

Performance Analysis of Substitutable Perishable Inventory System with Direct Demand in Supply Chain

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Abstract. This paper presents continuous review inventory systems in a simple chain with two different substitutable items in stock. The demand for the products follows independent Poisson with rates λ_1 and λ_2 respectively for product A and B. The common demand for both products at distributor follows Poisson distribution with rate λ_d . The operating policy at the lower echelon for the products are (si, Si) that is whenever the inventory level drops to 'si' on order for $Q_i = (Si - si)$ items is placed, the ordered items are received after a random time which is distributed as exponential. We assume that the demands accruing during the stock-out period are lost. The retailer replenishes the stock of products from the supplier which adopts (O,M) policy. The joint probability distribution of the inventory levels of the products, at retailer and the products at supplier are obtained in the steady state case. Various system performance measures are derived and the long run total expected inventory cost rate is calculated.

Keywords: Perishable inventory, Substitutable Product, Direct demand, Markov process (s,S) policy.

AMS Mathematics Subject Classification (2010): 90B05

1. Introduction

This paper investigates the substitutable product inventory control systems in supply chain. When different products are sold by a retailer, substitution between these products causes the retailers to manage their order quantities in a competitive environment. In this article, we examine the nature of continuous review inventory control system facing stochastic independent demand in supply chain. Substitutable product inventory problem was first studied by McGillivray and Silver [1] in the Economic Order Quantity (EOQ) context. Later, Parlar and Goyal [19] and Khouja and Mehrez and Rabinowitz [18] gave single-period formulations for an inventory system with two substitutable products independently of each other. In [21], Parlar proposed a Markov Decision Process model to find the optimal ordering policies for perishable and substitutable products from the point of view of one retailer. Parlar's study in [25] is a game theoretic analysis of the inventory control under substitutable demand. He modeled the two-product single period problem as a two-person nonzero-sum game and showed that there exists a unique Nash equilibrium.

Study on multi-echelon systems are much less compared to those on single location systems. The determination of optimal policies and the problems related to a multi-echelon systems are, to some extent, dealt by Veinott and Wagner [26] and Veinott[31].Sivazlian [27] discussed the stationary characteristics of a multi commodity single period inventory system. The terms multi-echelon or multi-level production distribution network and also synonymous with such networks (supply chain) when on items move through more than one steps before reaching the final customer. Inventory exist throughout the supply chain in various form for various reasons. At any manufacturing point they may exist as raw – materials, work-in process or finished goods.

The main objective for a multi-echelon inventory model is to coordinate the inventories at the various echelons so as to minimize the total cost associated with the entire multi-echelon inventory system. This is a natural objective for a fully integrated corporation that operates the entire system. It might also be a suitable objective when certain echelons are managed by either the suppliers or the retailers of the company. Multi-echelon inventory system has been studied by many researchers and its applications in supply chain management has proved worthy in recent literature.

As supply chains integrates many operators in the network and optimize the total cost involved without compromising as customer service efficiency. The first quantitative analysis in inventory studies started with the work of Harris [13]. Clark and Scarf [8] had put forward the multi-echelon inventory first. They analyzed a N-echelon pipelining system without considering a lot size. One of the oldest papers in the field of continuous review multi-echelon inventory system is written by Sherbrooke in 1968. Hadley and Whitin [12], Naddor [24] analyses various inventory Systems. HP's (Hawlett Packard) Strategic Planning and Modeling (SPaM) group initiated this kind of research in 1977.

Sivazlian and Stanfel [27] analyzed a two commodity single period inventory system. Kalpakam and Arivarignan [15] analyzed a multi-item inventory model with renewal demands under a joint replenishment policy. They assumed instantaneous supply of items and obtain various operational characteristics and also an expression for the long run total expected cost rate. Krishnamoorthy et al., [16] analyzed a two commodity continuous review inventory system with zero lead time. A two commodity problem with Markov shift in demand for the type of commodity required, is considered by Krishna moorthy and Varghese [17]. They obtain a characterization for limiting probability distribution to be uniform. Associated optimization problems were discussed in all these cases. However in all these cases zero lead time is assumed.

A Complete review on multi echelon inventory model was provided by Benito M. Beamon [6]. Sven Axsater [5] proposed an approximate model of inventory structure in SC. He assumed (S-1, S) polices in the Depot-Base systems for repairable items in the American Air Force and could approximate the average inventory and stock out level in bases.

Anbazhagan and Arivarignan [2,3] have analyzed two commodity inventory system under various ordering policies. Yadavalli et al., [32] have analyzed a model with joint ordering policy and varying order quantities. Yadavalli et al., [33] have considered a two commodity substitutable inventory system with Poisson demands and arbitrarily distributed lead time.

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In a very recent paper, Anbazhagan et al. [4] considered analysis of two commodity inventory system with compliment for bulk demand in which, one of the items for the major item, with random lead time but instantaneous replenishment for the gift item are considered. The lost sales for major item is also assumed when the items are out of stock. The above model is studied only at single location (Lower echelon). We extend the same in to multi-echelon structure (Supply Chain). The rest of the paper is organized as follows. The model formulation is described in section 2, along with some important notations used in the paper. In section 3, steady state analysis are done: Section 4 deals with the derivation of operating characteristics of the system. In section 5, the cost analysis for the operation. Section 6 concludes the paper.

2. Model

2.1. The problem description

The inventory control system considered in this paper is defined as follows. Two Substitutable finished products (A and B) are supplied from manufacturer to supplier which adopts (0,M) replenishment policy then the product is supplied to retailer who adopts (si,Si) policy. The demand at retailer node follows an independent Poisson distribution with rate λ_1 for one product A and λ_2 for product B. Also the common direct demand for both product at distributor follows independent Poisson process with rate λ_d . When the inventory of one of the product reaches zero the demand for the product is substitutable with the other product with probability p and similar argument for another product with probability q so that $p + q = 1$. The demands that occur when both the products are out of stock are assumed to be lost. The replacement of item in terms of product is made from supplier to retailer is administrated with exponential distribution having parameter $\mu > 0$. The maximum inventory level at retailer node for product A and B are S_i , and the recorder point is s_i and the ordering quantity is $Q (=S_i - s_i)$ items. The maximum inventory at supplier is $M (=nQ)$.

2.2. Notations and variables

We use the following notations and variables for the analysis of the paper.

Notations/ variable	Used For
$[C]_{i,j}$	The element of sub matrix at $(i,j)^{th}$ position of C
0	Zero matrix
λ_1, λ_2	Mean demand rate for products A and B at retailer node.
λ_d	Mean demand rate for both products at distributor.
μ	Mean replacement rate for both products A and B at retailer
S_1, S_2	Maximum inventory level products A and B at retailer
s_1, s_2	reorder level for products A and B at retailer
M	Maximum inventory level for both products A and B at supplier
H_1	Holding cost per item for product A at retailer
H_2	Holding cost per item for product B at retailer

3. Analysis

Let $I_1(t)$ and $I_2(t)$ denote the on hand Inventory levels of product A and B respectively at retailer and $I_d(t)$ denote the on hand inventory level of both products at supplier at time $t+$.

We define $I(t) = \{(I_1(t), I_2(t), I_d(t)) : t \geq 0\}$ as Markov process with state space of the initial state.

The infinitesimal generator matrix of this process $C = (a(i, j, k : l, m, n))_{(i, j, k)(l, m, n) \in E}$ can be obtained from the following arguments.

- The arrival of a demand for perishable product A at retailer make a state transition in the Markov process from (i, j, k) to $(i-1, j, k)$ with the intensity of transition $\lambda_1 + i\gamma > 0, i \neq 0$.
- The arrival of a demand for product B at retailer make a state transition in the Markov process from (i, j, k) to $(i, j-1, k)$ with the intensity of transition $\lambda_2 + j\gamma > 0, j \neq 0$.
- When the inventory level of perishable product A is zero, then the arrival of a demand for product A at retailer make a state transition in the Markov process from $(0, j, k)$ to $(0, j-1, k)$ with the intensity of transition $p\lambda_1 + \lambda_2 + j\gamma > 0$.
- When the inventory level of product B is zero, then the arrival of a demand for product B at retailer make a state transition in the Markov process from $(i, 0, k)$ to $(i-1, 0, k)$ with the intensity of transition $\lambda_1 + i\gamma + q\lambda_2 > 0$.
- The arrival of a direct demand for both products at distributor makes a state transition in the Markov process from (i, j, k) to $(i, j, k-Q)$ with the intensity of transition $\lambda_d > 0$.
- The replacement of inventory at retailer make a state transition in the Markov process from (i, j, k) to $(i+Q, j, k-Q)$ or (i, j, Q) to $(i, j+Q, k-Q)$ with the intensity of transition $\mu > 0$.

The infinitesimal generator C is given by

$$C = \begin{bmatrix} A & B & O & \cdots & O & O \\ O & A & B & \cdots & O & O \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ O & O & O & \cdots & A & B \\ B & O & O & \cdots & O & A \end{bmatrix}$$

The sub matrices A and B are given by

$$[A]_{m \times n} = \begin{cases} A_1 & n = m, & m = s_1 + 1, s_1 + 2, \dots, S_1, \\ A_2 & n = m - 1, & m = S_1, S_1 - 1, \dots, 1, \\ A_3 & n = m, & m = s_1, s_1 - 2, \dots, 1, \\ A_4 & n = m, & m = 0, \\ 0 & \text{otherwise} \end{cases}$$

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$$[B]_{m \times n} = \begin{cases} B_1 & m = n & m = S_1 + 1, S_1 - 1, \dots, 1, 0 \\ B_2 & m = n + Q & m = s_1, s_1 - 1, \dots, 1, 0, \\ 0 & \text{otherwise} \end{cases}$$

where

$$[A_1]_{m \times n} = \begin{cases} \lambda_1 + m\gamma_1 & n = m - 1 & m = S_1, S_1 - 1, \dots, S_1 \\ -(\lambda_1 + m\gamma_1 + \lambda_2 + n\gamma_2 + \lambda_d) & n = m, & m = s_1 + 1, s_1 + 2, \dots, S_1 \\ -(\lambda_1 + m\gamma_1 + \lambda_2 + n\gamma_2 + \lambda_d + \mu) & n = m, & m = 1, 2, \dots, s_1 \\ -(P(\lambda_1 + m\gamma_1) + \lambda_2 + \mu + \lambda_d) & n = m, & m = 0, \\ 0 & \text{otherwise} \end{cases}$$

$$[A_2]_{m \times n} = \begin{cases} \lambda_2 + n\gamma_2 & n = m & m = S_1, S_1 - 1, \dots, S_1 \\ P(\lambda_1 + m\gamma_1 + \lambda_2 + n\gamma_2) & n = m & m = 0 \\ 0 & \text{otherwise} \end{cases}$$

$$[A_3]_{m \times n} = \begin{cases} \lambda_1 + m\gamma_1 & n = m - 1 & m = S_1, S_1 - 1, \dots, S_1 \\ -(\lambda_1 + m\gamma_1 + \lambda_2 + n\gamma_2 + \mu + \lambda_d) & n = m, & m = s_1 + 1, s_1 + 2, \dots, S_1 \\ -(\lambda_1 + m\gamma_1 + \lambda_2 + n\gamma_2 + 2\mu + \lambda_d) & n = m, & m = 1, 2, \dots, s_1 \\ -(P\lambda_1 + m\gamma_1 + \lambda_2 + n\gamma_2 + 2\mu + \lambda_d) & n = m, & m = 0, \\ 0 & \text{otherwise} \end{cases}$$

$$[A_4]_{m \times n} = \begin{cases} \lambda_1 + m\gamma_1 & n = m - 1 & m = S_1, S_1 - 1, \dots, 1 \\ -(\lambda_1 + m\gamma_1 + q\lambda_2 + m\gamma_2 + \mu) & n = m, & m = s_1 + 1, s_1 + 2, \dots, S_1 \\ -(\lambda_1 + m\gamma_1 + q\lambda_2 + m\gamma_2 + 2\mu) & n = m, & m = 1, 2, \dots, s_1 \\ -2\mu & n = m, & m = 0, \\ 0 & \text{otherwise} \end{cases}$$

$$[B_1]_{m \times n} = \begin{cases} \lambda_d & m = n & m = S_1, S_1 - 1, \dots, 0 \\ 0 & \text{otherwise} \end{cases}$$

and

$$[B_2]_{m \times n} = \begin{cases} \mu & m = n + Q & m = S_1, S_1 - 1, \dots, 0 \\ 0 & \text{otherwise} \end{cases}$$

3.1. Steady state analysis

The structure of the infinitesimal matrix C, reveals that the state space E of the Markov process $\{I(t): t \geq 0\}$ is finite and irreducible. Let the limiting probability distribution of the inventory level process be

$$\prod_{i,j}^k = \lim_{t \rightarrow \infty} \Pr\{(I_1(t), I_2(t), I_d(t) = (i, j, k))\} \rightarrow \infty$$

where $\prod_{i,j}^k$ the steady state probability that the system be in state (i, j, k) .
 Let $\prod_{i,j}^k = \left\{ \prod_{i,j}^{nQ} \prod_{i,j}^{(n-1)Q} \dots \prod_{i,j}^{Qk} \right\}$ denote the steady state probability distribution. For each (i, j, k) , $\prod_{i,j}^k$ be obtained by solving the matrix equation $\prod C = 0$.
 By solving the above system of equations, together with normalizing condition $\sum_{i,j,k} \prod_{i,j}^k = 1$, the steady probability of all the system states are obtained.

4. Operating characteristic

In this section we derive some important system performance measures.

Average inventory level

The event I_1 , I_2 denote the average inventory level for products A and B respectively at retailer node and I_d denote the average inventory level at distributor node

$$\begin{aligned} \text{(i)} \quad I_1 &= \sum_{k=Q}^{nQ} \sum_{j=0}^{S_2} \sum_{i=0}^{S_1} i \cdot \prod_{i,j}^k \\ \text{(ii)} \quad I_2 &= \sum_{k=Q}^{nQ} \sum_{j=0}^{S_2} \sum_{i=0}^{S_1} j \cdot \prod_{i,j}^k \\ \text{(iii)} \quad I_d &= \sum_{k=Q}^{nQ} \sum_{j=0}^{S_2} \sum_{i=0}^{S_1} k \cdot \prod_{i,j}^k \end{aligned}$$

Mean reorder rate

Let R_1 , R_2 denote the mean reorder rate for products A and B, at retailer and R_d denote the mean reorder rate for products at distributor respectively,

$$\begin{aligned} \text{(i)} \quad R_1 &= (\lambda_1 + (s_1 + 1)\gamma_1) \sum_{k=Q}^{nQ} \sum_{j=0}^{S_2} \prod_{s_1+1,j}^k \\ \text{(ii)} \quad R_2 &= (\lambda_2 + (s_2 + 1)\gamma_2) \sum_{k=Q}^{nQ} \sum_{j=0}^{S_2} \prod_{s_2+1,j}^k \\ \text{(iii)} \quad R_d &= (\mu + \lambda_d) \sum_{i=0}^{S_1} \sum_{j=0}^{S_2} \prod_{i,j}^Q \end{aligned}$$

Shortage rate

Shortage occur at retailer only for main product. Let S_r be the shortage rate at retailer for main product, then

$$\text{(i)} \quad S_r = (\lambda_1 + \lambda_2) \sum_{k=Q}^{nQ} \prod_{0,0}^k$$

5. Cost analysis

In this section we impose a cost structure for the proposed model and analyze it by the criteria of minimization of long run total expected cost per unit time. The long run expected cost rate $TC(s_1, s_2, Q)$ is given by

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$$TC(s_1, s_2, Q) = I_1 \cdot H_1 + I_2 \cdot H_2 + I_d \cdot H_d + R_1 \cdot O_1 + R_2 \cdot O_2 + R_d \cdot O_d + S_r \cdot T_r$$

Although we have not proved analytically the convexity of the cost function $TC(s_1, s_2, Q)$ we try to do number of numerical examples to find the optimal cost.

6. Conclusion

This paper presents a sub suitable inventory system in supply chain with three independent demands. The model is analyzed within the framework of Markov processes. Joint probability distribution of inventory levels at DC and Retailer for both products are computed in the steady state. Various system performance measures are derived and the long-run expected cost is calculated.

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