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A Study on Face Integer Edge Cordial Graphs

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Abstract. In this paper, we investigate the face integer edge cordial labeling of duplication of each vertex by an edge in fan graph f_n for $n \geq 2$, planar graph G' is obtained from joining the outer vertex of the two copies of the planar graph G by a path of arbitrary length and $S'(K_{1,n})$.

Keywords: Integer edge cordial labeling, face integer edge cordial labeling, splitting graph.

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1. Introduction

A (p,q) planar graph G means a graph G = (V,E), where V is the set of vertices with |V| = p, E is the set of edges with |E| = q and F is the set of interior faces of G with |F| = number of interior faces of G. For standard terminology and notations related to graph theory we refer to Harary [3] and graph labeling we refer Gallian [2]. The concept of cordial labeling of graph was introduced by Cahit [1]. The concept of face edge product cordial labeling was introduced by Lawrence et al. in [4]. In [5], Nicholas et al. introduced the concept of integer edge cordial labeling of graphs. In [6], Mohamed Sheriff et al. introduced the concept of face integer edge cordial labeling of graph. The brief summaries of definition which are necessary for the present investigation are provided below.

2. Basic definitions

Definition 2.1. A complete bipartite graph $K_{1,n}$ is called a star and it has n+1 vertices and n edges.

Definition 2.2. The join of two graphs G and H is a graph G+H with $V(G+H) = V(G) \cup V(H)$ and $E(G+H) = E(G) \cup E(H) \cup \{ uv : u \in V(G) \text{ and } v \in V(H) \}$.

Definition 2.3. The graph $P_n + K_1$ is called a fan graph of n vertices. It denoted by f_n.

Definition 2.4. Duplication of a vertex v_k by a new edge $e = u_k w_k$ in a graph G produces a new graph G' such that $N(u_k) = \{v_k, w_k\}$ and $N(w_k) = \{v_k, u_k\}$.

Definition 2.5. For a graph G, the splitting graph $S'(G)$ of a graph G is obtained by adding a new vertex v' corresponding to each vertex v of G such that $N(v) = N(v')$.

Definition 2.6. $[-x,\ldots,x] = \{t \,/\, t$ is an integer and $|t| \le x\}$ and $[-x,\ldots,x]^* = [-x,\ldots,x] - \{0\}$.

Definition 2.7. For a planar graph G, an edge labeling function is defined as $g : E \rightarrow [-\frac{q}{2},\ldots,\frac{q}{2}]^*$ or $[-\frac{q}{2},\ldots,\frac{q}{2}]$ as q is even or odd be an injective map, which induces vertex labeling function $g^* : V(G) \rightarrow \{0,1\}$ such that $g^*(v) = 1$, if $\sum_{i=1}^{n} g(e_i) \ge 0$ and $g^*(v) = 0$ otherwise, where $e_1, e_2, \ldots, e_n$ are adjacent edges of the vertex v and face labeling function $g^{**} : F(G) \rightarrow \{0,1\}$ such that $g^{**}(f) = 1$, if $\sum_{i=1}^{m} g(e_i) \ge 0$ and $g^{**}(f) = 0$ otherwise, where $e_1,e_2,\ldots,e_m$ are edges of face f. g is called face integer edge cordial labeling of graph G if $|v_g(0)-v_g(1)| \le 1$ and $|f_g(0)-f_g(1)| \le 1$. $v_g(i)$ is the number of vertices of G having label i under g^* and $f_g(i)$ is the number of interior faces of G having label i under g^{**} for i = 0,1. A planar graph G is face integer edge cordial if it admits face integer edge cordial labeling.

3. Main theorems

Theorem 3.1. The duplication of each vertex by an edge in fan graph f_n is face integer edge cordial graph for $n \ge 2$.

Proof. Let f_n be the fan graph of n vertices. Let $v_1,v_2,\ldots,v_{n+1},e_1,e_2,\ldots,e_{2n-1}$ and $f_1,f_2,\ldots,f_{n-1}$ be the vertices, edges and an interior faces of G, where $e_i = v_1v_{i+1}$ for $i = 1,2,\ldots,n$ and $e_{n+i} = v_{i+1}v_{i+2}$ for $i = 1,2,\ldots,n-1$.

Let G be a duplication of each vertex by an edge in fan graph f_n. Let $v_1,v_2,\ldots,v_{n+1},u_1,u_2,\ldots,u_{n+1},w_1,w_2,\ldots,w_{n+1}$ be the vertices, $e_1,e_2,\ldots,e_{2n-1}$, e_{ij} for $i = 1,2,\ldots,n+1$ and $j = 1,2,3$ be the edges and $f_1, f_2, \ldots, f_{2n}$ be an interior faces of G, where $e_i = v_1v_{i+1}$ for $i = 1,2,\ldots,n$, $e_{n+i} = v_{i+1}v_{i+2}$ for $i = 1,2,\ldots,n-1$ and $e_{i1} = v_iu_i$, $e_{i2} = u_iw_i$, $e_{i3} = w_iv_i$ for $i = 1,2,\ldots,n+1$. $f_i = v_1v_{i+1}v_{i+2}v_1$ for $i = 1,2,\ldots,n-1$ and $f_{n-1+i} = v_iu_iw_iv_i$ for $i = 1,2,\ldots,n+1$,

Then $|V(G)| = 3n+3$, $|E(G)| = 5n+2$ and $|F(G)| = 2n$.

Case (i) : n is odd.

Define $g : E(G) \rightarrow [-(\frac{5n+1}{2}),\ldots,(\frac{5n+1}{2})]$ as follows.

$g(e_i) = i$	for $1 \le i \le \frac{n-1}{2}$;
$g(e_i) = 0$	for $i = \frac{n+1}{2}$;
$g(e_i) = \frac{n+1}{2} - i$	for $\frac{n+3}{2} \le i \le n$;
$g(e_i) = \frac{n+1}{2}$	for $i = n+1$;
$g(e_i) = g(e_{i-1}) + 1$	for $n+2 \le i \le \frac{3n-1}{2}$;

$g(e_i) = -n+1$ for $i = \frac{3n+1}{2}$;

$g(e_i) = g(e_{i-1}) + 1$ for $\frac{3n+3}{2} \le i \le 2n-1$

$g(e_{1j}) = n+j-1$ for $1 \le j \le 3$;

$g(e_{(i+1)j}) = g(e_{ij})+3$ for $1 \le i \le \frac{n+1}{2}$ and $1 \le j \le 3$;

$g(e_{ij}) = -n-j+1$ for $i = \frac{n+3}{2}$ and $1 \le j \le 3$;

$g(e_{(i+1)j}) = g(e_{ij})-3$ for $\frac{n+5}{2} \le i \le n$ and $1 \le j \le 3$;

Then induced vertex labels are

$g^*(v_i) = 1$ for $1 \le i \le \frac{n+1}{2}$;

$g^*(v_i) = 0$ for $\frac{n+3}{2} \le i \le n+1$;

$g^*(u_i) = 1$ for $1 \le i \le \frac{n+1}{2}$;

$g^*(u_i) = 0$ for $\frac{n+3}{2} \le i \le n+1$;

$g^*(w_i) = 1$ for $1 \le i \le \frac{n+1}{2}$;

$g^*(w_i) = 0$ for $\frac{n+3}{2} \le i \le n+1$.

Also the induced face labels are

$g^{**}(f_i) = 1$ for $1 \le i \le \frac{n-1}{2}$ and $g^{**}(f_i) = 0$ for $\frac{n+1}{2} \le i \le n-1$

$g^{**}(f_i) = 1$ for $n \le i \le \frac{3n-1}{2}$ and $g^{**}(f_i) = 0$ for $\frac{3n+1}{2} \le i \le 2n$

In view of the above defined labeling pattern we have $v_f(0) = v_f(1) = \frac{3n+3}{2}$ and $f_g(0) = f_g(1) = n$. Then $|v_g(0) - v_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$.

Thus G is face integer edge cordial graph for n is odd.

Case (ii) : n is even.

Define $g : \rightarrow [-(\frac{5n+2}{2}), \ldots, (\frac{5n+2}{2})]^*$ as follows.

$g(e_i) = i$ for $1 \le i \le \frac{n}{2}$;

$g(e_i) = \frac{n}{2} - i$ for $\frac{n+2}{2} \le i \le n$;

$g(e_i) = \frac{n+2}{2}$ for $i = n+1$;

$g(e_i) = g(e_{i-1}) + 1$ for $n+2 \le i \le \frac{3n-2}{2}$;

$g(e_i) = -n$ for $i = \frac{3n}{2}$;

$g(e_i) = -n+1$ for $i = \frac{3n+2}{2}$;

$g(e_i) = g(e_{i-1}) + 1$ for $\frac{3n+4}{2} \le i \le 2n-1$

$g(e_{1j}) = n+j$ for $1 \le j \le 3$;

$g(e_{(i+1)j}) = g(e_{ij})+3$ for $1 \le i \le \frac{n}{2}$ and $1 \le j \le 3$;

$g(e_{i1}) = \frac{5n+2}{2}$ for $i = \frac{n+2}{2}$;

$g(e_{i2}) = -\left(\frac{5n+2}{2}\right)$ for $i = \frac{n+2}{2}$;

$g(e_{i3}) = n$ for $i = \frac{n+2}{2}$;

$g(e_{ij}) = -n-j$ for $i = \frac{n+4}{2}$ and $1 \le j \le 3$;

$g(e_{(i+1)j}) = g(e_{ij})-3$ for $\frac{n+6}{2} \le i \le n$ and $1 \le j \le 3$;

Then induced vertex labels are

$g^*(v_i) = 1$ for $1 \le i \le \frac{n+2}{2}$;

$g^*(v_i) = 0$ for $\frac{n+4}{2} \le i \le n+1$

$g^*(u_i) = 1$ for $1 \le i \le \frac{n+2}{2}$;

$g^*(u_i) = 0$ for $\frac{n+4}{2} \le i \le n+1$;

$g^*(w_i) = 1$ for $1 \le i \le \frac{n}{2}$;

$g^*(w_i) = 0$ for $\frac{n+2}{2} \le i \le n+1$.

Also the induced face labels are

$g^{**}(f_i) = 1$ for $1 \le i \le \frac{n-2}{2}$ and $g^{**}(f_i) = 0$ for $\frac{n}{2} \le i \le n-1$

$g^{**}(f_i) = 1$ for $n \le i \le \frac{3n}{2}$ and $g^{**}(f_i) = 0$ for $\frac{3n+2}{2} \le i \le 2n$

In view of the above defined labeling pattern we have $v_f(1) = v_f(0)+1 = \frac{3n+4}{2}$ and $f_g(0) = f_g(1) = n$. Then $|v_g(0) - v_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$.

Thus G is face integer edge cordial graph for n is even.

Hence the duplication of each vertex by an edge in fan graph f_n is the face integer edge cordial graph for $n \ge 2$.

Theorem 3.2. The planar graph G′ is obtained from joining the outer vertex of the two copies of the planar graph G by a path of arbitrary length is face integer edge cordial graph.

Proof: Let G be the planar graph with n vertices, m edges and s interior faces.

Let G_1 and G_2 be the two copies of G. Let $v_1, v_2, \ldots, v_n$ be the vertices, $e_1, e_2, \ldots, e_m$ be the edges and $f_1, f_2, \ldots, f_s$ be the interior faces of G1 and $v'_1, v'_2, \ldots, v'_n$ be the vertices, e'_1,

$e'_2, \ldots, e'_m$ be the edges and $f'_1, f'_2, \ldots, f'_s$ be the interior faces of G2. Assume v_1 and v'_1 are an outer vertex of G1 and G2 respectively. Let G′ be the planar graph is obtained from joining the outer vertex of G1 and G2 by a path of arbitrary length k–1. Let $u_1, u_2, \ldots, u_k$ be the vertices and $e_{m+1}, e_{m+2}, \ldots, e_{m+k-1}$ be the edges of path P_k.

In G′, $w_1 = v_1$ and $w_k = v'_1$.

Then $|V(G')| = 2n+k-2$, $|E(G')| = 2m+k-1$ and $|F(G')| = 2s$.

Case (i) : k is odd.

Define $g : E(G) \to [-(\frac{2m+k-1}{2}), \ldots, (\frac{2m+k-1}{2})]^*$ as follows.

$g(e_i) = -i$ for $1 \le i \le m$;

$g(e'_i) = i$ for $1 \le i \le m$;

$g(e_{m+i}) = -\left(\frac{2m+k-1}{2}\right) + (i-1)$ for $1 \le i \le \frac{k-1}{2}$;

$g(e_{m+i}) = \frac{2m-k+1}{2} + i$ for $\frac{k+1}{2} \le i \le k-1$;

Then induced vertex labels are

$g^*(v_i) = 0$, for $1 \le i \le n$;

$g^*(w_i) = 0$, for $2 \le i \le \frac{k-1}{2}$ and $g^*(w_i) = 1$, for $\frac{k+1}{2} \le i \le k-1$;

$g^*(v'_i) = 1$, for $1 \le i \le n$;

Also the induced face labels are

$g^{**}(f_i) = 0$, for $1 \le i \le s$ and $g^{**}(f'_i) = 1$, for $1 \le i \le s$

In view of the above defined labeling pattern we have $v_f(1) = v_f(0)+1 = \frac{2n+k-1}{2}$ and $f_g(0) = f_g(1) = s$. Then $|v_g(0) - v_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$.

Thus G′ is face integer edge cordial graph for n is odd.

Case (ii) : n is even.

Define $g : E(G) \to [-(\frac{2m+k-2}{2}), \ldots, (\frac{2m+k-2}{2})]$ as follows.

$g(e_i) = -i$ for $1 \le i \le m$;

$g(e'_i) = i$ for $1 \le i \le m$;

$g(e_{m+i}) = -\left(\frac{2m+k-2}{2}\right) + (i-1)$ for $1 \le i \le \frac{k-2}{2}$;

$g(e_{m+i}) = 0$ for $i = \frac{k}{2}$;

$g(e_{m+i}) = \frac{2m-k}{2} + i$ for $\frac{k+2}{2} \le i \le k-1$;

Then induced vertex labels are

$g^*(v_i) = 0$, for $1 \le i \le n$;

$g^*(w_i) = 0$, for $2 \le i \le \frac{k}{2}$ and $g^*(w_i) = 1$, for $\frac{k+2}{2} \le i \le k-1$;

$g^*(v'_i) = 1$, for $1 \le i \le n$;

Also the induced face labels are

$g^{**}(f_i) = 0$, for $1 \le i \le s$ and $g^{**}(f'_i) = 1$, for $1 \le i \le s$

In view of the above defined labeling pattern we have $v_f(1) = v_f(0) = \frac{2n+k-2}{2}$ and $f_g(0) = f_g(1) = s$. Then $|v_g(0) - v_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$.

Thus G' is face integer edge cordial graph for n is odd.

The planar graph G' is obtained from joining the outer vertex of the two copies of the planar graph G by a path of arbitrary length is face integer edge cordial graph.

Theorem 3.3. The graph $S'(K_{1,n})$ is face integer edge cordial graph for $n \ge 2$.

Proof: Let $v,v_1,\ldots,v_n$ be the vertices of $K_{1,n}$. Let $G = S'(K_{1,n})$. Then $v,v_1,v_2,\ldots,v_n,v',v_1',\ldots,v_n'$ are the vertices, $e_1,e_2,\ldots,e_{3n}$ are the edges and $f_1,f_2,\ldots,f_{n-1}$ are the interior faces of G, where $e_i = v v_i'$, $e_{n+i} = vv_i$ and $e_{2n+i} = v_iv'$ for $1 \le i \le n$ and $f_i = vv_iv'v_{i+1}v$ for $1 \le i \le n-1$. Then $|V(G)| = 2n+2$, $|E(G)| = 3n$ and $|F(G)| = n-3$.

Case (i) : n is odd.

Define $g : E(G) \to [-(\frac{3n-1}{2}),\ldots,(\frac{3n-1}{2})]$ as follows.

$g(e_i) = -i$ for $1 \le i \le \frac{n+1}{2}$;

$g(e_i) = -\frac{n+1}{2}+i$ for $\frac{n+3}{2} \le i \le n$;

$g(e_{n+i}) = -\frac{n+1}{2} - i$ for $1 \le i \le \frac{n-1}{2}$

$g(e_{n+i}) = i$ for $\frac{n+1}{2} \le i \le n$

$g(e_{2n+i}) = -n- i$ for $1 \le i \le \frac{n-3}{2}$

$g(e_{2n+i}) = 0$ for $i = \frac{n-1}{2}$

$g(e_{2n+i}) = -\frac{3n-1}{2}$ for $i = \frac{n+1}{2}$

$g(e_{2n+i}) = \frac{3n+5}{2} -i$ for $\frac{n+3}{2} \le i \le n$

Then induced vertex labels are

$g^*(v) = 1$ and $g^*(v') = 1$

$g^*(v_i) = 0$, for $1 \le i \le \frac{n+1}{2}$ and $g^*(v_i) = 1$, for $\frac{n+3}{2} \le i \le n$;

$g^*(v_i') = 0$, for $1 \le i \le \frac{n+1}{2}$ and $g^*(v_i') = 1$, for $\frac{n+3}{2} \le i \le n$.

Also the induced face labels are

$g^{**}(f_i) = 0$, for $1 \le i \le \frac{n-1}{2}$ and $g^{**}(f_i) = 1$, for $\frac{n+1}{2} \le i \le n-1$

In view of the above defined labeling pattern we have $v_f(0) = v_f(1) = n+1$ and $f_g(0) = f_g(1) = \frac{n-1}{2}$. Then $|v_g(0) - v_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$.

Thus $S'(K_{1,n})$ is face integer edge cordial graph for n is odd.

Case (ii) : n is even.

Define g : E(G) $\to [-(\frac{3n}{2}),\ldots,(\frac{3n}{2})]^*$ as follows.

$g(e_i) = i$ for $1 \le i \le \frac{n}{2}$;

$g(e_i) = \frac{n}{2} - i$ for $\frac{n+2}{2} \le i \le n$;

$g(e_{n+1}) = n+1$

$g(e_{n+i}) = \frac{n}{2} + i$ for $2 \le i \le \frac{n}{2}$

$g(e_{n+i}) = -i$ for $\frac{n+2}{2} \le i \le n$

$g(e_{2n+1}) = \frac{n}{2} + 1$

$g(e_{2n+i}) = n+i$ for $2 \le i \le \frac{n}{2}$

$g(e_{2n+i}) = -\frac{n}{2} - i$ for $\frac{n+2}{2} \le i \le n$

Then induced vertex labels are

$g^*(v) = 1$ and $g^*(v') = 0$

$g^*(v_i) = 1$, for $1 \le i \le \frac{n}{2}$ and $g^*(v_i) = 0$, for $\frac{n+2}{2} \le i \le n$;

$g^*(v'_i) = 1$, for $1 \le i \le \frac{n}{2}$ and $g^*(v'_i) = 0$, for $\frac{n+2}{2} \le i \le n$

Also the induced face labels are

$g^{**}(f_i) = 1$, for $1 \le i \le \frac{n}{2}$ and $g^{**}(f_i) = 0$, for $\frac{n+2}{2} \le i \le n-1$

In view of the above defined labeling pattern we have $v_f(0) = v_f(1) = n+1$ and $f_g(0)+1 = f_g(1) = \frac{n}{2}$. Then $|v_g(0) - v_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$.

Thus $S'(K_{1,n})$ is face integer edge cordial graph for n is even.

Hence the graph $S'(K_{1,n})$ is face integer edge cordial graph for $n \ge 2$.

4. Conclusions

In this paper, we presented the face integer edge cordial labeling of duplication of each vertex by an edge in fan graph f_n for $n \ge 2$, planar graph G' is obtained from joining the outer vertex of the two copies of the planar graph G by a path of arbitrary length and $S'(K_{1,n})$.

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