≤ {ν_A (x) v ν_A (y) v λ } v { ν_B(x) v ν_B(y) vλ}

= $\nu_{(A\cap B)_M}$(x) v $\nu_{(A\cap B)_M}$(y) v λ (ii)

Also, $\mu_{(A\cap B)_M}$(rx) v λ ={ μ_{A_M}(rx) ∧ μ_{B_M} (rx)} v λ

≥ {μ_A (x) ∧μ } ∧ { μ_B(x) ∧μ } = $\mu_{(A\cap B)_M}$(x) ∧ μ (iii)

Similarly, $\nu_{(A\cap B)_M}$(rx)∧μ={ν_{A_M}(rx)vν_{B_M}(rx)}∧μ

≤{ν_A (x) v λ } v { ν_B(x) vλ} = $\nu_{(A\cap B)_M}$(x) v λ (iv)

From (i), (ii), (iii) and (iv), $(A\cap B)_M$ is a (λ,μ)-IFSM of M.

Theorem 3.3. The interjection of a family of (λ,μ)-IFSMs of R-module M is a (λ,μ)-IFSM of M

Proof: Let {V_i} be a family of (λ,μ)-IFSMs of M. And let A=$\bigcap_{i=1}^{n} V_i$. Let x,y in M. Then

μ_A(x +y) v λ =$\inf_{i\in I} \mu_{V_i}$(X + Y) vλ ≥ $\inf_{i\in I}$min {μ_{V_i}(X), μ_{V_i}(y) } ∧ μ

= min {$\inf_{i\in I}\mu_{V_i}$(X), $\inf_{i\in I}\mu_{V_i}$ (y)}∧ μ = min {μ_A (x), μ_A (y)}∧μ, for all x, y in M.

Now, ν_A(x +y) ∧μ =$\sup_{i\in I} \mu_{V_i}$ (X + Y) ∧μ ≤ $\sup_{i\in I}$max {μ_{V_i}(X), μ_{V_i}(y) } vλ

= max {$\sup_{i\in I} \mu_{V_i}$(X), $\sup_{i\in I} \mu_{V_i}$(y)}vλ= max {μ_A (x),μ_A (y)} vλ, for all x,y in M.

Also

μ_A(rx) v λ =$\inf_{i\in I} \mu_{V_i}$(rX) vλ ≥ $\inf_{i\in I}$ {μ_{V_i}(X)} ∧ μ = μ_A (x) ∧μ ,for all x in M and r in R.

Now, ν_A(rx) ∧μ =$\sup_{i\in I} \mu_{V_i}$ (rX) ∧μ ≤ $\sup_{i\in I}$ {μ_{V_i}(X)} vλ = $\sup_{i\in I}$ {μ_{V_i}(X)} vλ= μ_A (x) vλ ,for all x,y in M and r in R. That is, A is an (λ ,μ)-IFSM if M. Hence, the interjection of a family of (λ,μ)-IFSMs of R-module M is a (λ,μ)-IFSM of M.

Theorem 3.4. Let A = (μ_A, ν_A) be an IFS of an R-Module M. Then A is an (λ ,μ)-IFSM of M iff A^C is (λ,μ)-IAFSM of M.

Proof: Let A be (λ ,μ)-IFSM of M. To show that A^C=(ν_A , μ_A) is (λ ,μ)-IAFSM of M. Since μ_A (0) vλ =μ ,ν_A(0) ∧μ =λ, which implies that μ_A^c(0) ∧ μ = λ , ν_A^c (0) v λ = μ. Further μ_A(x +y) v λ ≥ min { μ_A(x), μ_A(y) } ∧μ implies that ν_A^c (x + y) vλ ≥ min { ν_A^c (x), ν_A^c (y) } ∧ μ. We know that

min{ ν_A^c (x), ν_A^c (y) }∧μ ≤ ν_A^c (x +y) vλ that is min {1-μ_A^c(x), 1-μ_A^c(y) } ∧ {1-λ } ≤ { 1-μ_A^c(x + y) } v {1-μ}

min {1-μ_A^c(x), 1-μ_A^c(y) ,(1-λ)} ≤ { 1- min{ μ_A^c (x + y),μ } }

1-max {μ_A^c(x) , μ_A^c(y), λ } ≤ 1-min { μ_A^c(x + y) ,μ }. Take complement on both sides, we get

max {μ_A^c(x), μ_A^c(y), λ } ≥ min { μ_A^c(x + y), μ } (i)

Similarly,

ν_A(x +y) ∧μ ≤ max { ν_A(x), ν_A(y) } vλ implies that μ_A^c (x + y) ∧μ ≤ max { μ_A^c (x), μ_A^c (y)} vλ.

We Know that max { μ_A^c (x), μ_A^c (y) } vλ ≤ μ_A^c (x +y) ∧μ that is

max {1-ν_A^c(x), 1-ν_A^c(y) } v {1-μ } ≥ min { 1- ν_A^c(x + y) ,1-λ}.

1-*min* {ν_A^c(x) , ν_A^c(y), μ } ≥ 1-max { ν_A^c(x + y) ,λ }. Take complement on both sides, we get *min* {ν_A^c(x) , ν_A^c(y), μ } ≤ { ν_A^c(x + y) vλ (ii)

Also we have μ_A(rx) v λ ≥ μ_A(x) ∧ μ that is ν_A^c (rx) v λ ≥ ν_A^c (x) ∧μ.

Also $\{1-\mu_A^c$ (rx)} v {1-μ} ≥ $\{1-\mu_A^c$ (x)}∧{ 1-λ} and 1-min { μ_A^c(rx),μ} ≥ 1- {$\mu_A^c(x)$ v λ}. Taking complement on both sides, μ_A^c (rx) ∧ μ ≤ μ_A^c (x) vλ.
Similarly, if ν_A(rx) ∧ μ ≤ ν_A(x) v λ that is μ_A^c (rx) ∧ μ ≤ μ_A^c (x) vλ.
Also $\{1-\nu_A^c$ (rx)} ∧ {1 -λ} ≤ $\{1-\nu_A^c$ (x) }v { 1-μ}, 1-max { ν_A^c (rx), λ} ≥ 1- { $\nu_A^c(x)$ ∧ μ}. Taking complement on both sides, ν_A^c (rx) v λ ≥ ν_A^c (x) ∧ μ. Hence, if A is an (λ ,μ)-IFSM of M iff A^C is (λ,μ)-IAFSM of M.

Theorem 3.5. Union of two (λ ,μ)-IFSMs of M is also a (λ ,μ)-IFSM of M
Proof : Let A and B be two (λ ,μ)-IFSMs of M, which implies that A^C and B^C are (λ ,μ)-IAFSMs of M.(by previous theorem). Therefore $A^c \cap B^c$ is a (λ,μ)-IAFSM of M, which implies that $(A \cup B)^c$ is (λ ,μ)-IAFSM of M. Therefore A ∪B is (λ ,μ)-IFSM of M.

Theorem 3.6. The union of family of (λ,μ)-IFSMs of R-module M is a (λ,μ)-IFSM of M
Proof: Let {V_i} be a family of (λ,μ)-IFSMs of M. And let A = $\bigcup_{i=i}^{n} V_i$. Let x and y in M. Then μ_A (0) v λ ={ $\mu_{\cup A_i}(0)$ } v λ = { $\mu_{A_1}(0)$ v $\mu_{A_2}(0)$ v $\mu_{A_3}(0)$ v…… $\mu_n(0)$ } v λ = μ v μ v μ v……v μ = μ ,ν_A (0) ∧ μ = { $\nu_{\cup A_i}(0)$ } ∧μ = { $\nu_{A_1}(0)$ ∧ $\nu_{A_2}(0)$ ∧ $\nu_{A_3}(0)$ ∧…… ∧$\nu_n(0)$ } ∧μ =λ∧ λ ∧ λ∧….∧ λ = λ and μ_A (x+y) v λ = $\sup_{i\in I} \mu_{V_i}$(x+y) v λ ≥ $\sup_{i\in I} max$ { $\mu_{V_i}(x), \mu_{V_i}(y)$ }∧ μ = max { $\sup_{i\in I}\mu_{V_i}(x)$, $\sup_{i\in I}\mu_{V_i}(y)$} ∧ μ = max { μ_A (x), μ_A (y)} ∧μ, for all x,y in M.
Now, ν_A (x+y) ∧ μ = $\inf_{i\in I} \nu_{V_i}$(x+y) ∧ μ ≤ $\inf_{i\in I} min$ {$\nu_{V_i}(x), \nu_{V_i}(y)$ }v λ= min { $\inf_{i\in I}\nu_{V_i}(x)$, $\inf_{i\in I}\nu_{V_i}(y)$} v λ = min { ν_A (x), ν_A (y)} v λ ,for all x,y in M. Also μ_A (rx) v λ = $\sup_{i\in I} \mu_{V_i}$(rx) v λ ≥ $\sup_{i\in I}${ $\mu_{V_i}(x)$ }∧ μ = μ_A (x) ∧μ, for all x in M and r in R. Now, ν_A (rx) ∧ μ = $\inf_{i\in I} \nu_{V_i}$(rx) ∧ μ ≤ $\inf_{i\in I}$ {$\nu_{V_i}(x)$} v λ = ν_A (x) v λ, for all x in M and r in R. That is, A is an (λ,μ)-IFSM of M. Hence, the union of family of (λ,μ)-IFSMs of M is a (λ,μ)-IFSM of M.

Theorem 3.7. If A is an (λ, μ)-IFSM of M, then ◊A is an (λ, μ)-IFSM of M.
Proof: Suppose if A is an (λ, μ)-IFSM of M, then μ_A (0) vλ =μ v λ,{ μ_A (0) } ∧ μ = { 1-μ } ∧ μ =λ for all x,y in M. And then μ_A (x+y) vλ ≥ min { μ_A (x), μ_A (y) } ∧ μ. Similarly $\{1-\mu_A$ (x+y)} ∧ μ ≥ min { μ_A (x), μ_A (y) } ∧ μ =$\{1-\mu_A$ (x+y)} ∧ {1- λ} = 1- { μ_A (x+y) vλ } ≤ 1-{ min { μ_A (x), μ_A (y) } ∧ μ = {1- μ_A (x) }v {1- μ_A (y) } v {1- μ} = {1- μ_A (x) }v {1- μ_A (y) } v λ also we know that μ_A (rx) vλ ≥ μ_A (x) ∧ μ. Similarly $\{1-\mu_A$ (rx)} ∧ μ ={1 -$\mu_A(rx)$} ∧ $\{1-\lambda\} = 1-\{\mu_A(rx) \vee \lambda\}$ ≤$1-\{\mu_A$ (x) ∧ μ } = 1- {μ_A(x)}v {1-μ } = 1-{ μ_A (x) }v λ. Hence ◊A is also an (λ, μ)-IFSM of M.

Definition 3.3. Strongest (λ,μ)-intuitionistic fuzzy relation.
Let A be an intuitionistic fuzzy set of M. Then the strongest fuzzy relation on M is V given by
μ_V (x,y) = min { $\mu_A(x), \mu_A$(y) } v λ = min { $\mu_A(x), \mu_A$(y) } ∧ μ,
ν_V (x,y) = max { $\nu_A(x), \nu_A$(y) } ∧ μ = max { $\nu_A(x), \nu_A$(y) } v λ for all x,y in M.
and μ_A(0) v λ = { $\sup_{i\in I} \mu_{A_i}$(0)} v λ, ν_A (0) ∧ μ ={ $\inf_{i\in I} \nu_{A_i}$(0) }∧ μ.

Theorem 3.8. Let A be an intuitionistic fuzzy subset of R-Module M and V be the strongest intuitionistic fuzzy relation on M. Then A is called (λ,μ)-IFSM of M if and only if V is an (λ,μ)-IFSM of M×M.

Proof: Suppose that A is (λ,μ)-IFSM of M. Then for any X=(x_1, x_2) and Y =(y_1, y_2) are in M×M, we have μ_V (x + y) v λ = μ_V [(x_1, x_2) + (y_1,y_2)]v λ = μ_V [(x_1+y_1 ,x_2 + y_2)]v λ = min { μ_A(x_1 +y_1),μ_A (x_2,y_2)} v λ ≥ min {min {μ_A (x_1) ∧ μ_A (y_1) } ∧ μ}, min {μ_A (x_2) ∧ μ_A (y_2) } ∧ μ} = min {min {μ_A (x_1), μ_A (x_2) }, min {μ_A (y_1), μ_A (y_2) }} ∧ μ =min{ μ_V (x), μ_V (y) } ∧ μ. Therefore, μ_V (x + y) v λ ≥ min{ μ_V (x), μ_V (y) } ∧ μ, for all x,y in M×M and ν_V (x + y) ∧ μ = ν_V [(x_1, x_2) + (y_1,y_2)] ∧ μ= ν_V [(x_1+y_1 ,x_2 + y_2)]∧ μ = max { ν_A(x_1 +y_1),ν_A (x_2,y_2)} ∧ μ ≤ max {max {ν_A (x_1) v ν_A (y_1) } v λ}, max { ν_A (x_2) v ν_A (y_2) } v λ } = max {max {ν_A (x_1) , ν_A (x_2) }, max {ν_A (y_1) , ν_A (y_2) }} v λ =max { ν_V (x), ν_V (y) } v λ. Therefore, ν_V (x + y) ∧ μ ≤ max{ ν_V (x), ν_V (y) } v λ, for all x,y in M×M.

Let r=(r_1,r_2) ∈ R×R and x=(x_1 , x_2) ∈ M×M. We have μ_V (rx) v λ = μ_V [(r_1, r_2). (x_1,x_2)]v λ = μ_V [(r_1.x_1 ,r_2.x_2)]v λ = min { μ_A(r_1.x_1),μ_A (r_2.x_2)} v λ ≥ min {μ_A (x_1) , μ_A (x_2) } ∧ μ} = { μ_V (x_1,x_2) } ∧ μ = μ_V (x) ∧ μ. Therefore, μ_V (rx) v λ ≥ { μ_V (x)} ∧ μ, for all x in M×M and r in R×R. Similarly, ν_V (rx) ∧ μ = ν_V [(r_1, r_2). (x_1,x_2)]∧ μ= ν_V [(r_1.x_1 ,r_2.x_2)]∧ μ= max { ν_A(r_1.x_1),ν_A (r_2.x_2)} ∧ μ ≤ max {ν_A (x_1), ν_A (x_2) } v λ } = { ν_V (x_1,x_2) } v λ = ν_V (x) vλ. Therefore, ν_V (rx) ∧μ ≤ { ν_V (x)} v λ, for all x in M×M and r in R×R. Therefore this proves that V is an (λ,μ)-IFSM of M×M. Conversely assume that V is an (λ ,μ)-IFSM of M×M, then for any x=(x_1 , x_2) and Y =(y_1 , y_2) are in M×M, we have min{ μ_A(x_1+y_1), μ_A(x_2+y_2)}v λ = μ_v(x_1+y_1,x_2+y_2) v λ = μ_v[(x_1,x_2)+(y_1,y_2)] v λ = μ_V(x + y) v λ ≥ min { μ_v(x), μ_v(y) } ∧ μ =min { μ_V (x_1,x_2), μ_V (y_1,y_2)} ∧ μ = min{min{ μ_A (x_1) , μ_A (x_2)},min{ μ_A (y_1) , μ_A (y_2)}}∧ μ.

If μ_A(x_1 +y_1) ≤ μ_A(x_2+y_2) and μ_A(x_1) ≤ μ_A(x_2), μ_A(y_1) ≤ μ_A(y_2), we get μ_A(x_1+y_1) v λ ≥ min { μ_A(x_1), μ_A(y_1)}∧μ for all x and y ∈ M×M. And max {ν_A(x_1+y_1), ν_A(x_2+y_2)}∧ μ = ν_v(x_1+y_1,x_2+y_2) ∧ μ = ν_v[(x_1,x_2)+(y_1,y_2)] ∧ μ = ν_V(x + y) ∧ μ ≤ max { ν_v(x), ν_v(y) } v λ =max { ν_V (x_1,x_2), ν_V (y_1,y_2)} v λ = max{max{ ν_A (x_1), ν_A (x_2)},max{ ν_A (y_1), ν_A (y_2)}} vλ. If ν_A(x_1 +y_1) ≥ ν_A(x_2+y_2) and ν_A(x_1) ≥ ν_A(x_2), ν_A(y_1) ≥ ν_A(y_2),We get ν_A(x_1+y_1) ∧ μ ≤ max { ν_A(x_1), ν_A(y_1)} ϵ v λ for all x and y ∈ M×M.

Also, let x=(x_1 , x_2) ∈ M×M and r=(r_1,r_2)∈ $R \times R$. To prove that,$\mu_V(xr)$vλ ≥ μ_V(x)∧μ and ν_V(xr) ∧μ ≤ ν_V(x) v λ. Take μ_V(xr) vλ = μ_V(x_1,x_2).(r_1,r_2)vλ = μ_V(x_1.r_1, x_2.r_2)vλ = min{ μ_A(x_1.r_1), μ_A(x_2.r_2)}v λ ≥ min { μ_A(x_1)∧μ, μ_A(x_2)∧μ } = μ_V(x_1,x_2) ∧μ there fore μ_V(xr) v λ ≥ μ_V(x) ∧ μ Similarly, ν_V(xr) ∧ μ = ν_V(x_1,x_2).(r_1,r_2) ∧ μ = ν_V(x_1.r_1, x_2.r_2) ∧ μ = max{ ν_A(x_1.r_1), ν_A(x_2.r_2)} ∧ λ ≤ max { ν_A(x_1)v λ , ν_A(x_2)v λ } = ν_V(x_1,x_2) v λ there fore ν_V(xr) ∧ μ ≤ ν_V(x) v λ. Hence the theorem.

Theorem 3.9. Let A be intuitinistic fuzzy subset of R. Then A is called (λ ,μ)-inuitionistic fuzzy submodule of an R-module M, if all non-empty A_α is a (λ,μ)-intuitionistic fuzzy submodule of M, for all αϵ (λ,μ].

Proof : Let μ_A be (λ,μ)-inuitionistic fuzzy submodule of an R-module M. μ_A(0) v λ = μ ≥ α.Let α ϵ(λ,μ] and x,y ϵ A_α. Then μ_A(x) ≥ α and μ_A(y) ≥ α thus μ_A(x +y) v λ ≥ min{μ_A(x), μ_A(y) } ∧μ ≥ α ∧ α∧ μ= α and so x+ y ϵ A_α. Now μ_A(rx) v λ ≥ μ_A(x) ∧μ ≥ α ∧ μ= α which implies that rx ϵ A_α. And ν_A(0) = λ ≤ α. Let α ϵ(λ,μ] and x,y ϵ A_α. Then ν_A(x) ≤ α and ν_A(y) ≤ α thus ν_A(x +y) ∧ μ ≤ max{ ν_A(x), ν_A(y) } v λ ≤ α v α v λ = α and so x+ y ϵ A_α. Now ν_A(rx) ∧ μ ≤ ν_A(x) v λ ≤ α v λ = α which implies that rx ϵ A_α. Therefore A_α is a (λ,μ)-inuitionistic fuzzy submodule of M.

Conversely, let A_α be (λ,μ)-inuitionistic fuzzy submodule of M for all α ϵ(λ,μ]. If there exist x,y ∈ M such that, μ_A(x +y) v λ < min{μ_A(x), μ_A(y) } ∧μ, then μ_A(x +y) < α. Hence x+ y ∉ A_α, which contradicts the fact that A_α is a (λ,μ)-inuitionistic fuzzy submodule of M. Hence μ_A(x +y) v λ ≥ min{μ_A(x), μ_A(y) } ∧μ for all x,y in M. Infact, suppose μ_A(rx) vλ ≤ μ_A(x) ∧μ = α then μ_A(rx) < α.(since μ >α).Thus rx ∉ A_α for all x ϵ M and rϵ R. This is contradicts. So μ_A(rx) v λ ≥ μ_A(x) ∧ μ. And if there exist x and y in M such that, ν_A(x +y) ∧ μ > max{ν_A(x), ν_A(y) } v λ, then ν_A(x +y) > α. Hence x+ y ∉ A_α, which contradicts the fact that A_α is a (λ,μ)-inuitionistic fuzzy submodule of M. Hence ν_A(x +y) ∧ μ ≤ max{ν_A(x), ν_A(y) } v λ for all x,y in M. Suppose ν_A(rx) ∧ μ ≥ ν_A(x) v λ = α then ν_A(rx) > α.(since λ < α). Thus rx ∉ A_α for all x ϵ M and rϵ R. This is contradicts. So ν_A(rx) ∧ μ ≤ ν_A(x) v λ. Therefore A is a (λ,μ)- inuitionistic fuzzy submodule of M.

Definition 3.4. Let A and B be two (λ,μ)-inuitionistic fuzzy submodules of an R-module M. Then there sum is defined as ($\mu_A + \mu_B$) (x) = $\sup_{x=a+b}$ {min { $\mu_A(a), \mu_B$ (b) }},for all x in M and

($\nu_A + \nu_B$) (x) = $\inf_{x=a+b}$ {max { $\nu(a), \nu_B$ (b) }},for all x in M

Theorem 3.10. Let A and B be two (λ,μ)-inuitionistic fuzzy submodules of an R-module M. Then the sum A+B is also an (λ,μ)-inuitionistic fuzzy submodules of M.

Proof : Clearly, ($\mu_A + \mu_B$) (0) = $\sup_{0=a+(-a)}$ {min { $\mu_A(a), \mu_B$ (-a)}}

= $\sup_{0=a+(-a)}$ {min { $\mu_A(a), \mu_B$ (a) }} = μ, (as min{μ_A(0),μ_B(0)}. Let x,y in M and r,s in R. Then ($\mu_A + \mu_B$) (rx +sy) v λ = $\sup_{rx+sy=a+b}$ {min { $\mu_A(a), \mu_B$ (b) }} v λ = {min{ μ_A(rx), μ_B(sy) }} v λ ≥ min{ μ_A(x), μ_B(y) } ∧μ. As both μ_A and μ_B are intuitionistc fuzzy submodule of M, therefore we have μ_A(rx +sy) v λ = $\sup_{rx+sy=a+b}$ {min { $\mu_A(a), \mu_A$ (b) }} v λ = {min{ μ_A(rx), μ_A(sy) }} v λ ≥ min{ μ_A(x), μ_A(y) } ∧μ and μ_B(rx +sy) v λ = $\sup_{rx+sy=a+b}$ {min { $\mu_B(a), \mu_B$ (b) }} v λ = {min{ μ_B(rx), μ_B(sy) }} v λ ≥ min{ μ_B(x), μ_B(y) } ∧μ. Adding these, we get ($\mu_A + \mu_B$) (rx +sy) v λ = { min { μ_A(x), μ_A(y) } ∧μ} + { min { μ_B(x), μ_B(y) } ∧μ} ≥ {min { μ_A(x) + μ_B(x), μ_A(y) + μ_B(y)}}∧ μ ≥ {min{($\mu_A + \mu_B$)(x), ($\mu_A + \mu_B$)(y) } ∧ μ}.Again by putting y = 0,We get ($\mu_A + \mu_B$) (rx) v λ ≥ { μ_A (x),μ_A(y) } ∧μ similarly we can get,

($\nu_A + \nu_B$) (0) = $\inf_{0=a+(-a)}$ {max { $\nu_A(a), \nu_B$ (-a) }} = $\inf_{0=a+(-a)}$ {max { $\nu_A(a), \nu_B$ (a) }} = λ, (as max{ν_A(0), ν_B (0)}.Let x,y in M and r,s in R.Then ($\nu_A + \nu_B$) (rx +sy) ∧μ = $\inf_{rx+sy=a+b}$ {max { $\nu_A(a), \nu_B$ (b) }} ∧ μ = {max{ ν_A(rx), ν_B(sy) }} v λ ≤ max{ ν_A(x), ν_B(y) } v λ. As both ν_A and ν_B are intuitionistc fuzzy submodule of M, therefore we have ν_A(rx +sy) ∧ μ = $\inf_{rx+sy=a+b}$ {max { $\nu_A(a), \nu_A$ (b) }} ∧ μ

= {max{ ν_A(rx), ν_A(sy) }} ∧ μ ≤ max{ ν_A(x), ν_A(y) } v λ and ν_B(rx +sy) ∧ μ = $\inf_{rx+sy=a+b}$ {max { $\nu_B(a), \nu_B$ (b) }} ∧ μ = {max{ ν_B(rx), ν_B(sy) }} ∧ μ ≤ max{ ν_B(x), ν_B(y) } v λ. Adding these, we get ($\nu_A + \nu_B$) (rx +sy) ∧ μ = { max { ν_A(x), ν_A(y) } v λ} + { max { ν_B(x), ν_B(y) } v λ} ≤ {max { ν_A(x) + ν_B(x), ν_A(y) + ν_B(y)}} v λ ≤ {max{($\nu_A + \nu_B$)(x), ($\nu_A + \nu_B$)(y) } v λ}.Again by putting y = 0, we get ($\nu_A + \nu_B$) (rx) ∧ μ ≤ { ν_A (x), ν_A(y) } v λ, from this we conclude that,vA + B is an (λ,μ)-inuitionistic fuzzy submodules of M.

Theorem 3.11. Let M be an R-module and A be a non-empty subset of an R-module M. Then A is a submodule of M if and only if B =<χ_A,$\bar{\chi}_A$> is an (λ,μ)-IFSM of , where χ_A is the (λ,μ)- characteristic function of A defined by $\chi_A(x)=\begin{cases}\mu, if\ x \in A\\ \lambda, if\ x \notin A\end{cases}$ and $\overline{\chi_A}(x) = 1-\chi_A(x)$ and $0 \le \lambda+\mu \le 1$.

Proof: Let M be a R –Module and A be a non –empty subset of M. First let A be a sub module of M. We have 0 in A. Therefore $\chi_A(0) \vee \lambda = \mu \vee \lambda = \mu$ and $\overline{\chi_A}(0) \wedge \mu = \lambda \wedge \mu = \lambda$. Take x and y in M and r in R.

Case (i) : If x and y in A then x + y, rx in A. Since A is a submodule of M, $\chi_A(0) = \chi_A(x) = \chi_A(y) = \chi_A(x+y) = \chi_A(rx) = \mu$ and $\overline{\chi_A}(0) = \overline{\chi_A}(x) = \overline{\chi_A}(y) = \overline{\chi_A}(x+y) = (\overline{\chi_A}(rx) = \lambda$ so $\chi_A(x+y) \vee \lambda \ge \min\{\chi_A(x), \chi_A(y)\} \wedge \mu$ for all x,y in M and $\chi_A(rx) \vee \lambda \ge \chi_A(x) \wedge \mu$ for all x in M and r in R. Also $\overline{\chi_A}(x+y) \wedge \mu \le \max\{\overline{\chi_A}(x), \overline{\chi_A}(y)\} \vee \lambda$ and $\overline{\chi_A}(rx) \wedge \mu \le \overline{\chi_A}(x) \vee \lambda$, for all x in M, r in R.

Case (ii): If x in A, y or r not in A (or x not in A, y or r in A) then x+y, r in R may or may not be in A is non-empty subset of M and r in R, so $\chi_A(x)= \chi_A(y)=\mu$, $\overline{\chi_A}(x)= \overline{\chi_A}(y)=\lambda$. Since B=< χ_A , $\overline{\chi_A}$> is an (λ,μ)-IFSM of M, We have $\chi_A(x+y) \vee \lambda \ge \min\{\chi_A(x), \chi_A(y)\} \wedge \mu = \min\{\mu,\mu\} \wedge \mu = \mu$ for x,y in M and $\chi_A(rx) \vee \lambda \ge \chi_A(x) \wedge \mu = \mu \wedge \mu = \mu$ for all x in M and r in R. Therefore $\chi_A(x+y) = \chi_A(rx)=\mu$. And, $\overline{\chi_A}(x+y) \wedge \mu \le \max\{\overline{\chi_A}(x), \overline{\chi_A}(y)\} \vee \lambda = \max\{\lambda,\lambda\}=\lambda$, and $\overline{\chi_A}(rx) \wedge \mu \le \overline{\chi_A}(x) \vee \lambda = \lambda \vee \lambda = \lambda$ for x in M and r in R. Therefore $\overline{\chi_A}(x+y) = \overline{\chi_A}(rx)=\lambda$. Hence x+y and rx in A,so Ais a submodule of M.

Definition 3.5. Let M and N be two R-Modules. Let f:M→N be any function. Let A be a (λ,μ)-intuitionistic fuzzy submodule of M, and V be an (λ,μ)-intuitionistic fuzzy submodule of of N is defined by

$\mu_V(x) \vee \lambda = \begin{cases}\sup(\mu_A(x) \vee \lambda, for\ x \epsilon f^{-1}(y)\\ 0 \quad , otherwise\end{cases}$ and

$\nu_V(x) \wedge \mu = \begin{cases}\inf(\nu_A(x)\ \mu\ \mu, for\ x \epsilon f^{-1}(y)\\ 0 \quad , otherwise\end{cases}$ and, for x in M and y in N, then A is called pre image of V under F and is denoted by $f^{-1}(V)$.

Definition 3.6. Let M and N be any two R-modules. Let f: M→N be any function. Let A be a (λ,μ)-intuitionistic fuzzy submodule of M, then the anti-image of A under f is the (λ,μ)-intuitionistic fuzzy subset

f_(A)= $(\mu_V(y) \vee \lambda, \nu_V(y) \wedge \mu)$,where

$f\_(x)=\begin{cases}(y \in N, \inf(\mu_A(x) \vee \lambda), \sup(\nu_A(x) \wedge \mu))\ ; for\ x \in f^{-1}(y)\\ 0 \quad ; otherwise\end{cases}$

Theorem 3.12. Let M and N be any two R-modules. Let f:M→N be a mapping from M to N. Then

(i) If A is a (λ,μ)-IFSM, then $f(A^c) = (f\_(A))^c$

(ii) If B is a (λ,μ)-IFSm, then $f^{-1}(B^C) = (f^{-1}(B))^C$

Proof: Let A be a (λ,μ)-IFSM, then for each y in N, we have,

$\{f(A^c)(y) =$

$\{\sup_{x\epsilon f^{-1}(y)}(\mu_{A^c}(x) \wedge \mu), \inf_{x\epsilon f^{-1}(y)}(\nu_{A^c}(x) \vee \lambda)\} = \{\sup_{x\epsilon f^{-1}(y)}(\nu_A(x) \vee \lambda, \inf_{x\epsilon f^{-1}(y)}(\mu_A(x) \wedge \mu\}$

$=(f\_(A^c))^c(y)$. Thus $f(A^c)= (f\_(A))^c$.

Let B be a (λ,μ)-IFSM, then for each x in M, we have,

$f^{-1}(B^c)(x) = B^c(f(x)) = \{\nu_B(f(x))\wedge\mu , \mu_B(f(x))\vee\lambda\} =$
$\{\nu_{f^{-1}(B)}(x)\wedge\mu , (\mu_{f^{-1}(B)}(x)\vee\lambda\} = (f^{-1}(B))^c$. Thus, $f^{-1}(B^c) = (f^{-1}(B))^c$.

Theorem 3.13. Let M and N be any two R-modules. Let f:M→N be a mapping from M to N. Then
(i) If B is a (λ,μ)-IAFSM of N, then $f^{-1}(B)$ is (λ,μ) –IAFSM of M.
(ii) If A is a (λ,μ)-IAFSM of M, then $f\_(A)$ is (λ,μ) –IAFSM of N.
Proof: Let B be a (λ,μ)-IAFSM of N, then B^c is (λ,μ)-IFSM of N and so $f^{-1}(B^c)$ is (λ,μ)-IFSM of M. That is $(f^{-1}(B))^c$ is (λ,μ)-IFSM of M.Hence $f^{-1}(B)$ is (λ,μ) –IAFSM of M.
Let A be a (λ,μ) –IAFSM of M,then A^c is (λ,μ) –IFSM of M and $f^{-1}(A^c)$ is (λ,μ) –IFSM of N. Since $f(A^c) = (f\_(A))^c$ implies that $(f\_(A))^c$ *is* (λ,μ) –IFSM of N. Hence $f\_(A)$ is (λ,μ)-IAFSM of N.

REFERENCES

1. B.Anitha and D.Sivakumar, On (λ,μ)-anti-fuzzy subrings, *International Journal of Computer Applications*, 75(8) (2013) 45-47.
2. B.Anitha and D.Sivakumar, t-norm (λ,μ)-fuzzy subrings and t-norm (λ,μ)-fuzzy ideals, *International Journal of Algebra*, 5(22) (2011) 1065-1072.
3. K.T.Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 20(1) (1986) 87-96.
4. S.K.Bhakat and P.Das, $(\in, \in \vee q)$-fuzzy group, *Fuzzy Sets and Systems*, 80 (1996) 359-368.
5. C.V.Naegoita and D.A.Ralescue, *Application of fuzzy sets and system analysis,* BirKhauser, Basel, 1975.
6. V.Saravanan and D.Sivakumar, A study on intuitionistic fuzzy subsemiring of a semiring, *Elixir Appl. Math.*, 46 (2012) 8552-8556.
7. Saravanan, B.Anitha and D.Sivakumar, Homomorphism and anti-homomorphism of intuitionistic (λ,μ)-fuzzy sub ring of a ring, to appear in *American Journal of Mathematical Science and Applications.*
8. P.K.Sharma, Anti fuzzy submodule of a module, to appear in *Advances in Fuzzy Sets and fuzzy Systems.*
9. P.K.Sharma, (α, β)-cut of intuitionistic fuzzy modules, *International Journal of Mathematical Science and Applications*, 1(3) (2011) 1447-1454.
10. P.K.Sharma, Intuitionistic fuzzy module over intuitionostic fuzzy ring, *Internatinal Journal of Fuzzy Mathematics and Systems*, 2(2) (2012) 131-140.
11. P.K.Sharma, On intuitionistic anti-fuzzy submodule of a module, *International Journal of Fuzzy Mathematics and Systems*, 2(1) (2012) 127-132.
12. P.K.Sharma, t-intuitionistic fuzzy quotient modules, *International Journal of Fuzzy Mathematics and Systems*, 2(1) (2012) 37-45.
13. W.B.Vasantha Kandasamy, *Smarandache Fuzzy Algebra*, American Research Press, Rehoboth.
14. V.Veeramani and K.Arjunan, Some properties of intuitionistic fuzzy normal subrings, *Applied Mathematical Science*, 2(1) (2012) 37-4.
15. Wang-jin-Liu, Fuzzy invariant subgroups and fuzzy ideals, *Fuzzy Sets and System*, 8 (1982) 133-139.

16. B,Yao, (λ,μ)-fuzzy subring and (λ,μ)-fuzzy ideals, *The Journal of Fuzzy Mathematics*, 15 (4) (2007) 981-987.
17. L.A.Zadeh, Fuzzy set, *Information and Control*, 8 (1965) 338-353.