L(3,2,1)-Labeling of the Jahangir Graph

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Abstract. An L(3,2,1)-labeling is a simplified model for the channel assignment problem. It is a natural generalization of the widely studied L(2,1)-labeling. An L(3,2,1)-labeling of a graph \(G\) is a function \(f\) from the vertex set \(V(G)\) to the set of positive integers such that for any two vertices \(x, y\), if \(d(x, y) = 1\), then \(|f(x) - f(y)| \geq 3\); if \(d(x, y) = 2\), then \(|f(x) - f(y)| \geq 2\); if \(d(x, y) = 3\), then \(|f(x) - f(y)| \geq 1\). The L(3,2,1)-labeling number \(K_{3,2,1}(G)\) of \(G\) is the smallest positive integer \(k\) such that \(G\) has an L(3,2,1)-labeling with \(k\) as the maximum label. In this paper we determine the L(3,2,1)-labeling number of the Jahangir graph \(J_{4,m}\).

Keywords: L(3,2,1)-labeling, Jahangir graph

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1. Introduction

Griggs and Yeh defined the L(2,1)-labeling of a graph \(G = (V, E)\) as a function \(f\) which assigns every \(x, y \in V\), a label from the set of positive integers such that \(|f(x) - f(y)| \geq 2\) if \(d(x, y) = 1\) and \(|f(x) - f(y)| \geq 1\) if \(d(x, y) = 2\) [1].

L(2,1)-labeling has been widely studied in recent years. Chartand et al. introduced the radio-labeling of graphs; this was motivated by the regulations for the channel assignments in the channel assignment problem [3]. Radio-labeling takes into consideration the diameter of the graph, and as a result, every vertex is related.

Practically, interference among channels may go beyond two levels. L(3,2,1)-labeling [2,4] naturally extends from L(2,1)-labeling, taking into consideration vertices which are within a distance of three apart; however, it remains less difficult than radio-labeling.

In this chapter we determine the L(3,2,1)-labeling number of the Jahangir graph \(J_{4,m}\).

Definition 1.1. Let \(G = (V,E)\) be a graph and \(f\) be a mapping \(f: V \rightarrow \mathbb{N}\). \(f\) is an \(L(3,2,1)\)-labeling of \(G\) if, for all \(x,y \in V\),
The \( L(3,2,1) \)-number, \( K_3(G) \), of a graph \( G \) is the smallest natural number \( k \) such that \( G \) has an \( L(3,2,1) \)-labeling with \( k \) as the maximum label. An \( L(3,2,1) \)-labeling of a graph \( G \) is called a minimal \( L(3,2,1) \)-labeling of \( G \) if, under the labeling, the highest label of any vertex is \( K_3(G) \).

Note: If 1 is not used as a vertex label in an \( L(3,2,1) \)-labeling of a graph, then every vertex label can be decreased by one to obtain another \( L(3,2,1) \)-labeling of the graph. Therefore in a minimal \( L(3,2,1) \)-labeling 1 will necessarily appear as a vertex label.

Definition 1.4: The Jahangir graph \( J_n,m \) for \( m \geq 3 \) is a graph on \( (nm + 1) \) vertices. That is, a graph consisting of a cycle \( C_{nm} \) with one additional vertex which is adjacent to \( m \) vertices of \( C_{nm} \) at distance \( n \) to each other on \( C_{nm} \).

Theorem 2.1: \( K_3(J_{4,m}) = 2m + 2 \) for all \( m \geq 5 \) where \( J_{4,m} \) is the Jahangir graph.

Proof: Let \( G = (V, E) \) be the Jahangir graph \( J_{4,m} \) with the vertex set
\[
V = \{u_0, u_1, u_2, \ldots, u_m, v_1, v_2, \ldots, v_{3m}\}
\]
and the edge set
\[
E = \{(u_i, u_j) : 1 \leq i, j \leq m \} \cup \{ (u_i, v_j) : 0 \leq i \leq m - 1 \} \cup \{ (v_j, v_{j+1}) : 1 \leq i \leq 3m - 1 \} \cup \{ (v_i, u_j) : 1 \leq i \leq m - 1 \}.
\]

Let \( f \) be a minimal \( L(3,2,1) \)-labeling of the Jahangir graph \( J_{4,m} \). We have \( d(u_0, u_i) = 1 \) for all \( 1 \leq i \leq m \); \( d(u_i, u_j) = 2 \) for all \( 1 \leq i \leq j \leq m \) with \( i \neq j \); \( d(u_i, v_{j+1}) = 1 \) for all \( 1 \leq i \leq m - 1 \); \( d(v_i, u_j) = 1 \) for all \( 1 \leq i \leq 3m - 1 \) with \( i \neq 3, 6, \ldots, 3m \); \( d(v_i, v_{i+1}) = 4 \) for all \( 1 \leq i \leq m - 1 \).

Since the diameter \( \text{diam}(G) \) is greater than three, \( f \) is not injective. Since \( f \) is minimal, any one of the vertices of \( G \) should have label 1. Let \( f(u_0) = 1 \). We have \( d(u_0, u_i) = 1 \) for all \( 1 \leq i \leq m \), without loss of generality we can assume \( f(u_i) \geq 4 \).

As the distance between any two vertices of \( u_i \) is two, their labels should differ by at least 2. Since \( f(u_i) \geq 4 \) and there are \( (m-1) \) remaining vertices of \( u_i \), the label of one of the vertices of \( u_i \) should be greater than or equal to \( 2(m-1) + 4 \).

Hence \( K_3(J_{4,m}) \geq 2m + 2 \).

Next we prove that \( K_3(J_{4,m}) \leq 2m + 2 \). Define
\[
f(u_i) = \begin{cases} 
1 & \text{if } i = 0 \\
2(i + 1) & \text{if } 1 \leq i \leq m 
\end{cases}
\]
\[
f(v_i) = \begin{cases} 
2 & \text{if } i = 2, 5, 8, \ldots, (3m - 1) \\
5 & \text{if } i = 6, 9, 12, \ldots, (3m - 3) \\
7 & \text{if } i = 10, 13, \ldots, (3m - 2) \\
9 & \text{if } i = 3m \\
11 & \text{if } i = 4, 7
\end{cases}
\]
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As per the labeling, the max \( f(u) : u \in V \) = 2m + 2.
Hence \( K_3(J_{4,m}) = 2m + 2 \) for all \( m \geq 5 \).

L(3,2,1) - labeling of \( J_{4,5} \)

![Figure 2.2: (a) \( (K_3(J_{4,5})) = 12 \)](image)

L(3,2,1) - labeling of \( J_{4,8} \)

![Figure 2.2: (b) \( (K_3(J_{4,8})) = 18 \)](image)

REFERENCES