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L(3,2,1)-Labeling of the Jahangir Graph

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Abstract. An L(3,2,1)-labeling is a simplified model for the channel assignment problem. It is a natural generalization of the widely studied L(2,1)-labeling. An L(3,2,1)-labeling of a graph G is a function f from the vertex set V(G) to the set of positive integers such that for any two vertices x,y, if d(x,y) = 1, then $|f(x) - f(y)| \geq 3$; if d(x,y) = 2, then $f(x) - f(y)| \geq 2$; if d(x,y) = 3,then $|f(x) - f(y)| \geq 1$. The L(3,2,1)-labeling number $K_3(G)$ of G is the smallest positive integer k such that G has an L(3,2,1)-labeling with k as the maximum label. In this paper we determine the L(3,2,1)-labeling number of the Jahangir graph $J_{4,m}$.

Keywords: L(3,2,1)-labeling, Jahangir graph

AMS Mathematics Subject Classification (2010): 05C78

1. Introduction

Griggs and Yeh defined the L(2, 1)-labeling of a graph G = (V, E) as a function f which assigns every x, $y \in V$, a label from the set of positive integers such that $|f(x) - f(y)| \geq 2$ if d(x, y) = 1 and $|f(x) - f(y)| \geq 1$ if d(x, y) = 2 [1].

L(2, 1)-labeling has been widely studied in recent years. Chartand et al. introduced the radio-labeling of graphs; this was motivated by the regulations for the channel assignments in the channel assignment problem [3]. Radio-labeling takes into consideration the diameter of the graph, and as a result, every vertex is related.

Practically, interference among channels may go beyond two levels. L(3, 2, 1)-labeling [2,4] naturally extends from L(2, 1)-labeling, taking into consideration vertices which are within a distance of three apart; however, it remains less difficult than radio-labeling.

In this chapter we determine the L(3,2,1)-labeling number of the Jahangir graph $J_{4,m}$.

Definition 1.1. Let G = (V,E)be a graph and f be a mapping f: $V \rightarrow \mathbf{N}$. f is an ***L(3,2,1)-labeling*** of G if, for all $x,y \in V$,

$$|f(x) - f(y)| \geq \begin{cases} 3 & \text{if} \quad d(x, y) = 1 \\ 2 & \text{if} \quad d(x, y) = 2 \\ 1 & \text{if} \quad d(x, y) = 3 \end{cases}$$

Definition 1.2. The ***L(3,2,1)-number***, $K_3(G)$, of a graph G is the smallest natural number k such that G has an L(3,2,1)-labeling with k as the maximum label. An L(3,2,1)-labeling of a graph G is called a minimal L(3,2,1)-labeling of G if, under the labeling, the highest label of any vertex is $K_3(G)$.

Note : If 1 is not used as a vertex label in an L(3, 2, 1)-labeling of a graph, then every vertex label can be decreased by one to obtain another L(3, 2, 1)-labeling of the graph. Therefore in a minimal L(3, 2, 1)-labeling 1 will necessarily appear as a vertex label.

Definition 1.3. A ***cycle*** in a graph G is a sequence of distinct vertices $(u_0, u_1, u_2, \ldots, u_{(n-1)})$ where u_i and $u_{(i+1)}$ are adjacent for all i = 0, 1, 2,…, (n − 2) and $u_{(n-1)}$ and u_0 are adjacent. A cycle with n vertices is denoted by C_n.

Definition 1.4: The ***Jahangir graph*** $J_{n,m}$ for $m \geq 3$ is a graph on (nm + 1) vertices. That is, a graph consisting of a cycle C_{nm} with one additional vertex which is adjacent to m vertices of C_{nm} at distance n to each other on C_{nm}.

Theorem 2.1: $K_3(J_4, m) = 2m + 2$ for all $m \geq 5$ where $J_{4,m}$ is the Jahangir graph.

Proof : Let G = (V, E) be the Jahangir graph $J_{4,m}$ with the vertex set $V = \{u_0, u_1, u_2, \ldots, u_m, v_1, v_2, \ldots, v_{3m}\}$ and the edge set $E = \{u_0u_i : 1 \leq i \leq m\} \cup \{u_{i+1}v_{3i+1} : 0 \leq i \leq m-1\} \cup \{v_iv_{i+1} : 1 \leq i \leq 3m-1 \text{ with } i \neq 3,6,\ldots,3m\} \cup \{v_{3i}u_j, u_j v_{3i+1} : 1 \leq i \leq m-1, j = 2,3,4,\ldots,m\}$.

Let f be a minimal L(3,2,1)-labeling of the Jahangir graph $J_{4,m}$. we have $d(u_0, u_i) = 1$ for all $1 \leq i \leq m$; $d(u_i, u_j) = 2$ for all $1 \leq i, j \leq m$ with $i \neq j$; $d(u_{i+1}, v_{3i+1}) = 1$ for all $1 \leq i \leq m-1$; $d(v_i, v_{i+1}) = 1$ for all $1 \leq i \leq 3m-1$ with $i \neq 3,6,\ldots,3m$; $d(u_1, v_{3i+2}) = 4$ for all $1 \leq i \leq m-2$.

Since the diam(G) is greater than three, f is not injective. Since f is minimal, any one of the vertices of G should have label 1. Let $f(u_0) = 1$. we have $d(u_0, u_i) = 1$ for all $1 \leq i \leq m$, without loss of generality we can assume $f(u_1) \geq 4$.

As the distance between any two vertices of u_i is two, their labels should differ by atleast 2. Since $f(u_1) \geq 4$ and there are (m–1) remaining vertices of u_i, the label of one of the vertices of u_i should be greater than or equal to 2(m– 1)+ 4.

Hence $K_3(J_{4,m}) \geq 2m + 2$.

Next we prove that $K_3(J_{4,m}) \leq 2m + 2$. Define

$$f(u_i) = \begin{cases} 1 & \text{if } i = 0 \\ 2(i+1) & \text{if } 1 \leq i \leq m \end{cases}$$

$$f(v_i) = \begin{cases} 2 & \text{if } i = 2, 5, 8, \ldots, (3m-1) \\ 5 & \text{if } i = 6, 9, 12, \ldots, (3m-3) \\ 7 & \text{if } i = 1 \ \& \ i = 10, 13, \ldots, (3m-2) \\ 9 & \text{if } i = 3, 3m \\ 11 & \text{if } i = 4, 7 \end{cases}$$

As per the labeling, the max $\{f(u) : u \in V\} = 2m + 2$.
Hence $K_3(J_{4,m}) = 2m + 2$ for all $m \geq 5$.

L(3,2,1) - labeling of $J_{4,5}$

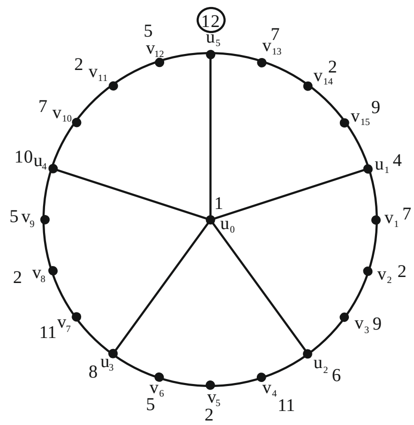

Figure 2.2: (a) $(K_3(J_{4,5})) = 12$

L(3,2,1) - labeling of $J_{4,8}$

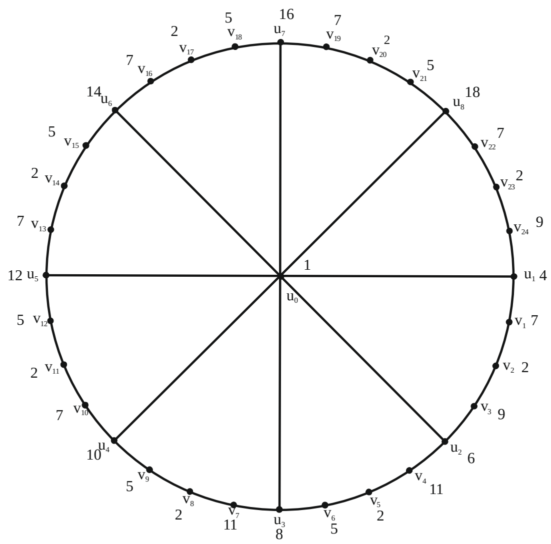

Figure 2.2: (b) $(K_3(J_{4,8})) = 18$

REFERENCES

1. J.R.Griggs and R.K.Yeh, Labeling graphs with a condition at distance two, *SIAM J. Discrete Math.*, 5 (1992) 586 -596.
2. D.S.T.Ramesh and S.Sarathi, L(3,2,1)-labeling of some special graphs, *Archimedes J. Math.*, 2(1) (2002) 31-38.

3. G.Chartrand, D.Erwin, F.Harary and P.Zhang, Radio labeling of graphs, *Bull Inst. Combin. Appl.*, 33 (2001) 77- 85.
4. J.Clipperton, J.Gehrtz, Z.Szaniszlo and D.Torkornoo, L(3,2,1)-labeling of simple graphs, VERUM, Valparaiso University, 2005.
5. Sk.Amanathulla and M.Pal, L(3,2,1)-and L (4,3,2,1)-labeling problems on circular-arc graphs, *Int. J. Control Theory Appl.*, 9 (2016) 869-884.
6. Sk.Amanathulla and M.Pal, l(3,2,1)-labeling problems on permutation graphs, *Transylvanian Review*, XXV (2017).
7. Sk.Amanathulla and M.Pal, L(3,2,1)-and L(4,3,2,1)-labeling problems on interval graphs, *AKCE International Journal of Graphs and Combinatorics*, 14 (3) (2017) 205-215.