Intern. J. Fuzzy Mathematical Archive
Vol. 15, No. 2, 2018, 253-262
ISSN: 2320 –3242 (P), 2320 –3250 (online)
Published on 30 April 2018
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/ijfma.v15n2a17

International Journal of
Fuzzy Mathematical
Archive

Knot Matrix

R. Selvarani

K.L.N. College of Engineering, Pottapalayam, Sivagangai District
Sivagangai-630612. Email: selvaklnce@gmail.com

Received 6 March 2018; accepted 21 April 2018

Abstract. In this paper, we define a knot matrix from knot diagram and derive an algorithm for knot matrix. Also, we define a signed addition modulo 2, which satisfied knotable matrix.

Keywords: Knot diagram, Knot matrix and signed addition modulo 2

AMS Mathematics Subject Classification (2010): 57Q45

1. Introduction

Brauer [1] introduced algebras, known as Brauer's algebras, in connection with the issue of decay of a tensor item representation into irreducible ones. These algebras have a basis consisting of undirected graphs. Wenzl [2] obtained the structure of these algebras D_{n+1} by making use of conditional expectations and by an inductive procedure from the structure of D_{n-1} and D_n. Parvathi and Kamaraj [3] introduced signed Brauer's algebra, which has a basis consisting of signed diagrams. Kamaraj and Mangayarkarasi [4] introduced knot diagrams using Brauer graphs without horizontal edges and also used two types of knots only. Kamaraj and Selvarani [5] introduced knot in Z*. Kamaraj and Selvarani [6] introduced an edge crossable matrix of order nxn. We are motivated to introduce a nxn matrices in {0,1,-1}. We call them knot matrix.

2. Preliminaries

2.1. Brauer's algebras

Definition 1.1.1. [1, 2] A graph has 2n vertices and n edges, the 2n vertices are arranged in two lines of n vertices each point has exactly one degree. The collection of all this type of diagrams is called a **Brauer diagram** (or) **Brauer graph**. It is denoted by D_n.

Example 1.1.2.

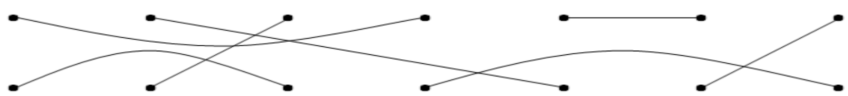


Figure 1:

Definition 1.1.3. [1,2] Let D_n be a Brauer diagram. Let D_{n1}, D_{n2} be two diagrams of D_n.

Then the composition of $D_{n1} \circ D_{n2}$ is defined as

(i). D_{n1} is arranged in the upper diagram.

(ii). D_{n2} is arranged in the lower diagram.

(iii). Lower points of D_{n1} is joined to the corresponding upper points of D_{n2}

(iv). Remove the cycles after the joining

(v). we get the new diagram. It is denoted by $D_{n1} \circ D_{n2}$

The multiplication of D_{n1} and D_{n2} is defined by setting

$D_{n1}D_{n2} = \delta^{n(D_{n1}, D_{n2})} D_{n1} \circ D_{n2}$

Remark 1.1.4. $n(D_{n1}, D_{n2})$ means that number of removing the closed cycles in $D_{n1}D_{n2}$.

Definition 1.1.5. [1,2] Let F be field with $\delta \in F$. The **Brauer algebra** $D_n(\delta)$ is an associative *F*-algebra with a linear basis which consists of all Brauer elements of diagrams.

Result 1.1.6. The dimension of $D_n(\delta)$= (2n-1)(2n-3)…3.1

1.2. Signed Brauer's Algebras

Definition 1.2.1. A Brauer graph which has directions is called a **signed Brauer graph**. It is denoted by $\overline{D}_n$.

Example 1.2.2. In $\overline{D}_8$

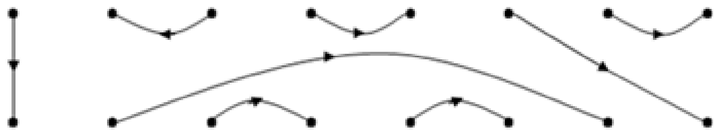


Figure 2:

Remark 1.2.3. An edge having $\downarrow$ is called a positive vertical edge. An edge having $\rightarrow$ is called a positive horizontal edge . A positive horizontal edge (or)vertical edge is called positive sign. An edge having $\uparrow$ is called a negative vertical edge. An edge having $\leftarrow$ is called a negative horizontal edge. A negative horizontal edge (or)negative vertical edge is called negative sign.

1.3. Knot graph [4]

Definition 1.3.1. Let S_n be the symmetric group of order n and let $\pi \in S_n$. Then π can be represented as a graph which is an element of Brauer graph. Let $E(\pi)$ denote the set of all edges in the graph representation of π. We use the symbol e_i to represent the edges $(i, \pi(i))$, $\forall\, i = 1,2,\cdots,n$

Let $E(\pi) = \{e_i = (i, \pi(i));\ i = 1,2,\ldots n\}$

A_π is denoted as $A_\pi \subseteq E(\pi) \times E(\pi)$ where

$A_\pi = \{a_{ij} = (e_{i,}\ e_j) : i \le j,\ e_i, e_j \in E(\pi)\}$

A_π can be written as $\{a_{11,}\ a_{12,}\ a_{13}, \ldots a_{1n,} a_{22,} a_{23}, a_{24}, \ldots a_{2n} \ldots a_{n-1n,}\ a_{nn}\}$

$B_\pi = \{b_{ij} = a_{ij} \in A_\pi : \pi(i) > \pi(j)\}$

Definition 1.3.2. Let S_n be the symmetric group of order n and $\pi \in S_n$. A **knot graph** of order n is a special graph which is defined from π as follows: π can be represented by a graph, which is an element of Brauer graph.

(i) If $i < j$ and $\pi(i) < \pi(j)$, then the edges are drawn in usual Brauer graph.

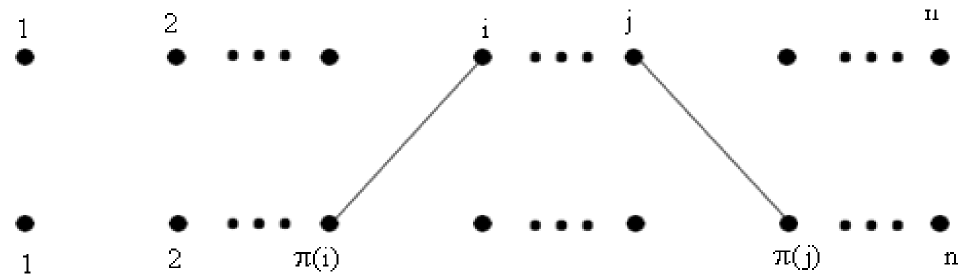


Figure 3:

(ii) If $i < j$ and $\pi(i) > \pi(j)$, then the edges are drawn in two cases as shown below

In case 1, $(i, \pi(i))$ is the higher edge and $(j, \pi(j))$ is the lower edge. It can also be said that the edge $(j, \pi(j))$ is lower than the edge $(i, \pi(i))$.

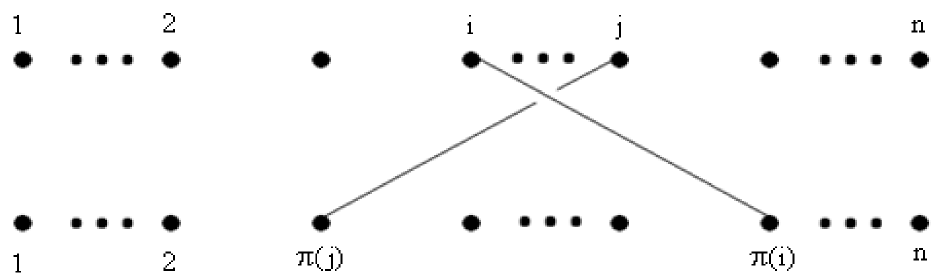


Figure 4:

In case 2, the edge $(j, \pi(j))$ is higher than $(i, \pi(i))$ or else $(i, \pi(i))$ is lower than $(j, \pi(j))$

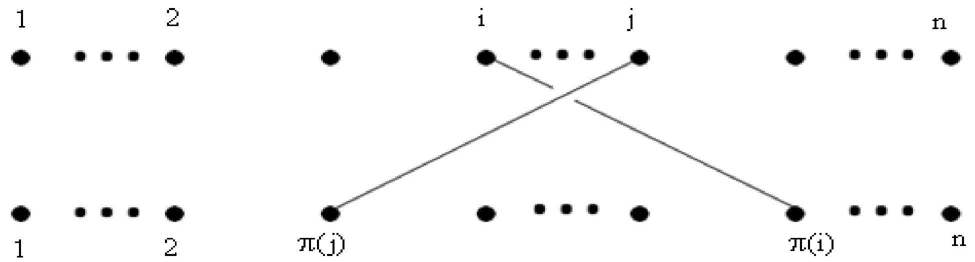


Figure 5:

The above graph is called a knot graph of order n.

Definition 1.3.3. A knot mapping $f_\pi : A_\pi \to \{-1, 0, 1\}$

$$f_\pi(e_i, e_j) = \begin{cases} 0 & \text{if } \pi(i) < \pi(j) \\ 1 & \text{if } \pi(j) > \pi(i) \quad \& \, e_i \text{ is higher than } e_j \\ -1 & \text{if } \pi(j) > \pi(i) \quad \& \, e_i \text{ is lower than } e_j \end{cases}$$

Definition 1.3.4. $|B_\pi|$ is called the number of knot in π.

Result 1.3.5. The number of knot mapping of π is $2^{|B_\pi|}$

Example 1.3.6. The number of knot graph of S_2

Let $\pi_1, \pi_2 \in S_2$

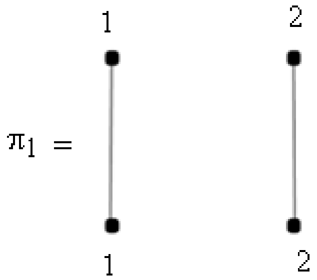


Figure 6:

$|B_{\pi_1}| = 0$

$|B_{\pi_2}| = 1$, therefore the number of knot of π_2 is $2^1 = 2$

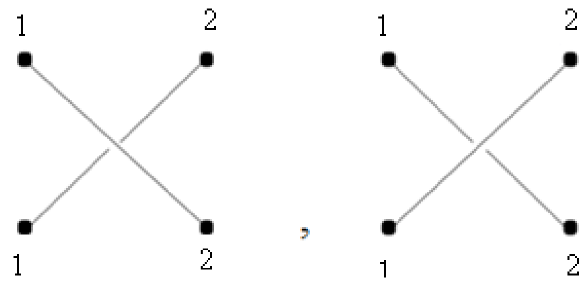
$\pi_2 =$ 

Figure 7:

Therefore the number of knot graph of S_2

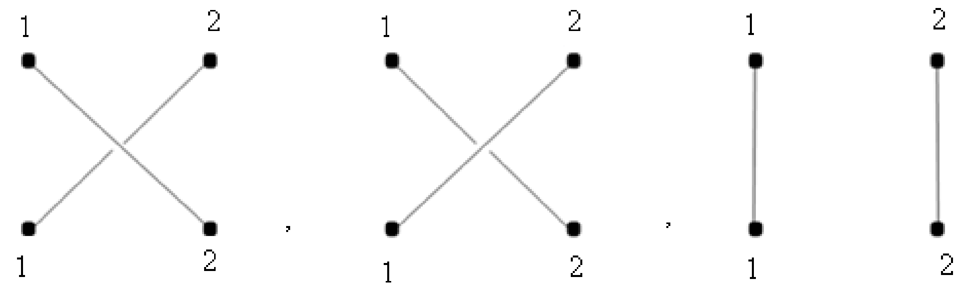


Figure 8:

1.4. Generalized Knot symmetric algebras in Z*[5]

$S_\pi = \{(s_1, s_2, \cdots s_\beta) : s_i = (1,-1)^k \text{ (or)} (-1, 1)^l\}$ where k and l are integers

Definition 1.4.1. If $(s_1, s_2, \cdots s_\beta) \in S_\pi$, then s_i is called **knots in π.**

Definition 1.4.2. If $s_i = (1, -1)^k$, then s_i is called **Type I knots in π**.

Definition 1.4.3. If $s_i = (-1, 1)^l$, then s_i is called **Type II knots in π.**

Definition 1.4.4. If $s_i = (1,-1)^k \text{ (or)} (-1, 1)^l$ and $k = 1 \text{ (or) } l = 1$, then s_i is called **knot in π.**

1.5. Edge crossing matrix [6]

Definition 1.5.1. If $i < j$ and $\pi(i) > \pi(j)$, then e_i **crosses** e_j.

Otherwise, we say that e_i does not cross e_j.

Definition 1.5.2. $f_\pi : A_\pi \to \{0, 1\}$ is defined as

$$f_\pi(a_{ij}) = \begin{cases} 0, & \text{if } e_i \text{ does not cross } e_j \\ 1, & \text{if } e_i \text{ crosses } e_j \end{cases}; \text{ where } \pi \in S_n$$

Definition 1.5.3. M_π is defined as

$M_\pi = (f_\pi(a_{ij}))_{i,j=1,2,...,n}$; $\pi \in S_n$; M_π is called an **edge crossing matrix**

6. Knot matrix

6.1.1. Types of Knots

Let $\pi \in S_n$ and $a_{ij} = (e_i, e_j)$

Case 1 : $f(a_{ij}) = 0$

Case 2:

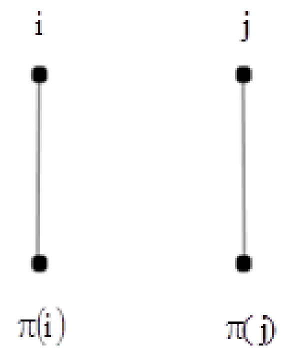


Figure 8:

If $f(a_{ij}) = +1$ is called a **Positive Knot** in π

Example 6.1.2. Positive Knot in π

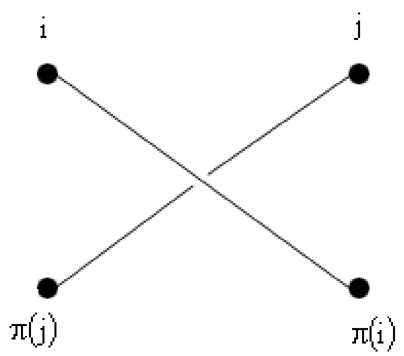


Figure 9:

Case 3: If $f(a_{ij}) = -1$ is called a **Negative Knot** in π

Example 6.1.3. Negative Knot in π

Knot Matrix

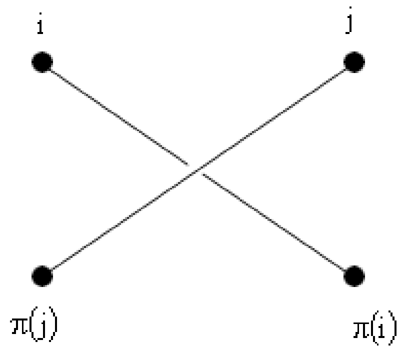


Figure 10:

6.1.2. Knot mapping

Define $f : A_\pi \to \{0, +1, -1\}$ such that

$$f(a_{ij}) = \begin{cases} 0 & \text{no knot between } e_i \text{ and } e_j \\ +1 & e_i \text{is upper than } e_j \text{(i.e., Positive knot)} \\ -1 & e_i \text{is upper than } e_j \text{(i.e.. Negative knot)} \end{cases}$$

6.1.3. Knot matrix

Let $\pi \in S_n$

Define $M_\pi^f = \left(f_\pi(a_{ij})\right)_{i,j=1,2...n}$

Definition 6.1.4.

$P(e_i) = \{e_j : e_i \text{ is upper then } e_j\}$

$|p(e_i)|$ *is called number of positive knot of* e_i

Definition 6.1.5.

$N(e_i) = \{e_j : e_i \text{ is lower then } e_j\}$

$|N(e_i)|$ *is called number of negative knot of* e_i

Properties 6.1.6.

- Sum of positive values of i^{th} row = No of positive knot of $e_{i.}$
- Sum of Negative values of i^{th} column = No of negative knot of $e_{i.}$
- Knot matrix is a Skew Symmetric.
- Det(A) = 0.
- Sum of the trace value is zero.
- All the eigen values are zero.

Example 6.1.7.

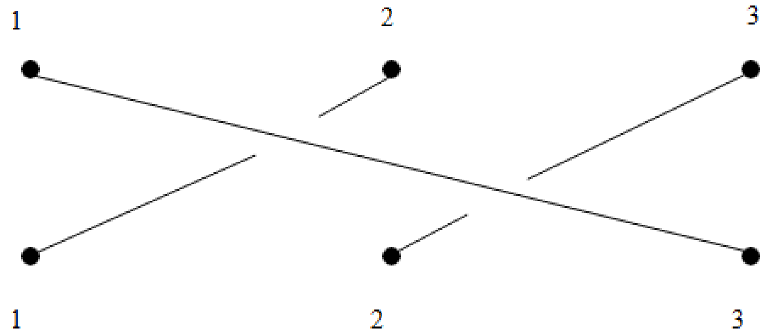


Figure 11:

In S_3 , $E(\pi) = \{(1,3)(1,2)\}$

$A_\pi = E(\pi) \times E(\pi)$

$A_\pi = \{(e_1,e_1),(e_1,e_2),(e_1,e_3),(e_2,e_1),(e_2,e_2),(e_2,e_3),(e_3,e_1),(e_3,e_2),(e_3,e_3)\}$

$$M_\pi = \begin{pmatrix} 0 & +1 & +1 \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

6.1.8. Algorithm to derive a Knot matrix from a given knot diagram

Let $A_\pi = (a_{ij})$; π *is a knot diagram.*

1. Set $i \leftarrow 1, \quad j \leftarrow 1$
2. If $i = j$, $a_{ij} \leftarrow 0$; otherwise $a_{ij} = \begin{cases} +1 & \text{if } e_i \text{ is upper than } e_j \\ -1 & \text{if } e_i \text{ is lower than } e_j \\ 0 & \text{if no knot between } e_i \text{ and } e_j \end{cases}$
3. If $j \le n,\ j \leftarrow j+1$ and go to step to2 ; if $i \le n$, *then* $i \leftarrow i+1,\ j \leftarrow 1$ and
   go to step 2; Otherwise go to step 4.
4. **Stop.**

Definition 6.1.9. If $M_\pi : \pi \in S_n$ be a square matrix, then M_π is called a Knotable matrix

Remark 6.1.10. Any Skew Symmetric in {0, +1, -1} is Knotable Matrix.

Result 6.1.11.
(i)The addition of two knotable matrix need not be knotable matrix.

Example 6.1.12.

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ -1 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 1 \\ -2 & -1 & 0 \end{pmatrix}$$

(ii)The product of two knotable matrix need not be knotable matrix.

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ -1 & -1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

6.2. Signed addition modulo 2

We define a binary operation * called as signed addition modulo 2 on {0, -1, 1} as follows.

*	0	1	-1
0	0	1	-1
1	1	0	0
-1	-1	0	0

Definition 6.2.1.

If $A = (a_{ij})_{n\times n}$ and $B = (b_{ij})_{n\times n}$, then define $A * B = (c_{ij})_{n\times n}$; where $c_{ij} = a_{ij} * b_{ij}$

Definition 6.2.2. Let $A = (a_{ij}) \in M_{\pi}$, $B = (b_{ij}) \in M_{\sigma}$, $C = (c_{ij}) = A * B \in M_{\pi * \sigma}$

Remark 6.2.3.

If $a, b \in \{0, -1, 1\}$; $a \neq b$; $a \neq 0$ and $b \neq 0$ then $a = -b$.

Theorem 6.2.4. If A and B is a knotable Matrix then A*B is also a knotable Matrix.

Proof:

Claim 1: $C_{ii} = 0$

By definition $C_{ii} = a_{ii} * b_{ii}$

= 0(since A and B are skew symmetric matrix)

Claim 2: $C_{ij} + C_{ji} = 0$

It is enough to prove that $a_{ij} * b_{ij} + a_{ji} * b_{ji} = 0$

Case 1:

$a_{ij} = b_{ij} \neq 0$

$a_{ij} * b_{ij} = a_{ij} * a_{ij} = 0$

Similarly

$a_{ji} * b_{ji} = a_{ji} * a_{ji} = 0$

That is

$a_{ij} * b_{ij} + a_{ji} * b_{ji} = 0$+0=0

Case 2:

$a_{ij} \neq b_{ij}$ & $a_{ij} \neq 0$ & $b_{ij} \neq 0$

By remark2

$a_{ij} = -b_{ij}$

$a_{ij} * b_{ij} = a_{ij} * (-a_{ij}) = 0$

Similarly

$a_{ji} * b_{ji} = a_{ji} * (-a_{ji}) = 0$

That is

$a_{ij} * b_{ij} + a_{ji} * b_{ji} = 0$+0=0

Case 3:

$a_{ij} \neq b_{ij}$ & $a_{ij} \neq 0$ & $b_{ij} = 0$

$a_{ij} * b_{ij} = a_{ij} * 0 = a_{ij}$

Similarly

$a_{ji} * b_{ji} = a_{ji} * 0 = a_{ji}$

That is

$a_{ij} * b_{ij} + a_{ji} * b_{ji} = a_{ij} * a_{ji} = 0$

Case 4:

$a_{ij} \neq b_{ij}$ & $a_{ij} = 0$ & $b_{ij} \neq 0$

The proof is similar to the previous case.
Hence A* B is knotable matrix.

Result 6.2.5. Signed addition modulo 2 is a knotable matrix.

REFERENCES

1. R.Brauer, Algebras which are connected with semi-simple continuous groups, *Ann. Math.*, 38 (1937) 854–872 .
2. H.Wenzl, The structure of Brauer's centralizer algebra, *Ann. Math.*, 128 (1988) 173–193.
3. M.Parvathi and M.Kamaraj, Signed Brauer's algebras, *Commun. Algebra.*, 26(3) (1998) 839–855.
4. M.Kamaraj and R.Mangayarkarasi, Knot symmetric algebras, *Res. J. Pure Algebra.*, 1(6) (2011) 141–151.
5. M.Kamaraj and R.Selvarani, Generalized knot symmetric algebras in Z*, *International Journal of Scientific and Engineering Research*, 6(3) (2015) 73- 80.
6. M.Kamaraj and R.Selvarani, Edge crossing matrix, *Global Journal of Pure and Applied Mathematics*, 12(2) (2016) 61-64.