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Face Sum Divisor Cordial Graphs

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Abstract. In this paper, we investigate the face sum divisor cordial labeling of switching of any vertex in cycle C_n, switching of a pendent vertex in path P_n and $S'(K_{1,n})$.

Keywords: Sum divisor cordial labeling, face sum divisor cordial labeling, switching of a vertex.

AMS Mathematics Subject Classification (2010): 05C78

1. Introduction

We begin with simple, finite, planar, undirected graph. A (p,q) planar graph G means a graph G = (V,E), where V is the set of vertices with |V| = p, E is the set of edges with |E| = q and F is the set of interior faces of G with |F| = number of interior faces of G. For standard terminology and notations related to graph theory we refer to Harary [4] while for number theory we refer to Burton [2]. A graph labeling is the assignment of unique identifiers to the edges and vertices of a graph. For a dynamic survey on various graph labeling problems along with an extensive bibliography we refer to Gallian [3]. In [1], Cahit introduced the concept of cordial labeling of graph. Varatharajan et al. [7] introduced the concept of divisor cordial labeling of graphs. The concept of sum divisor cordial labeling was introduced by Lourdusamy et al. [6]. Lawrence et al. introduced the concept of face product cordial labeling of graphs in [5]. Motivated by the concept of face product cordial labeling and sum divisor cordial labeling, we introduce new type of labeling which is called a face sum divisor cordial labeling of graph. The present work is focused on some new families of face sum divisor cordial labeling of switching of a pendent vertex in path P_n, switching of any vertex in cycle C_n and $S'(K_{1,n})$. We will provide brief summary of definitions and other information which are necessary for the present investigations.

2. Basic definitions

Definition 2.1. Let a and b be two integers. If a divides b means that there is a positive integer k such that b = ka. It is denoted by a|b. If a does not divide b, then we denote $a \nmid b$.

Definition 2.2. Let G = (V(G), E(G)) be a simple graph and f : V(G)→{1,2,...,|V(G)|} be a bijection. For each edge uv, assign the label 1 if f(u)|f(v) or f(v)|f(u) and the label 0 otherwise. The function f is called a divisor cordial labeling if $|e_f(0)-e_f(1)| \leq 1$. A graph with a divisor cordial labeling is called a divisor cordial graph.

Definition 2.3. Let $G = (V(G), E(G))$ be a simple graph and $f : V(G) \to \{1,2,\ldots, |V(G)|\}$ be a bijection. For each edge uv, assign the label 1 if $2|(f(u)+f(v))$ and the label 0 otherwise. The function f is called a sum divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$. A graph which admits a sum divisor cordial labeling is called a sum divisor cordial graph.

Definition 2.4. A vertex switching G_v of a graph G is obtained by taking a vertex v of G, removing the entire edges incident with v and adding edges joining v to every vertex which are not adjacent to v in G.

Definition 2.5. For a graph G, the splitting graph $S'(G)$ of a graph G is obtained by adding a new vertex v' corresponding to each vertex v of G such that $N(v) = N(v')$.

Definition 2.6. A complete bipartite graph $K_{1,n}$ is called a star and it has n+1 vertices and n edges.

Definition 2.7. A face sum divisor cordial labeling of a graph G with vertex set V is a bijection f from V(G) to $\{1,2,\ldots, |V(G)|\}$ such that an edge uv is assigned the label 1 if 2 divides f(u)+f(v) and 0 otherwise and for face f is assigned the label 1 if 2 divides $f(u_1)+f(u_2)+\ldots+ f(u_k)$ and 0 otherwise, where $u_1,u_2,\ldots,u_k$ are vertices corresponding to the face. Also the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1 and the number of faces labeled with 0 and the number of faces labeled with 1 differ by at most 1. A graph with a face sum divisor cordial labeling is called a face sum divisor cordial graph.

3. Main theorems

Theorem 3.1. Switching of any vertex in cycle C_n admits face sum divisor cordial labeling for $n \geq 5$.

Proof: Let $v_1,v_2,\ldots,v_n$ be the successive vertices of C_n. G_v denotes the graph, which is obtained by switching of a vertex v of C_n. Without loss of generality let the switched vertex be v_1. Let G be a graph G_{v_1}. Then $v_1,v_2,\ldots,v_n$ are vertices, $e_1,e_2,\ldots,e_{2n-5}$ are edges and $f_1,f_2,\ldots,f_{n-4}$ are the interior faces of G. $e_i = v_1v_{i+2}$, for $1 \leq i \leq n-3$, $e_{n-3+i} = v_{i+1}v_{i+2}$, for $1 \leq i \leq n-2$ and $f_i = v_1v_{i+2}v_{i+3}v_1$ for $1 \leq i \leq n-4$. Then $|V(G)| = n$, $|E(G)| = 2n-5$ and $|F(G)| = n-4$.

Define $g : V(G) \to \{1, 2, 3, \ldots, n\}$ as follows

Case 1 : n = 5.

$g(v_1) = 1$, $g(v_2) = 2$, $g(v_3) = 4$, $g(v_4) = 3$ and $g(v_5) = 5$.

Then induced edge labels are

$g^*(e_1) = 0$, $g^*(e_2) = 1$, $g^*(e_3) = 1$, $g^*(e_4) = 0$ and $g^*(e_5) = 1$.

Also the induced face label is

$g^{**}(f_1) = 1$.

In view of the above defined labeling pattern we have

$e_f(0)+1 = e_f(1) = 3$ and $f_g(0)+1 = f_g(1) = 1$.

Then $|e_g(0) - e_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$,

Thus switching of any one vertex in cycle C_n is face sum divisor cordial graph for $n = 5$.

Case 2: $n = 6$.

$g(v_1) = 1$, $g(v_2) = 2$, $g(v_3) = 4$, $g(v_4) = 3$ and $g(v_{i+4}) = g(v_i)+4$, for $1 \le i \le n - 4$.

Then induced edge labels are

$g^*(e_1) = 0$, $g^*(e_2) = 1$, $g^*(e_3) = 1$, $g^*(e_4) = 1$, $g^*(e_5) = 0$, $g^*(e_6) = 1$ and $g^*(e_7) = 0$.

Also the induced face labels are

$g^{**}(f_1) = 1$ and $g^{**}(f_2) = 0$.

In view of the above defined labeling pattern we have

$e_f(0)+1 = e_f(1) = 4$ and $f_g(0) = f_g(1) = 1$.

Then $|e_g(0) - e_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$,

Thus switching of any one vertex in cycle C_n is face sum divisor cordial graph for $n = 6$.

Case 3: $n > 6$.

Sub Case 3.1: $n \equiv 0,1,2 (\bmod 4)$

$g(v_1) = 1$, $g(v_2) = 2$, $g(v_3) = 4$, $g(v_4) = 3$ and $g(v_{i+4}) = g(v_i)+4$, for $1 \le i \le n - 4$.

Then induced edge labels are

$g^*(e_1) = g^*(e_4) = 0$, $g^*(e_2) = g^*(e_3) = 1$ and $g^*(e_{i+4}) = g^*(e_i)$, for $1 \le i \le n - 7$.

$g^*(e_{2i+n-4}) = 1$ and $g^*(e_{2i+n-3}) = 0$, for $1 \le i \le \frac{n-2}{2}$, if $n \equiv 0,2 (\bmod 4)$.

$g^*(e_{2i+n-4}) = 1$, for $1 \le i \le \frac{n-1}{2}$ and $g^*(e_{2i+n-3}) = 0$, for $1 \le i \le \frac{n-3}{2}$, if $n \equiv 1 (\bmod 4)$.

Also the induced face labels are

$g^{**}(f_{2i-1}) = 1$ and $g^{**}(f_{2i}) = 0$, for $1 \le i \le \frac{n-4}{2}$, if $n \equiv 0,2 (\bmod 4)$.

$g^{**}(f_{2i-1}) = 1$, for $1 \le i \le \frac{n-3}{2}$ and $g^{**}(f_{2i}) = 0$, for $1 \le i \le \frac{n-5}{2}$, if $n \equiv 1 (\bmod 4)$.

In view of the above defined labeling pattern we have

$e_f(0)+1 = e_f(1) = n-2$ and $f_g(0)+1 = f_g(1) = \frac{n-3}{2}$, if $n \equiv 1 (\bmod 4)$.

$e_f(0)+1 = e_f(1) = n-2$ and $f_g(0) = f_g(1) = \frac{n-4}{2}$, if $n \equiv 2 (\bmod 4)$.

$e_f(0) = e_f(1)+1 = n-2$ and $f_g(0) = f_g(1) = \frac{n-4}{2}$, if $n \equiv 0 (\bmod 4)$.

Then $|e_g(0) - e_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$,

Thus switching of any one vertex in cycle C_n is face sum divisor cordial graph for $n \equiv 0,1,2 (\bmod 4)$.

Sub Case 3.2: $n \equiv 3 (\bmod 4)$

$g(v_1) = 1$, $g(v_2) = 2$, $g(v_3) = 4$, $g(v_4) = 3$, $g(v_{i+4}) = g(v_i) + 4$, for $2 \le i \le n-5$ and $g(v_n) = n$.

Then induced edge labels are

$g^*(e_1) = g^*(e_4) = 0$, $g^*(e_2) = g^*(e_3) = 1$ and $g^*(e_{i+4}) = g^*(e_i)$, for $1 \le i \le n - 7$.

$g^*(e_{2i+n-3}) = 1$ and $g^*(e_{2i+n-2}) = 0$, for $1 \le i \le \frac{n-3}{2}$. $g^*(e_{2n-5}) = 0$.

Also the induced face labels are

$g^{**}(f_{2i-1}) = 1$, for $1 \le i \le \frac{n-3}{2}$ and $g^{**}(f_{2i}) = 0$, for $1 \le i \le \frac{n-5}{2}$.

In view of the above defined labeling pattern we have $e_f(0) = e_f(1)+1 = n-2$ and $f_g(0)+1= f_g(1) = \frac{n-3}{2}$. Then $|e_g(0) - e_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$.

Thus switching of any one vertex in cycle C_n is face sum divisor cordial graph for $n \equiv 3(\text{mod } 4)$.

Hence switching of any one vertex in cycle C_n is face sum divisor graph for $n \ge 5$.

Example 3.1. Switching of a vertex v_1 in cycle C_8 and its face sum divisor cordial labeling is shown in figure 3.1.

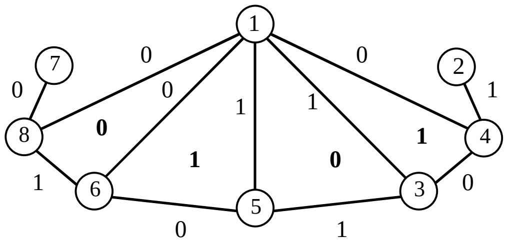

Figure 3.1:

Theorem 3.2. Switching of a pendent vertex in path P_n is face sum divisor cordial graph for $n \ge 4$.

Proof: Let $v_1, v_2, \ldots, v_n$, be the vertices of path P_n. v_1 and v_n are pendent vertex of path P_n. Without loss of generality, let the switched vertex be v_1. The graph G is obtained by switching of a pendent vertex v_1 in path P_n.

The $v_1,v_2,\ldots,v_n$ are vertices, $e_1,e_2,\ldots,e_{2n-4}$ are edges and $f_1,f_2,\ldots,f_{n-3}$ are the interior faces of G. $e_i = v_1v_{i+2}$, for $1 \le i \le n-2$, $e_{n-2+i} = v_{i+1}v_{i+2}$, for $1 \le i \le n-2$ and $f_i = v_1v_{i+2}v_{i+3}v_1$ for $1 \le i \le n-4$. Then $|V(G)| = n$, $|E(G)| = 2n-4$ and $|F(G)| = n-3$.

Define $g : V(G) \to \{1, 2, 3, \ldots, n\}$ as follows

Case 1: $n = 4$.

$g(v_1) = 1$, $g(v_2) = 2$, $g(v_3) = 4$ and $g(v_4) = 3$.

Then induced edge labels are

$g^*(e_1) = 0$, $g^*(e_2) = 1$, $g^*(e_3) = 1$ and $g^*(e_4) = 0$.

Also the induced face label is

$g^{**}(f_1) = 1$.

In view of the above defined labeling pattern we have

$e_f(0) = e_f(1) = 2$ and $f_g(0)+1 = f_g(1) = 1$.

Then $|e_g(0) - e_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$.

Thus switching of a pendent vertex in path P_n is face sum divisor cordial graph for $n = 4$.

Case 2: $n = 5$.

$g(v_1) = 5$, $g(v_2) = 1$, $g(v_3) = 3$, $g(v_4) = 2$ and $g(v_5) = 4$.

Then induced edge labels are

$g^*(e_1) = 1$, $g^*(e_2) = 0$, $g^*(e_3) = 0$, $g^*(e_4) = 1$, $g^*(e_5) = 0$ and $g^*(e_6) = 1$.

Also the induced face labels are

$g^{**}(f_1) = 1$ and $g^{**}(f_2) = 0$.

In view of the above defined labeling pattern we have

$e_f(0) = e_f(1) = 3$ and $f_g(0) = f_g(1) = 1$.

Then $|e_g(0) - e_g(1)| \leq 1$ and $|f_g(0) - f_g(1)| \leq 1$.

Thus switching of a pendent vertex in path P_n is face sum divisor cordial graph for $n = 5$.

Case 3: $n > 5$.

Sub Case 3.1: $n \equiv 0(\text{mod } 4)$

$g(v_1) = 1$, $g(v_2) = 2$, $g(v_3) = 4$, $g(v_4) = 3$ and $g(v_{i+4}) = g(v_i)+4$, for $1 \leq i \leq n - 4$.

Then induced edge labels are

$g^*(e_1) = g^*(e_4) = 0$, $g^*(e_2) = g^*(e_3) = 1$ and $g^*(e_{i+4}) = g^*(e_i)$, for $1 \leq i \leq n - 6$.

$g^*(e_{2i+n-3}) = 1$ and $g^*(e_{2i+n-2}) = 0$, for $1 \leq i \leq \frac{n-2}{2}$.

Also the induced face labels are

$g^{**}(f_{2i-1}) = 1$, for $1 \leq i \leq \frac{n-2}{2}$ and $g^{**}(f_{2i}) = 0$, for $1 \leq i \leq \frac{n-4}{2}$.

In view of the above defined labeling pattern we have $e_f(0) = e_f(1) = n-2$ and $f_g(0)+1 = f_g(1) = \frac{n-2}{2}$. Then $|e_g(0) - e_g(1)| \leq 1$ and $|f_g(0) - f_g(1)| \leq 1$.

Thus switching of a pendent vertex in path P_n is face sum divisor cordial graph for $n \equiv 0(\text{mod } 4)$.

Sub Case 3.2: $n \equiv 1(\text{mod } 4)$

$g(v_1) = n$, $g(v_2) = 1$, $g(v_3) = 3$, $g(v_4) = 2$, $g(v_5) = 4$ and $g(v_{i+4})=g(v_i)+4$, for $2 \leq i \leq n-4$.

Then induced edge labels are

$g^*(e_1) = g^*(e_4) = 1$, $g^*(e_2) = g^*(e_3) = 0$ and $g^*(e_{i+4}) = g^*(e_i)$, for $1 \leq i \leq n - 6$.

$g^*(e_{2i+n-3}) = 1$, for $1 \leq i \leq \frac{n-1}{2}$ and $g^*(e_{2i+n-2}) = 0$, for $1 \leq i \leq \frac{n-3}{2}$.

Also the induced face labels are

$g^{**}(f_{2i-1}) = 1$, for $1 \leq i \leq \frac{n-3}{2}$ and $g^{**}(f_{2i}) = 0$, for $1 \leq i \leq \frac{n-3}{2}$.

In view of the above defined labeling pattern we have $e_f(0) = e_f(1) = n-2$ and $f_g(0) = f_g(1) = \frac{n-3}{2}$. Then $|e_g(0) - e_g(1)| \leq 1$ and $|f_g(0) - f_g(1)| \leq 1$.

Thus switching of a pendent vertex in path P_n is face sum divisor cordial graph for $n \equiv 1(\text{mod } 4)$.

Sub Case 3.3: $n \equiv 2(\text{mod } 4)$

$g(v_1) = n$, $g(v_2) = 1$, $g(v_3) = 2$, $g(v_4) = 4$, $g(v_5) = 3$, $g(v_6) = 5$ and $g(v_{i+4}) = g(v_i)+4$, for $3 \leq i \leq n-4$.

Then induced edge labels are

$g^*(e_1) = g^*(e_2) = 1$, $g^*(e_3) = g^*(e_4) = 0$ and $g^*(e_{i+4}) = g^*(e_i)$, for $1 \leq i \leq n - 6$.

$g^*(e_{2i+n-3}) = 0$, for $1 \leq i \leq \frac{n-2}{2}$ and $g^*(e_{2i+n-2}) = 1$, for $1 \leq i \leq \frac{n-2}{2}$.

Also the induced face labels are

$g^{**}(f_{2i-1}) = 1$, for $1 \leq i \leq \frac{n-2}{2}$ and $g^{**}(f_{2i}) = 0$, for $1 \leq i \leq \frac{n-4}{2}$.

In view of the above defined labeling pattern we have $e_f(0) = e_f(1) = n–2$ and $f_g(0)+1= f_g(1) = \frac{n-3}{2}$. Then $|e_g(0) − e_g(1)| \le 1$ and $|f_g(0) − f_g(1)| \le 1$.

Thus switching of a pendent vertex in path P_n is face sum divisor cordial graph for $n \equiv 2(\text{mod } 4)$.

Sub Case 3.4: $n \equiv 3(\text{mod } 4)$

$g(v_1) = n–1$, $g(v_2) = 1$, $g(v_3) = 3$, $g(v_4) = 2$, $g(v_5) = 4$, $g(v_6) = 5$, $g(v_7) = 7$ and $g(v_{i+4}) = g(v_i)+4$, for $4 \le i \le n–4$.

Then induced edge labels are

$g^*(e_1) = g^*(e_4) = 0$, $g^*(e_2) = g^*(e_3) = 1$ and $g^*(e_{i+4}) = g^*(e_i)$, for $1 \le i \le n – 6$.

$g^*(e_{2i+n−3}) = 1$, for $1 \le i \le \frac{n-1}{2}$ and $g^*(e_{2i+n−2}) = 0$, for $1 \le i \le \frac{n-3}{2}$.

Also the induced face labels are

$g^{**}(f_{2i−1}) = 0$, for $1 \le i \le \frac{n-3}{2}$ and $g^{**}(f_{2i}) = 1$, for $1 \le i \le \frac{n-3}{2}$.

In view of the above defined labeling pattern we have $e_f(0) = e_f(1) = n–2$ and $f_g(0)= f_g(1) = \frac{n-3}{2}$. Then $|e_g(0) − e_g(1)| \le 1$ and $|f_g(0) − f_g(1)| \le 1$.

Thus switching of a pendent vertex in path P_n is face sum divisor cordial graph for $n \equiv 3(\text{mod } 4)$.

Therefore switching of a pendent vertex in path P_n is face sum divisor graph for $n \ge 4$.

Example 3.2. Switching of a pendent vertex of path P_6 and its face sum divisor cordial labeling is shown in figure 3.2.

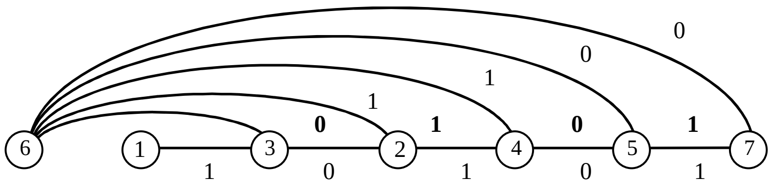

Figure 3.2:

Theorem 3.3. The graph $S'(K_{1,n})$ is face sum divisor cordial graph for $n \ge 2$.

Proof: Let $v,v_1,…,v_n$ be the vertices of $K_{1,n}$. Let $G = S'(K_{1,n})$. Then $v,v_1,…,v_n$, $v', v'_1,…, v'_n$ are the vertices, $e_1,e_2,…,e_{3n}$ are the edges and $f_1,f_2,…,f_{n–1}$ are the interior faces of G, where $e_i = v'v_i$, $e_{n+i} = v_iv$ and $e_{2n+i} = v\,v'_i$ for $1 \le i \le n$ and $f_i = v'v_ivv_{i+1}v'$ for $1 \le i \le n–1$. Then $|V(G)| = 2n+2$, $|E(G)| = 3n$ and $|F(G)| = n–3$.

Case 1: $n = 2$.

$g(v') = 1$, $g(v) = 2$, $g(v_1) = 3$, $g(v_2) = 4$, $g(v'_1) = 5$ and $g(v'_2) = 6$.

Then induced edge labels are

$g^*(e_1) = g^*(e_4) = g^*(e_6) = 1$ and $g^*(e_2) = g^*(e_3) = g^*(e_5) = 0$.

Also the induced face label is

$g^{**}(f_1) = 1$.

In view of the above defined labeling pattern we have

$e_f(0) = e_f(1) = 3$ and $f_g(0)+1 = f_g(1) = 1$.

Then $|e_g(0) - e_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$.

Thus $S'(K_{1,n})$ is face sum divisor cordial graph for n = 2.

Case 2: n = 3.

$g(v') = 1$, $g(v) = 2$, $g(v_1) = 3$, $g(v_2) = 5$, $g(v_3) = 4$ and $g(v_i') = n+2+i$, for $1 \le i \le 3$.

Then induced edge labels are

$g^*(e_1) = g^*(e_2) = g^*(e_6) = g^*(e_7) = g^*(e_9) = 1$ and $g^*(e_3) = g^*(e_4) = g^*(e_5) = g^*(e_8) = 0$.

Also the induced face labels are

$g^{**}(f_1) = 0$ and $g^{**}(f_2) = 1$.

In view of the above defined labeling pattern we have

$e_f(0) + 1 = e_f(1) = 5$ and $f_g(0) = f_g(1) = 1$.

Then $|e_g(0) - e_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$.

Thus $S'(K_{1,n})$ is face sum divisor cordial graph for n = 3.

Case 3: $n \ge 4$.

Sub Case 3.1: $n \equiv 0,1,2 \pmod 4$

$g(v') = 1$, $g(v) = 2$, $g(v_1) = 3$, $g(v_2) = 4$, $g(v_3) = 6$, $g(v_4) = 5$, $g(v_{i+4}) = g(v_i) + 4$, for $1 \le i \le n-4$ and $g(v_i') = n+2+i$, for $1 \le i \le n$.

Then induced edge labels are

$g^*(e_1) = g^*(e_4) = 0$, $g^*(e_2) = g^*(e_3) = 1$ and $g^*(e_{i+4}) = g^*(e_i)$, for $1 \le i \le n-4$.

$g^*(e_{n+1}) = g^*(e_{n+4}) = 1$, $g^*(e_{n+2}) = g^*(e_{n+3}) = 0$ and $g^*(e_{n+4+i}) = g^*(e_{n+i})$, for $1 \le i \le n-4$.

$g^*(e_{2n+2i-1}) = 0$, for $1 \le i \le \frac{n}{2}$ and $g^*(e_{2n+2i}) = 1$, for $1 \le i \le \frac{n}{2}$, if $n \equiv 0,2 \pmod 4$.

$g^*(e_{2n+2i-1}) = 1$, for $1 \le i \le \frac{n+1}{2}$ and $g^*(e_{2n+2i}) = 0$, for $1 \le i \le \frac{n-1}{2}$, if $n \equiv 1 \pmod 4$.

Also the induced face labels are

$g^{**}(f_{2i-1}) = 1$, for $1 \le i \le \frac{n}{2}$ and $g^{**}(f_{2i}) = 0$, for $1 \le i \le \frac{n-2}{2}$, if $n \equiv 0,2 \pmod 4$.

$g^{**}(f_{2i-1}) = 1$, for $1 \le i \le \frac{n-1}{2}$ and $g^{**}(f_{2i}) = 0$, for $1 \le i \le \frac{n-1}{2}$, if $n \equiv 1 \pmod 4$.

In view of the above defined labeling pattern we have

$e_f(0) = e_f(1) = \frac{3n}{2}$ and $f_g(0)+1 = f_g(1) = \frac{n}{2}$, if $n \equiv 0,2 \pmod 4$.

$e_f(0)+1 = e_f(1) = \frac{3n+1}{2}$ and $f_g(0) = f_g(1) = \frac{n-1}{2}$, if $n \equiv 1 \pmod 4$.

Then $|e_g(0) - e_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$.

Thus $S'(K_{1,n})$ is face sum divisor cordial graph for $n \equiv 0,1,2 \pmod 4$.

Sub Case 3.2: $n \equiv 3 \pmod 4$

$g(v') = 1$, $g(v) = 2$, $g(v_1) = 3$, $g(v_2) = 5$, $g(v_3) = 4$, $g(v_4) = 6$, $g(v_{i+4}) = g(v_i) + 4$, for $1 \le i \le n-4$ and $g(v_i') = n+2+i$, for $1 \le i \le n$.

Then induced edge labels are

$g^*(e_1) = g^*(e_2) = 1$, $g^*(e_3) = g^*(e_4) = 0$ and $g^*(e_{i+4}) = g^*(e_i)$, for $1 \le i \le n-4$.

$g^*(e_{n+1}) = g^*(e_{n+2}) = 0$, $g^*(e_{n+3}) = g^*(e_{n+4}) = 1$ and $g^*(e_{n+4+i}) = g^*(e_{n+i})$, for $1 \le i \le n-4$.

$g^*(e_{2n+2i-1}) = 1$, for $1 \le i \le \frac{n+1}{2}$ and $g^*(e_{2n+2i}) = 0$, for $1 \le i \le \frac{n-1}{2}$.

Also the induced face labels are

$g^{**}(f_{2i-1}) = 0$ and $g^{**}(f_{2i}) = 1$, for $1 \le i \le \frac{n-1}{2}$.

In view of the above defined labeling pattern we have

$e_f(0)+1 = e_f(1) = \frac{3n+1}{2}$ and $f_g(0) = f_g(1) = \frac{n-1}{2}$.

Then $|e_g(0) - e_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$.

Thus $S'(K_{1,n})$ is face sum divisor cordial graph for $n \equiv 3 \pmod 4$.

Hence the graph $S'(K_{1,n})$ is face integer edge cordial graph for $n \ge 2$.

Example 3.3. The graph $S'(K_{1,4})$ and its face sum divisor cordial labeling is shown in figure 3.3.

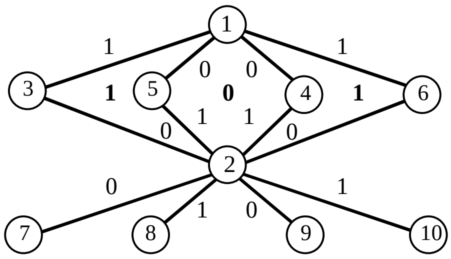

Figure 3.3:

4. Conclusions

In this paper, we presented the face sum divisor cordial labeling of switching of any vertex in cycle C_n, switching of a pendent vertex in path P_n and $S'(K_{1,n})$.

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