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A Note on Characterization of Intuitionistic Fuzzy Bi-Ideals of Near Rings

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Abstract. In this paper, we introduce the notion of intuitionistic fuzzy bi-ideals of near-rings. We give some characterizations of intuitionistic fuzzy bi-ideals of near-rings.

Keywords: Near-rings, Bi-ideals, Fuzzy bi-ideals, Intuitionistic fuzzy set, Intuitionistic fuzzy subring, Intuitionistic fuzzy ideal, Intuitionistic fuzzy bi-ideal.

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1. Introduction

The notion of intuitionistic fuzzy set (IFS) was introduced by Atanassov [2] as a generalization of notion of fuzzy sets. The concept of near-rings was introduced by Pilz [9]and that of quasi-ideal in near ring was introduced by Yakabe [12]. The notion of bi-ideals was introduced by Chelvam and Ganesan [4].

In this paper we study the intuitionistic fuzzification of the notion of bi-ideals in near-rings. We give characterizations of intuitionistic fuzzy bi-ideals in near-rings.

A ***near-ring*** is a non empty set N with two binary operations "+" and "." such that

(i) (N,+) is a group not necessarily abelian
(ii) (N, .) is a semi group
(iii) $(x + y).z = x.z + y.z$, for all $x, y, z \in$ N.

Precisely speaking it is a right near-ring because it satisfies the right distributive law. If the condition (iii) is replace by $z(x + y) = z.x + z.y$ for all $x , y , z \in$N, then it is called left near- ring. We denote xy instead of $x.y$. A near-ring N is called ***zero symmetric*** if $x.0 = 0$ for all $x \in$N.

Given two subsets A and B of N, the product AB is defined as

$$AB = \{ab | a\in A, b\in B\}$$

A subgroup S of (N, +) is called ***left (right) N-subgroup*** of N if NS $\subseteq$ S (SN$\subseteq$S). A subgroup M of (N, +) is called ***subnear-ring*** of N if MM $\subseteq$ M. A subnear-ring M is called ***invariant*** in N if MN $\subseteq$ NM $\subseteq$ M.

2. Preliminaries

Definition 2.1. An ***ideal*** of a near-ring N is a subset I of N such that

(i) (I , +) is normal subgroup of (N , +)

(ii) I N $\subseteq$ I

(iii) $y(x+i) - yx \in$ I for all $x, y \in$ N and $i \in$ I

Note that I is right ideal of N if I satisfies (i) and (ii), and I is left ideal of N if I satisfies (i) and (iii).

Definition 2.2. A subgroup Q of N is called a ***quasi-ideal*** of N if QN$\cap$NQ$\cap$N*Q$\subseteq$Q.

Definition 2.3. A subgroup B of N is called a ***bi-ideal*** of N if BNB$\cap$(BN)*B$\subseteq$B.

Definition 2.4. Let X be a non-empty set. A mapping μ :X→ [0, 1] is a fuzzy set in X. The complement of μ, denoted by μ^c, is the ***fuzzy set*** in X given by $\mu^c(x) = 1 - \mu(x)$ for all $x \in$X. For any I $\subseteq$ X, χ_I denote the characteristic function of I.

Definition 2.5. For any fuzzy set μ in X and $r \in [0, 1]$, we define two sets, U $(\mu, r) = \{x \in X \mid \mu(x) \geq r\}$ and L$(\mu, r) = \{x \in X \mid \mu(x) \leq r\}$, which are called an ***upper and lower r-level cut*** of μ respectively and can be used to the characterization of μ.

Definition 2.6. A fuzzy set μ in N is a ***fuzzy subnear-ring*** of N if for all $x, y \in$N,

(i) $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$ and

(ii) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$.

Definition 2.7. A fuzzy set μ in N is a ***fuzzy bi-ideal*** of N if for all $x, y \in$N,

(i) $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$ and

(ii) $\mu(xyz) \geq \min\{\mu(x), \mu(z)\}$ for all $x, y, z \in$N.

3. Intuitionistic fuzzy sets and bi-ideals

Definition 3.1. An ***intuitionistic fuzzy set*** A in a non-empty set X is an object having the formA=$\{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$, where the functions μ_A:X→ [0,1] and ν_A:X → [0, 1]denote the degree of membership and the degree of non-membership of each element $x \in$X to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in$X.

For the sake of simplicity, we shall use the symbol A = (μ_A, ν_A) for the intuitionistic fuzzy set A = $\{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$.

Definition 3.2 An intuitionistic fuzzy set A = (μ_A, ν_A) of a group (G, +) is said to be an ***intuitionistic fuzzy subgroup*** of G if for all $x,y \in$G

(i) $\mu_A(x + y) \geq min\{\mu_A(x), \mu_A(y)\}$

(ii) $\mu_A(-x) = \mu_A(x)$

(iii) $\nu_A(x + y) \leq max\{\nu_A(x), \nu_A(y)\}$

(iv) $\nu_A(-x) = \nu_A(x)$

Equivalently, $\mu_A(x - y) \geq min\{\mu_A(x), \mu_A(y)\}$ and $\nu_A(x - y) \leq max\{\nu_A(x), \nu_A(y)\}$ for all $x,y \in$ G.
Let A = (μ_A, ν_A) and B = (μ_B, ν_B) be two intuitionistic fuzzy subset of a near-ring N. We define the product of A and B as AB = (μ_{AB} , ν_{AB}). If S ⊆ N, then, we define the characteristic function χ_S on N is defined as

$$\chi_s(x) = \begin{cases} (1,0) \text{ if } x \in S \\ (0,1) \text{ if } x \in N\backslash S \end{cases}$$

The characteristic function on N is χ_N and χ_N (x) = (1, 0) for all $x \in$ N

Definition 3.3. Let A be an intuitionistic fuzzy set of a universe set X. Then ***(α,β)-cut*** of A is a crisp set $C_{(\alpha,\beta)}$ (A) of the intuitionistic fuzzy set A is given by $C_{(\alpha,\beta)}(A)=\{x:x\in X$ such that $\mu_A(x) \geq \alpha$, $\nu_A(x) \leq \beta\}$ where $\alpha,\beta\in[0,1]$ with $\alpha+\beta \leq 1$.

Definition 3.4. An intuitionistic fuzzy set A = (μ_A, ν_A) in N is an ***intuitionistic fuzzy subnear-ring*** of N if for all x, $y \in$ N,

(v) $\mu_A(x - y) \geq min\{\mu_A(x), \mu_A(y)\}$
(vi) $\mu_A(xy) \geq min\{\mu_A(x), \mu_A(y)\}$
(vii) $\nu_A(x - y) \leq max\{\nu_A(x), \nu_A(y)\}$
(viii) $\nu_A(xy) \leq max\{\nu_A(x), \nu_A(y)\}$.

Definition 3.5. An intuitionistic fuzzy set A = (μ_A, ν_A) in N is an ***intuitionistic fuzzy bi-ideal*** of N if for all x, y, $n \in$N,

(i) $\mu_A(x - y) \geq min\{\mu_A(x), \mu_A(y)\}$
(ii) $\mu_A(xny) \geq min\{\mu_A(x), \mu_A(y)\}$
(iii) $\nu_A(x - y) \leq max\{\nu_A(x), \nu_A(y)\}$
(iv) $\nu_A(xny) \leq max\{\nu_A(x), \nu_A(y)\}$.

Theorem 3.6. If A and B be two IFBI's of a near-ring N, then A ∩ B is IFBI of near-ring N.
Proof. Let A= (μ_A,ν_A) and B = (μ_B,ν_B) be two IFBI's of a near-ring N. Let x,y∈ A ∩ B be any element.

Then $\mu_{A\cap B}(x-y) = Min\{\mu_A(x-y) , \mu_B(x-y)\}$
$\geq Min\{Min\{\mu_A(x), \mu_A(y)\}, Min\{\mu_B(x), \mu_B(y)\}\}$
$= Min\{Min\{\mu_A(x), \mu_B(x)\}, Min\{\mu_A(y), \mu_B(y)\}\}$
$= Min\{\mu_{A\cap B}(x) , \mu_{A\cap B}(y)\}$

Thus $\mu_{A\cap B}(x-y) \geq Min\{\mu_{A\cap B}(x) , \mu_{A\cap B}(y)\}$
Similarly, we can show that $\nu_{A\cap B}(x-y) \leq Max\{\nu_{A\cap B}(x) , \nu_{A\cap B}(y)\}$
Next, let x , y ∈ A ∩ B and n ∈ N be any element, then
$\mu_{A\cap B}$ (xny) = $Min\{\mu_A(xny) , \mu_B(xny)\}$
$\geq Min\{Min\{\mu_A(x), \mu_A(y)\}, Min\{\mu_B(x), \mu_B(y)\}\}$
$= Min\{ Min\{\mu_A(x), \mu_B(x)\}, Min\{\mu_A(y), \mu_B(y)\}\}$
$= Min \{ \mu_{A\cap B} (x) , \mu_{A\cap B} (y)\}$

Thus $\mu_{A\cap B}$ (xny) $\geq Min \{ \mu_{A\cap B} (x) , \mu_{A\cap B} (y)\}$

Similarly, we can show that $\nu_{A\cap B}(xny) \leq Max\{\nu_{A\cap B}(x), \nu_{A\cap B}(y)\}$
Hence $A \cap B$ is IFBI of ring N.

Corollary 3.7. Intersection of an arbitrary family of IFBI's of a near-ring N is again a IFBI of N.

Theorem 3.8. Let A be IFLI and B be IFRI of a near-ring N, then $A \cap B$ is IFBI of near-ring N.

Proof. Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be two IFBI's of a near-ring N.
Let $x, y \in A \cap B$ be any element. Then

$$\begin{aligned}\mu_{A\cap B}(x-y) &= Min\{\mu_A(x-y), \mu_B(x-y)\}\\ &\geq Min\{Min\{\mu_A(x), \mu_A(y)\}, Min\{\mu_B(x), \mu_B(y)\}\}\\ &= Min\{Min\{\mu_A(x), \mu_B(x)\}, Min\{\mu_A(y), \mu_B(y)\}\}\\ &= Min\{\mu_{A\cap B}(x), \mu_{A\cap B}(y)\}\end{aligned}$$

Thus $\mu_{A\cap B}(x-y) \geq Min\{\mu_{A\cap B}(x), \mu_{A\cap B}(y)\}$
Similarly, we can show that $\nu_{A\cap B}(x-y) \leq Max\{\nu_{A\cap B}(x), \nu_{A\cap B}(y)\}$
Further, let $x, y \in A \cap B$ and $n \in N$, then

$$\mu_{A\cap B}(xny) = Min\{\mu_A(xny), \mu_B(xny)\} \quad (1)$$

But $\mu_A(xny) \geq \mu_A((xn)y) \geq \mu_A(y)$ and $\mu_B(xny) \geq \mu_B(x(ny)) \geq \mu_B(x)$ implies that

$$Min\{\mu_A(xny), \mu_B(xny)\} \geq Min\{\mu_A(y), \mu_B(x)\} \quad (2)$$

As $A \cap B \subseteq A$ and $A \cap B \subseteq B$. So $\mu_{A\cap B}(y) \leq \mu_A(y)$ and $\mu_{A\cap B}(x) \leq \mu_B(x)$

$$\Rightarrow Min\{\mu_A(y), \mu_B(x)\} \geq Min\{\mu_{A\cap B}(y), \mu_{A\cap B}(x)\} \quad (3)$$

From (1), (2) and (3), we get $\mu_{A\cap B}(xny) \geq Min\{\mu_{A\cap B}(y), \mu_{A\cap B}(x)\}$
Similarly, we can show that $\nu_{A\cap B}(xny) \leq Max\{\nu_{A\cap B}(y), \nu_{A\cap B}(x)\}$
Hence $A \cap B$ is IFBI of ring N.

Theorem 3.9. Let A be IFS of a near-ring N, then A is IFBI of N if $C_{\alpha,\beta}(A)$ is bi-ideal of N, for all $\alpha,\beta \in [0,1]$ with $\alpha + \beta \leq 1$, where $\mu_A(0) \geq \alpha$ and $\nu_A(0) \leq \beta$

Proof: Let A be IFBI of a near-ring N. Then by definition of (α, β) –cut of A, we have
$C_{\alpha,\beta}(A) = \{ x \in N : \mu_A(x) \geq \alpha, \nu_A(x) \leq \beta \}$
Since $\mu_A(0) \geq \alpha$, $\nu_A(0) \leq \beta \Rightarrow C_{\alpha,\beta}(A) \neq \emptyset$.
Let $x, y \in C_{\alpha,\beta}(A)$ be any elements, then
$\mu_A(x) \geq \alpha, \mu_A(y) \geq \alpha, \nu_A(x) \leq \beta., \nu_A(y) \leq \beta$
$\Rightarrow Min\{\mu_A(x), \mu_A(y)\} \geq \alpha$ and $Max\{\nu_A(x), \nu_A(y)\} \leq \beta$
Now $\mu_A(x-y) \geq Min\{\mu_A(x), \mu_A(y)\} \geq \alpha$ and $\nu_A(x-y) \leq Max\{\nu_A(x), \nu_A(y)\} \leq \beta$
Next, let $x, y \in C_{\alpha,\beta}(A)$ and $n \in N$ be any element. Then
$\mu_A(xny) \geq Min\{\mu_A(x), \mu_A(y)\} \geq \alpha$ and $\nu_A(xny) \geq Max\{\mu_A(x), \mu_A(y)\} \leq \beta$
$\Rightarrow xny \in C_{\alpha,\beta}(A)$. Hence $C_{\alpha,\beta}(A)$ is bi-ideal of N.

Theorem 3.10. If every bi-ideal of a near-ring N is a ideal of N, then every IFBI of N is IFI of N.

Proof. Let A be IFBI of N. Then by theorem (3.9), $C_{\alpha,\beta}(A)$ be bi-ideal of N, for all α, $\beta \in [0,1]$ with $\alpha + \beta \leq 1$, which implies that $C_{\alpha,\beta}(A)$ be ideal of N, for all α, $\beta \in [0,1]$ with $\alpha+\beta \leq 1$ and A is IFI of N.

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