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Symmetries and Similarity Reductions of (2+1)-Dimensional Equal Width Wave Equation

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Abstract. We have employed the symmetries and similarity reductions of (2+1)-dimensional equal width wave equation

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1. Introduction

A simple model equation is the Korteweg-de Vries (KdV) equation

$$v_t + 6vv_x + \delta v_{xxx} = 0 \tag{1.1}$$

which describe the long waves in shallow water. Its modified version is,

$$u_t - 6u^2 u_x + u_{xxx} = 0 \tag{1.2}$$

And again there is Miura transformation

$$v = u^2 + u_x \ , \tag{1.3}$$

between the KdV equation (1.1) and its modified version (1.2).

In 2002, Liu and Yang studied the bifurcation properties of generalized KdV equation (GKdVE)

$$U_t + au^n u_x + u_{xxx} = 0 \ , \ a \in \mathbb{R} \ , \ n \in Z^+ \tag{1.4}$$

Gungor and Winternitz transformed the Generalized Kadomtsev-Petviashvili Equation (GKPE)

$$(u_t + p(t)uu_x + q(t)u_{xxx})_x + \sigma(y,t)u_{yy} + a(y,t)u_y + b(y,t)u_{xy} + c(y,t)u_{xx} + e(y,t)u_x + f(y,t)u + h(y,t) \tag{1.5}$$

to its canonical form and established conditions on the coefficient functions under which (1.5) has an infinite dimensional symmetry group having a Kac-Moody-Virasoro structure.

In this chapter, we discuss the symmetry reductions of the (2+1)-dimensional Equal Width Wave equation as,

$$u_t + uu_x - \mu(u_{xxt} + u_{yyt}). \tag{1.6}$$

2. The symmetry group and lie algebra of equal width wave equation

If (1.6) is invariant under a one parameter Lie group of Point transformations (Bluman and Kumei , Olver)

$$x^* = x + \epsilon\, \xi(x, y, t; u) + O(\epsilon^2), \quad (1.7)$$
$$y^* = y + \epsilon\, \eta(x, y, t; u) + O(\epsilon^2), \quad (1.8)$$
$$t^* = t + \epsilon\, \tau(x, y, t; u) + O(\epsilon^2), \quad (1.9)$$
$$u^* = u + \epsilon\, \emptyset\, (x, y, t; u) + O(\epsilon^2), \quad (1.10)$$

Then the third prolongation $pr^3(v)$ of the corresponding Vector field

$$V=\varepsilon(x,y,t;u)\frac{\partial}{\partial x}+\eta(x,y,t;u)\frac{\partial}{\partial y}+\tau(x,y,t;u)\frac{\partial}{\partial t}+\phi(x,y,t;u)\frac{\partial}{\partial u} \quad (1.11)$$

Satisfies

$$pr^3(v)\Omega(x,y,t;u)|\Omega(x,y,t;u=0) = 0 \quad (1.12)$$

The determining equations are obtained from (1.12) and solved for the infinitesimals ε, η, τ and $\emptyset$. They are as follows

$$\varepsilon = k_1, \quad (1.13)$$
$$\eta = k_2, \quad (1.14)$$
$$\tau = k_3 + tk_4, \quad (1.15)$$
$$\emptyset = -uk_4. \quad (1.16)$$

At this stage, we construct the symmetry generators corresponding to each of the constants involved.

Totally there are four generators given by

$$V_1 = \partial_x ,$$
$$V_2 = \partial_y ,$$
$$V_3 = \partial_t,$$
$$V_4 = t\partial_t - u\partial_u \quad (1.17)$$

The symmetry generators found in Eq. (1.17) form a closed Lie Algebra whose commutation table is shown below.

Table 1: Commutator Table

$[v_i, v_j]$	V_1	V_2	V_3	V_4
V_1	0	0	0	0
V_2	0	0	0	0
V_3	0	0	0	V_3
V_4	0	0	0	0

The commutation relations of the Lie algebra, determined by V_1,V_2,V_3 and V_4 are shown in the above table.

3. Reductions of (2+1)-dimensional equal width wave equation by one-dimensional subalgebras

Case 1: $v_1 = \partial_x$

The characteristic equation associated with this generator is

$$\frac{dx}{1} = \frac{dy}{0} = \frac{dt}{0} = \frac{du}{0}$$

We integrate the characteristic equation to get three similarity variables,

$$y=r,\ t=s, \text{ and } u=W(r,s). \quad (1.18)$$

Using these similarity variables in Eq.(1.6) can be recast in the form

$$Ws - \mu(Wrrs) = 0. \quad (1.19)$$

Case 2: $v_2 = \partial ys$

The characteristic equation associated with this generator is

$$\frac{dx}{0} = \frac{dy}{1} = \frac{dt}{0} = \frac{du}{0}$$

Following the standard procedure we integrate the characteristic equation to get three similarity variables,

$$X=r,\ y=s \text{ and } u=W(r,s) \qquad (1.20)$$

Using these similarity variables in Eq.(1.6) can be recast in the form

$$W_s + WW_r - \mu(WW_{rrs})=0 \qquad (1.21)$$

Case 3: $v_3=\partial t$

The characteristic equation associated with this generator is

$$\frac{dx}{0} = \frac{dy}{0} = \frac{dt}{1} = \frac{du}{0}$$

Following the standard procedure we integrate the characteristic equation to get three similarity variables,

$$X=r,\ y=s \text{ and } u=W(r,s) \qquad (1.22)$$

Using these similarity variables in Eq.(1.6) can be recast in the form

$$WW_r=0 \qquad (1.23)$$

Case 4: $v_4=t\partial t-u\partial u$

The characteristic equation associated with this generator is

$$\frac{dx}{0} = \frac{dy}{0} = \frac{dt}{t} = \frac{du}{-u}$$

Following the standard procedure we integrate the characteristic equation to get three similarity variables,

$$X=r,\ t=s \text{ and } u=W(r,s)\, t^{-1}. \qquad (1.24)$$

4. Reductions of (2+1)-dimensional equal width wave equation by two-dimensional abelian subalgebras

Case I: Reduction under v_1 and v_2.

The transformed v_2 is

$$\tilde{V}_2 = \partial_r$$

The characteristic equation for $\tilde{V}_2$ is

$$\frac{dr}{1} = \frac{ds}{0} = \frac{dW}{0}$$

Integrating this equation as before leads to new variables

$s = \zeta$ and $W = R(\zeta)$

which reduce Eq. (1.19) to

$$R(\zeta) = 0 \qquad (1.25)$$

Case II: Reduction under V_1 and V_3

The transformed V_3 is

$$\tilde{V}_3 = \partial_s$$

The characteristic equation for $\tilde{V}_3$ is

$$\frac{dr}{0} = \frac{ds}{1} = \frac{dW}{0}$$

Integrating this equation as before leads to new variable

$r = \zeta$ and $W = R(\zeta)$.

which satisfies Eq. (1.19)

Case III: Reduction under V_1 and V_4

The transformed V_1 is

$\tilde{V}_1 = \partial_r$

The characteristic equation for $\tilde{V}_1$ is

$$\frac{dr}{1} = \frac{ds}{0} = \frac{dW}{0}$$

Integrating this equation as before leads to new variables

$s = \zeta$ and $W = R(\zeta)$.

which reduce eq. (1.21) to

$$R - \mu R_{\zeta\zeta} = 0 \tag{1.26}$$

Case IV: Reduction under V_2 and V_3

The transformed V_3 is

$$\tilde{V}_3 = \partial_s$$

The characteristic equation for $\tilde{V}_3$ is

$$\frac{dr}{0} = \frac{ds}{1} = \frac{dW}{0}$$

Integrating this equation as before leads to new variables

$r = \zeta$ and $W = R(\zeta)$.

which reduce Eq.(1.21) to

$$RR_\zeta = 0. \tag{1.27}$$

Case V: Reduction under V_2 and V_4

The transformed V_2 is

$$\tilde{V}_2 = \partial_s$$

The characteristic equation for $\tilde{V}_2$ is

$$\frac{dr}{0} = \frac{ds}{1} = \frac{dW}{0}$$

Integrating this equation as before leads to new variables

$r = \zeta$ and $W = R(\zeta)$

which reduce Eq. (1.23) to

$$R - RR_\zeta - \mu R_{\zeta\zeta} = 0 \tag{1.28}$$

5. Conclusions

In this paper,

(i) A (2+1)-dimensional Equal width wave equation, $u_t + uu_x - \mu\left(u_{xxt} + u_{yyt}\right) = 0$ where $\mu \epsilon \mathbb{R}$ is subjected to Lie's classical method.

(ii) Equation (1.6) admits a four-dimensional symmetry group.

(iii) It is established that the symmetry generators form a closed Lie algebra.

(iv) Classification of symmetry algebra of (1.6) into one-and two-dimensional sub-algebras is carried out.

(v) Systematic reduction to (1+1)-dimensional PDE and then to first order ODEs are performed using one dimensional and two dimension solvable Abelian sub-algebras.

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