Intuitionistic Fuzzy Multisets in Medical Diagnosis

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Abstract. In this paper, sine inverse distance among intuitionistic fuzzy multi sets is proposed and some of its properties are discussed herein. The concept of the above method is an essential tool for dealing with uncertainties and shortcomings that affect the existing methods. Implementation of medical diagnosis is presented to find out the disease impacting the patient.

Keywords: Intuitionistic fuzzy set, intuitionistic fuzzy multi set, sine inverse distance, medical diagnosis

AMS Mathematics Subject Classification (2010): 03E72, 03F55, 62P10, 92C50

1. Introduction

Kumbakonam being a temple city with lots of temples and mutts attracts thousands of pilgrims and tourists every day. The dumped wastes left out by them are causing diseases. Further, Kumbakonam is surrounded by a lot of ponds and tanks. These water bodies get their water from several inlets. But in the absence of proper outlets to drain the water, the water gets contaminated and it becomes stagnated too. And it is crystal clear that the stagnated water becomes the breeding ground for mosquitoes to spread diseases like malaria, dengue and elephantiasis and so on and so forth. In many areas of Kumbakonam the prime cause of disease and decease is mosquito. This created an urge to carry out research in the medical field. By introducing innovative methods in the research, the diseases can be diagnosed instantly and infallibly.

A number of real life problems in engineering, medical sciences, social sciences, economics etc., involve imprecise data and their solution involves the use of mathematical principles based on uncertainty and imprecision. Such uncertainties are being dealt with the help of topics like probability theory, fuzzy set theory [20], rough set theory [12] etc., Healthcare industry has been trying to complement the services offered by conventional clinical decision making systems with the integration of fuzzy logic techniques in them. As it is not an easy task for a clinician to derive a fool proof diagnosis it is advantageous to automate few initial steps of diagnosis which would not
A. Edward Samuel and R. Narmadhagnanam

require intervention from an expert doctor. Intuitionistic fuzzy set possesses all attributes necessary to encode medical knowledge base and capture medical inputs.

As medical diagnosis demands large amount of information processing, large portion of which is quantifiable, also intuitive thought process involve rapid unconscious data processing and combines available information by law of average, the whole process offers low intra and inter personal consistency. So contradictions, inconsistency and fuzziness should be accepted as unavoidable as they are integrated in the behavior of biological systems as well as in their characterization. To model an expert doctor it is imperative that it should not disallow uncertainty as it would be then inapt to capture fuzzy or incomplete knowledge that might lead to the danger of fallacies due to misplaced precision.

As medical diagnosis contains lots of uncertainties and increased volume of information available to physicians from new medical technologies, the process of classifying different sets of symptoms under a single name of disease becomes difficult. In some practical situations, there is the possibility of each element having different truth membership and false membership functions. So, intuitionistic fuzzy sets play a vital role in medical diagnosis.


In this paper, by using the notion of intuitionistic fuzzy multi set, it was provided an exemplary for medical diagnosis. In order to make this, a novel method was executed.

Rest of the article is structured as follows. In Section 2, the basic definitions are briefly presented. Section 3 deals with proposed definitions and some of its properties. Sections 4, 5 and 6 contain methodology, algorithm and case study related to medical diagnosis respectively. Conclusion is given in Section 7.
Intuitionistic Fuzzy Multisets in Medical Diagnosis

2. Basic definitions

Definition 2.1. [20] Let \( X \) be a non-empty set. A fuzzy set \( A \) drawn from \( X \) is defined as \( A = \{(x, \mu_A(x)) : x \in X \} \) where \( \mu_A(x) : X \rightarrow [0,1] \) is the membership function for the fuzzy set \( A \).

Definition 2.2. [2] Let \( X \) be a non-empty set. An intuitionistic fuzzy set \( A \) in \( X \) is an object having the form \( A = \{(x, \mu_A(x), \nu_A(x)) : x \in X \} \) where the functions \( \mu_A(x) : X \rightarrow [0,1] \) and \( \nu_A(x) : X \rightarrow [0,1] \) define the degree of membership and degree of non-membership respectively of the element \( x \in X \) to the set \( A \), which is a subset of \( X \), and every element \( x \in X \), \( 0 \leq \mu_A(x) + \nu_A(x) \leq 1 \). The value \( \pi_A(x) = 1 - (\mu_A(x) + \nu_A(x)) \) is called the intuitionistic fuzzy index or hesitation margin of \( x \) in \( A \). \( \pi_A(x) \) is the degree of indeterminacy of \( x \) in the intuitionistic fuzzy set \( A \) and \( \pi_A(x) \in [0,1] \) i.e., \( \pi_A(x) : X \rightarrow [0,1] \) and \( 0 \leq \pi_A(x) \leq 1 \) for every \( x \in X \). \( \pi_A(x) \) expresses the lack of knowledge of whether \( x \) belongs to intuitionistic fuzzy set \( A \) or not.

Definition 2.3. [14] Let \( X \) be a nonempty set. An Intuitionistic Fuzzy Multiset (IFMS) \( A \) in \( X \) is characterized by two function namely count membership function \( MC \) and count non membership function \( NMC \) such that \( QMC : X \rightarrow Q \) and \( NMC : X \rightarrow Q \) where \( Q \) is the set of all crisp multi sets in \([0,1]\). Hence, for any \( x \in X \), \( MC(x) \) is the crisp multi set from \( [0,1] \) whose membership sequence is defined as \( (\mu^1_A(x), \mu^2_A(x), \ldots \mu^n_A(x)) \) where \( \mu^1_A(x) \geq \mu^2_A(x) \geq \ldots \geq \mu^n_A(x) \) and the corresponding non membership sequence \( NMC(x) \) is defined as \( (\nu^1_A(x), \nu^2_A(x), \ldots \nu^n_A(x)) \) where the non membership can be either decreasing or increasing function. Such that \( 0 \leq \mu^i_A(x) + \nu^i_A(x) \leq 1 \), \( \forall x \in X \) and \( i = 1, 2, \ldots, p \). Therefore, an IFMS \( A \) is given by \( A = \{(x, (\mu^i_A(x), \mu^2_A(x), \ldots \mu^n_A(x)), (\nu^i_A(x), \nu^2_A(x), \ldots \nu^n_A(x))) : x \in X \} \) where \( \mu^i_A(x) \geq \mu^2_A(x) \geq \ldots \geq \mu^n_A(x) \)

3. Proposed definition

Definition 3.1. Let \( A = \{(x, (\mu^i_A(x), \mu^2_A(x), \ldots \mu^n_A(x)), (\nu^i_A(x), \nu^2_A(x), \ldots \nu^n_A(x))) : x \in X \} \) and \( B = \{(x, (\mu^i_B(x), \mu^2_B(x), \ldots \mu^n_B(x)), (\nu^i_B(x), \nu^2_B(x), \ldots \nu^n_B(x))) : x \in X \} \) be two intuitionistic fuzzy multi sets. Then the sine inverse distance is defined as

\[
SID_{IFMS}(A, B) = 2p \sum_{i=1}^{p} \sin^{-1} \left[ \frac{\pi}{4} \left( \frac{\mu^i_A(x) - \mu^i_B(x) + \nu^i_A(x) - \nu^i_B(x)}{5pn} \right) \right]
\]

Proposition 1.
(i) \( SID_{IFMS}(A, B) \geq 0 \)
(ii) \( SID_{IFMS}(A, B) = SID_{IFMS}(B, A) \)
(iii) If \( A \subseteq B \subseteq C \) then \( SID_{\text{IFMS}}(A,C) \geq SID_{\text{IFMS}}(A,B) \) and \( SID_{\text{IFMS}}(A,C) \geq SID_{\text{IFMS}}(B,C) \)

Proof:
(i) The proof is straightforward
(ii) The proof is straightforward
(iii) It was well known that,
\[
\mu_i^A(x) \leq \mu_i^B(x) \leq \mu_i^C(x)
\]
\[
\nu_i^A(x) \geq \nu_i^B(x) \geq \nu_i^C(x)
\]
\( \because A \subseteq B \subseteq C \)

Hence,
\[
|\mu_i^A(x) - \mu_i^B(x)| \leq |\mu_i^A(x) - \mu_i^C(x)|
\]
\[
|\mu_i^B(x) - \mu_i^C(x)| \leq |\mu_i^A(x) - \mu_i^C(x)|
\]
\[
|\nu_i^A(x) - \nu_i^B(x)| \leq |\nu_i^A(x) - \nu_i^C(x)|
\]
\[
|\nu_i^B(x) - \nu_i^C(x)| \leq |\nu_i^A(x) - \nu_i^C(x)|
\]

Here, the sine inverse distance is an increasing function.
\( \therefore SID_{\text{IFMS}}(A,C) \geq SID_{\text{IFMS}}(A,B) \) and \( SID_{\text{IFMS}}(A,C) \geq SID_{\text{IFMS}}(B,C) \)

4. Methodology
In this section, it was presented an application of intuitionistic fuzzy multi set in medical diagnosis. In a given pathology, suppose \( S \) is a set of symptoms, \( D \) is a set of diseases and \( P \) is a set of patients and let \( Q \) be an intuitionistic fuzzy multi relation from the set of patients to the symptoms i.e., \( Q(P \rightarrow S) \) and \( R \) be an intuitionistic fuzzy relation from the set of symptoms to the diseases i.e., \( R(S \rightarrow D) \) and then the methodology involves three main jobs:
1. Determination of symptoms
2. Formulation of medical knowledge based on intuitionistic fuzzy sets
3. Determination of diagnosis on the basis of new method in intuitionistic fuzzy multi sets

5. Algorithm
Step 1: The Symptoms of the patients are given to obtain the patient-symptom relation \( Q \) and are noted in Table 1
Step 2: The medical knowledge relating the symptoms with the set of disease under consideration are given to obtain the symptom-disease relation \( R \) and are noted in Table 2.
Step 3: The computation \( T \) of the relation of patients and diseases is found using (1) and are noted in Table 3.
Step 4: Finally, the minimum value from Table 3 of each row was selected to find the
Intuitionistic Fuzzy Multisets in Medical Diagnosis

possibility of the patient affected with the respective disease and then it was concluded that the patient \( P_k \) \((k = 1, 2, 3, 4)\) was suffering from the disease \( D_r \) \((r = 1, 2, 3, 4)\).

6. Case study [14]

Let there be four patients \( P = \{P_1, P_2, P_3, P_4\}\) and the set of symptoms \( S = \{\text{Temperature, Cough, Throat pain, Headache, Body pain}\}\). The intuitionistic fuzzy multi relation \( Q(P \rightarrow S) \) is given as in Table 1. Let the set of diseases \( D = \{\text{Viral fever, Tuberculosis, Typhoid, Throat disease}\}\). The intuitionistic fuzzy relation \( R(S \rightarrow D) \) is given as in Table 2.

### Table 1: The Relation between Patients and Symptoms (using step 1)

<table>
<thead>
<tr>
<th>Q</th>
<th>Temperature</th>
<th>Cough</th>
<th>Throat pain</th>
<th>Headache</th>
<th>Body pain</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_1 )</td>
<td>(0.6, 0.2)</td>
<td>(0.4, 0.3)</td>
<td>(0.1, 0.7)</td>
<td>(0.5, 0.4)</td>
<td>(0.2, 0.6)</td>
</tr>
<tr>
<td></td>
<td>(0.7, 0.1)</td>
<td>(0.3, 0.6)</td>
<td>(0.2, 0.7)</td>
<td>(0.6, 0.3)</td>
<td>(0.3, 0.4)</td>
</tr>
<tr>
<td></td>
<td>(0.5, 0.4)</td>
<td>(0.4, 0.4)</td>
<td>(0.0, 0.8)</td>
<td>(0.7, 0.2)</td>
<td>(0.4, 0.4)</td>
</tr>
<tr>
<td>( P_2 )</td>
<td>(0.4, 0.5)</td>
<td>(0.7, 0.2)</td>
<td>(0.6, 0.3)</td>
<td>(0.3, 0.7)</td>
<td>(0.8, 0.1)</td>
</tr>
<tr>
<td></td>
<td>(0.3, 0.4)</td>
<td>(0.6, 0.2)</td>
<td>(0.5, 0.3)</td>
<td>(0.6, 0.3)</td>
<td>(0.7, 0.2)</td>
</tr>
<tr>
<td></td>
<td>(0.5, 0.4)</td>
<td>(0.8, 0.1)</td>
<td>(0.4, 0.4)</td>
<td>(0.2, 0.7)</td>
<td>(0.5, 0.3)</td>
</tr>
<tr>
<td>( P_3 )</td>
<td>(0.1, 0.7)</td>
<td>(0.3, 0.6)</td>
<td>(0.8, 0.0)</td>
<td>(0.3, 0.6)</td>
<td>(0.4, 0.4)</td>
</tr>
<tr>
<td></td>
<td>(0.2, 0.6)</td>
<td>(0.2, 0.0)</td>
<td>(0.7, 0.1)</td>
<td>(0.2, 0.7)</td>
<td>(0.3, 0.7)</td>
</tr>
<tr>
<td></td>
<td>(0.1, 0.9)</td>
<td>(0.1, 0.7)</td>
<td>(0.8, 0.1)</td>
<td>(0.2, 0.6)</td>
<td>(0.2, 0.7)</td>
</tr>
<tr>
<td>( P_4 )</td>
<td>(0.5, 0.4)</td>
<td>(0.4, 0.5)</td>
<td>(0.2, 0.7)</td>
<td>(0.5, 0.4)</td>
<td>(0.4, 0.6)</td>
</tr>
<tr>
<td></td>
<td>(0.4, 0.4)</td>
<td>(0.3, 0.3)</td>
<td>(0.1, 0.6)</td>
<td>(0.6, 0.3)</td>
<td>(0.5, 0.4)</td>
</tr>
<tr>
<td></td>
<td>(0.5, 0.3)</td>
<td>(0.1, 0.7)</td>
<td>(0.0, 0.7)</td>
<td>(0.3, 0.6)</td>
<td>(0.4, 0.3)</td>
</tr>
</tbody>
</table>

### Table 2: The relation among symptoms and diseases (using step 2)

<table>
<thead>
<tr>
<th>R</th>
<th>Viral Fever</th>
<th>Tuberculosis</th>
<th>Typhoid</th>
<th>Throat disease</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>(0.8, 0.1)</td>
<td>(0.2, 0.7)</td>
<td>(0.5, 0.3)</td>
<td>(0.1, 0.7)</td>
</tr>
<tr>
<td>Cough</td>
<td>(0.2, 0.7)</td>
<td>(0.9, 0.0)</td>
<td>(0.3, 0.5)</td>
<td>(0.3, 0.6)</td>
</tr>
<tr>
<td>Throat pain</td>
<td>(0.3, 0.5)</td>
<td>(0.7, 0.2)</td>
<td>(0.2, 0.7)</td>
<td>(0.8, 0.1)</td>
</tr>
<tr>
<td>Headache</td>
<td>(0.5, 0.3)</td>
<td>(0.6, 0.3)</td>
<td>(0.2, 0.6)</td>
<td>(0.1, 0.8)</td>
</tr>
<tr>
<td>Body ache</td>
<td>(0.5, 0.4)</td>
<td>(0.7, 0.2)</td>
<td>(0.4, 0.4)</td>
<td>(0.1, 0.8)</td>
</tr>
</tbody>
</table>

### Table 3: Sine inverse distance (using step 3 and step 4)

<table>
<thead>
<tr>
<th>T</th>
<th>Viral Fever</th>
<th>Tuberculosis</th>
<th>Typhoid</th>
<th>Throat disease</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_1 )</td>
<td>0.3066</td>
<td>0.7086</td>
<td><strong>0.2682</strong></td>
<td>0.7656</td>
</tr>
<tr>
<td>( P_2 )</td>
<td>0.5580</td>
<td><strong>0.3444</strong></td>
<td>0.4572</td>
<td>0.6774</td>
</tr>
<tr>
<td>( P_3 )</td>
<td>0.6768</td>
<td>0.5640</td>
<td>0.5142</td>
<td><strong>0.2304</strong></td>
</tr>
<tr>
<td>( P_4 )</td>
<td>0.3192</td>
<td>0.6714</td>
<td><strong>0.1932</strong></td>
<td>0.7146</td>
</tr>
</tbody>
</table>

From Table 3, it is obvious that, if the doctor agrees, then \( P_1 \) and \( P_4 \) suffers from Typhoid, \( P_2 \) suffers from Tuberculosis and \( P_3 \) suffers from Throat disease.
7. Conclusion
In this paper, it was analyzed the relationship between the set of symptoms found with the patients and the set of diseases and employed a novel method (sine inverse distance) to find out the disease possibly affected the patient. The technique considered in this study was more reliable to handle medical diagnosis problems quiet comfortably. In future, these methods can be enhanced to other types of intuitionistic fuzzy sets also.

REFERENCES
Intuitionistic Fuzzy Multisets in Medical Diagnosis

