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On Two Arithmetic-Geometric Banhatti Indices of Certain Dendrimer Nanostars

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Abstract. Chemical Graph Theory is a branch of Mathematical Chemistry whose focus of interest is to finding topological indices of chemical graphs which correlate well with chemical properties of the chemical molecules. We propose the arithmetic-geometric Banhatti index and multiplicative arithmetic-geometric Banhatti index of a molecular graph. In this paper, we compute these Banhatti topological indices of certain infinite classes of dendrimer nanostars.

Keywords: Arithmetic-geometric Banhatti index, multiplicative arithmetic-geometric Banhatti index, dendrimer nanostar.

AMS Mathematics Subject Classification (2010): 05C05, 05C07, 05C12, 05C35

1. Introduction

Let G be a finite, simple connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The edge connecting the vertices u and v will be denoted by uv. Let $d_G(e)$ denote the degree of an edge $e = uv$ in G is defined by $d_G(e)$ and $d_G(e) = d_G(u) + d_G(v) – 2$. We refer to [1] for undefined term and notation.

A molecular graph or a chemical graph is a finite, simple graph such that its vertices correspond to the atoms and the edges to the bonds. A topological index is a numerical parameter mathematically derived from the graph structure. Numerous topological indices are useful for establishing correlations between the structure of a molecular compound and its physico-chemical properties, see [2].

Recntly, Kulli [3], introduced the geometric-arithmetic Banhatti index of a graph G and it s defined as

$$GAB(G)=\sum_{ue}\frac{2\sqrt{d_G(u)d_G(e)}}{d_G(u)+d_G(e)}.$$

where ue means that the vertex u and edge e are incident in G. Recently some topological indices were studied, for example, in [4, 5, 6, 7, 8, 9, 10, 11, 12, 13].

Very recently, Kulli [4] proposed the multiplicative geometric-arithmetic Banhatti index of a graph G and it is defined as

$$GAIIB(G)=\prod_{ue}\frac{2\sqrt{d_G(u)d_G(e)}}{d_G(u)+d_G(e)}.$$

Recently, some multiplicative topological indices were studied, for example, in [14, 15, 16, 17, 18, 19, 20,21].

Motivated by the definition of the geometric-arithmetic Banhatti index, we introduce the arithmetic-geometric Banhatti index and multiplicative arithmetic-geometric index of a molecular graph as follows:

The arithmetric-geometric Banhatti index of a molecular graph G is defined as

$$AGB(G)=\sum_{ue}\frac{d_G(u)+d_G(e)}{2\sqrt{d_G(u)d_G(e)}}=\sum_{uv\in E(G)}\left[\frac{d_G(u)+d_G(e)}{2\sqrt{d_G(u)d_G(e)}}+\frac{d_G(v)+d_G(e)}{2\sqrt{d_G(v)d_G(e)}}\right]. \quad (1)$$

The multiplicative arithmetic- geometric Banhatti index of a molecular graph G is defined as

$$AGBII(G)=\prod_{ue}\frac{d_G(u)+d_G(e)}{2\sqrt{d_G(u)d_G(e)}}=\prod_{uv\in E(G)}\left|\frac{d_G(u)+d_G(e)}{2\sqrt{d_G(u)d_G(e)}}+\frac{d_G(v)+d_G(e)}{2\sqrt{d_G(v)d_G(e)}}\right| \quad (2)$$

We consider some families of dendrimer nanostars, see [22]. In this paper, the arithmetic-geometric Banhatti index and multiplicative arithmetic-geometric Banhatti index of certain families of dendrimer nanostars are computed.

2. Results for dendrimer nanostars $D_1[n]$

In this section, we consider a family of dendrimer nanostars with n growth stages, denoted by $D_1[n]$, where $n\geq 0$. The molecular graph of $D_1[n]$ with 4 growth stages is depicted in Figure 1.

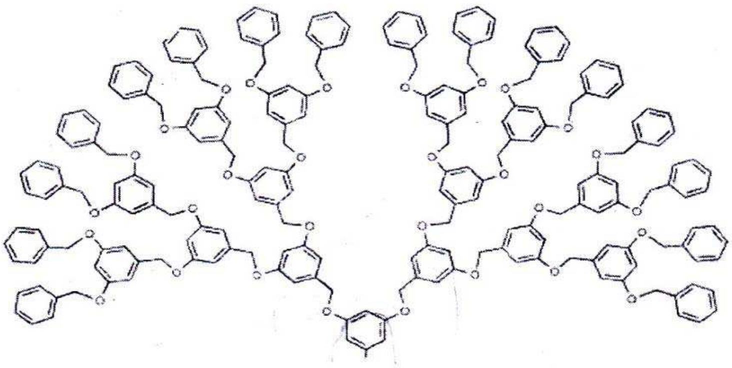

Figure 1: The molecular graph of $D_1[4]$

Let $G = D_1[n]$ be the chemical graph in the family of dendrimer nanostar. By calculation, we obtain that G has $18\times 2^n-11$ edges. We obtain that the edge set $E(D_1[n])$ can be divided into three partitions as

$E_{13} = \{uv\in E(G) \mid d_G(u) = 1, d_G(v) = 3\}$, $\quad |E_{13}| = 1$.

$E_{22} = \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_{22}| = 6 \times 2^n - 2$.
$E_{23} = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_{23}| = 12 \times 2^n - 10$.
Then the edge degree partition of G is given in Table 1.

$d_G(u)\, d_G(v) \backslash uv \in E(G)$	(1,3)	(2, 2)	(2, 3)
$d_G(e)$	2	2	3
Number of edges	1	$6 \times 2^n - 2$	$12 \times 2^n - 10$

Table 1: Edge degree partition of G

Theorem 1. The arithmetic-geometric Banhatti index of a dendrimer nanostar $D_1[n]$ is given by

$$AGB(D_1[n]) = \left(\frac{5}{2\sqrt{6}} + 2\right)12\times 2^n + \frac{3}{2\sqrt{2}} - \frac{45}{2\sqrt{6}} - 14.$$

Proof: Let G be the graph of a dendrimer nanostar $D_1[n]$. By using equation (1) and Table 1, we deduce

$$AGB(D_1[n]) = \sum_{uv\in E(G)}\left[\frac{d_G(u)+d_G(e)}{2\sqrt{d_G(u)d_G(e)}} + \frac{d_G(v)+d_G(e)}{2\sqrt{d_G(v)d_G(e)}}\right]$$

$$= \left(\frac{1+2}{2\sqrt{1\times 2}} + \frac{3+2}{2\sqrt{3\times 2}}\right) + \left(\frac{2+2}{2\sqrt{2\times 2}} + \frac{2+2}{2\sqrt{2\times 2}}\right)(6\times 2^n - 2) + \left(\frac{2+3}{2\sqrt{2\times 3}} + \frac{3+3}{2\sqrt{3\times 3}}\right)(12\times 2^n - 10)$$

$$= \left(\frac{5}{2\sqrt{6}} + 2\right)12\times 2^n + \frac{3}{2\sqrt{2}} - \frac{45}{2\sqrt{6}} - 14.$$

Theorem 2. The multiplicative arithmetic-geometric index of a dendrimer nanostar $D_1[n]$ is given by

$$AGBII(D_1[n]) = \left(\frac{3}{2\sqrt{2}} + \frac{5}{2\sqrt{6}}\right)^1 \times 2^{6\times 2^n - 2} \times \left(\frac{5}{2\sqrt{6}} + 1\right)^{12\times 2^n - 10}.$$

Proof: Let G be the graph of a dendrimer nanostar $D_1[n]$. By using equation (2) and Table 1, we deduce

$$AGBII(D_1[n]) = \prod_{uv\in E(G)}\left[\frac{d_G(u)+d_G(e)}{2\sqrt{d_G(u)d_G(e)}} + \frac{d_G(v)+d_G(e)}{2\sqrt{d_G(v)d_G(e)}}\right].$$

$$= \left(\frac{1+2}{2\sqrt{1\times 2}} + \frac{3+2}{2\sqrt{3\times 2}}\right)^1 \times \left(\frac{2+2}{2\sqrt{2\times 2}} + \frac{2+2}{2\sqrt{2\times 2}}\right)^{6\times 2^n - 2} \times \left(\frac{2+3}{2\sqrt{2\times 3}} + \frac{3+3}{2\sqrt{3\times 3}}\right)^{12\times 2^n - 10}$$

$$= \left(\frac{3}{2\sqrt{2}} + \frac{5}{2\sqrt{6}}\right)^1 \times 2^{6\times 2^n - 2} \times \left(\frac{5}{2\sqrt{6}} + 1\right)^{12\times 2^n - 10}.$$

3. Dendrimer nanostars $D_3[n]$

In this section, we consider a family of dendrimer nanostars with n growth stages, denoted by $D_3[n]$, where $n \geq 0$. The molecular graph of $D_3[n]$ with three growth stages is depicted in Figure 2.

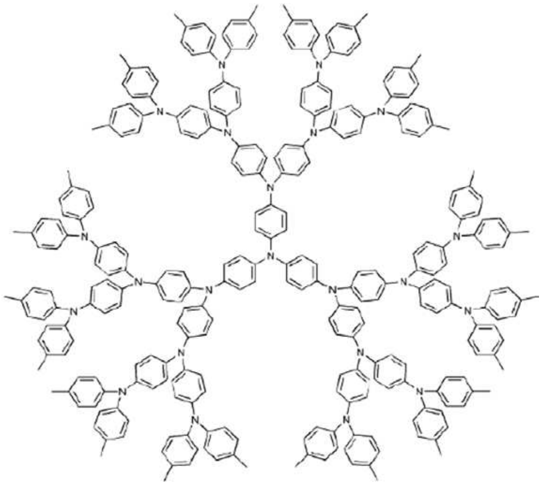

Figure 2: The molecular graph of $D_3[3]$

Let $G = D_3[n]$ be the chemical graph in the family of dendrimer nanostars. By calculation, we obtain that G has $24\times2^{n+1} - 24$ edges. Also by calculation, we obtain that the edge set $E(D_3[n])$ can be divided into 4 partitions as

$E_{13} = \{uv \in E(G) \mid d_G(u) =1, d_G(v) = 3\}$, $|E_{13}| = 3\times2^n$.
$E_{22} = \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_{22}| = 12\times2^n - 6$.
$E_{23} = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_{23}| = 24\times2^n - 12$.
$E_{33} = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_{33}| = 9\times2^n - 6$.

Then the edge degree partition of $D_3[n]$ is given in Table 2.

$d_G(u), d_G(v)\backslash uv \in E(G)$	(1, 3)	(2, 2)	(2, 3)	(3, 3)
$d_G(e)$	2	2	3	4
Number of edges	3×2^n	$12\times2^n - 6$	$24\times2^n - 12$	$9\times2^n - 6$

Table 2: Edge degree partition of G

Theorem 3. The arithmetic-geometric Banhatti index of a dendrimer nanostar $D_3[n]$ is given by

$$AGB(D_3[n]) = \left(\frac{3}{2\sqrt{2}} + \frac{5}{2\sqrt{6}}\right)3\times2^n + \left(\frac{5}{2\sqrt{6}} + 2\right)(24\times2^n - 12) + \frac{7}{2\sqrt{3}}(9\times2^n - 6)$$

Proof: Let G be the graph of a dendrimer nanostar $D_3[n]$. By using equation (1) and using Table 2, we derive

$$AGB(D_3[n]) = \sum_{uv\in E(G)}\left[\frac{d_G(u)+d_G(e)}{2\sqrt{d_G(u)d_G(e)}}+\frac{d_G(v)+d_G(e)}{2\sqrt{d_G(v)d_G(e)}}\right]$$

$$=\left(\frac{1+2}{2\sqrt{1\times 2}}+\frac{3+2}{\sqrt{3\times 2}}\right)3\times 2^n+\left(\frac{2+2}{2\sqrt{2\times 2}}+\frac{2+2}{2\sqrt{2\times 2}}\right)(12\times 2^n-6)$$

$$+\left(\frac{2+3}{2\sqrt{2\times 3}}+\frac{3+3}{\sqrt{3\times 3}}\right)(24\times 2^n-12)+\left(\frac{3+4}{2\sqrt{3\times 4}}+\frac{4+3}{2\sqrt{4\times 3}}\right)(9\times 2^n-6)$$

$$=\left(\frac{3}{2\sqrt{2}}+\frac{5}{2\sqrt{6}}\right)3\times 2^n+\left(\frac{5}{2\sqrt{6}}+2\right)(24\times 2^n-12)+\frac{7}{2\sqrt{3}}(9\times 2^n-6).$$

Theorem 4. The multiplicative sum connectivity Banhatti index of a dendrimer nanostar $D_3[n]$ is given by

$$AGBII(D_3[n])=\left(\frac{3}{2\sqrt{2}}+\frac{5}{2\sqrt{6}}\right)^{3\times 2^n}\times 2^{12\times 2^n-6}\times\left(\frac{5}{2\sqrt{6}}+1\right)^{24\times 2^n-12}\times\left(\frac{7}{2\sqrt{3}}\right)^{9\times 2^n-6}.$$

Proof: Let G be the graph of a dendrimer nanostar $D_3[n]$. By using equation (2) and Table 2, we derive

$$AGBII(D_3[n])=\prod_{uv\in E(G)}\left[\frac{d_G(u)+d_G(e)}{2\sqrt{d_G(u)d_G(e)}}+\frac{d_G(v)+d_G(e)}{2\sqrt{d_G(v)d_G(e)}}\right]$$

$$=\left(\frac{1+2}{2\sqrt{1\times 2}}+\frac{3+2}{2\sqrt{3\times 2}}\right)^{3\times 2^n}\times\left(\frac{2+2}{2\sqrt{2\times 2}}+\frac{2+2}{2\sqrt{2\times 2}}\right)^{12\times 2^n-6}$$

$$\times\left(\frac{2+3}{2\sqrt{2\times 3}}+\frac{3+3}{2\sqrt{3\times 3}}\right)^{24\times 2^n-12}\times\left(\frac{3+4}{2\sqrt{3\times 4}}+\frac{4+3}{2\sqrt{4\times 3}}\right)^{9\times 2^n-6}$$

$$=\left(\frac{3}{2\sqrt{2}}+\frac{5}{2\sqrt{6}}\right)^{3\times 2^n}\times 2^{12\times 2^n-6}\times\left(\frac{5}{2\sqrt{6}}+1\right)^{24\times 2^n-12}\times\left(\frac{7}{2\sqrt{3}}\right)^{9\times 2^n-6}.$$

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