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Closure Fuzzy Ideals in Distributive Lattices

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Abstract. In this paper, we introduce the concept of closure fuzzy ideals in a distributive lattice. The set of all closure fuzzy ideals of a lattice is isomorphic to the class of fuzzy ideals of the lattice of all dominator ideals. Moreover, a one to one correspondence between the class of all prime closure fuzzy ideals of a lattice and the prime fuzzy ideals of the lattice of all dominator ideals is established.

Keywords: Closure ideal, fuzzy ideal, closure fuzzy ideal

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1. Introduction

The concept of ideals of lattices developed by Hashimot [3]. Latter, Grätzer and Schmidit [2] studied on ideal theory of lattice. Mandelker [4], introduced the theory of relative annihilators and he characterized distributive lattices in terms of there relative annihilators. Rao [6], studied the concept of closure ideals in distributive lattices with the help of annihilators.

On the other hand, many papers on fuzzy algebras have been published since Rosenfeld [7] introduced the concept of fuzzy group in 1971. Many papers on fuzzy sets appeared showing the importance of the concept and its applications to logic, set theory, group theory, ring theory, real analysis, topology, measure theory etc. Uncertain data in many important applications in the areas such as economics, engineering, environment, medical sciences and business management could be caused by data randomness, information incompleteness, limitations of measuring instrument, delayed data updates etc. Liu [5] initiated the study of fuzzy subrings, and fuzzy ideals of a ring. Yuan and Wu [9] introduced the notion of fuzzy ideals and fuzzy congruences of distributive lattices. Swamy and Raju [8] studied properties of fuzzy ideals and congruences of lattices.

In this paper, we introduce the concept of closure fuzzy ideal in a distributive lattice. It is proved that the set of closure fuzzy ideals forms a complete distributive lattice. Moreover, the set of all closure fuzzy ideals of a lattice is isomorphic to the class of fuzzy ideals of the lattice of all dominator ideals. Finally, a one to one correspondence between the class of all prime closure fuzzy ideals of a lattice and the prime fuzzy ideals of the lattice of all dominator ideals is established.

2. Preliminaries

Throughout this paper L stands for the distributive lattice with least element 0 unless it is specified. We refer to G. Birkhoff [1] for the elementary properties of lattices.

Definition 2.1. *[6] For any $a \in L$, the dominator of a is defined as:*

$$(a)^e = \{x \in L: (a]^* \subseteq (x]^*\}.$$

The dominator $(a)^e$ of any element a of L is an ideal of L.

In [6], Rao observed the following. In a distributive lattice L with 0 the set of all dominator ideals of the form $(a)^e$ can be made into a lattice $(A^e(L), \cap, \underline{\vee})$ called the lattice of dominator ideals of L with least element $(0)^e$. For two dominator $(a)^e$ and $(b)^e$ their supremum in $A^e(L)$ is $(a)^e \, \underline{\vee} \, (b)^e = (a \vee b)^e$ and also their infimum in $A^e(L)$ is $(a)^e \cap (b)^e = (a \wedge b)^e$.

For an ideal I in L

$$\alpha(I) = \{(a)^e: a \in I\}$$

is an ideal in $A^e(L)$ and the set

$$\beta(J) = \{a \in L: (a)^e \in J\}$$

is an ideal of L when J is any ideal in $A^e(L)$. An ideal I of L is called a closure ideal ideal if $\beta\alpha(I) = I$.

Remember that, for any set A, a function $\mu: A \to ([0,1], \wedge, \vee)$ is called a fuzzy subset of A, where $[0,1]$ is a unit interval, $\alpha \wedge \beta = min\{\alpha, \beta\}$ and $\alpha \vee \beta = max\{\alpha, \beta\}$ for all $\alpha, \beta \in [0,1]$.

Definition 2.2. *[7] Let μ and θ be fuzzy subsets of a set A. Define the fuzzy subsets $\mu \cup \theta$ and $\mu \cap \theta$ of A as follows: for each $x \in A$,*

$$(\mu \cup \theta)(x) = \mu(x) \vee \theta(x) \text{ and } (\mu \cap \theta)(x) = \mu(x) \wedge \theta(x).$$

Then $\mu \cup \theta$ and $\mu \cap \theta$ are called the union and intersection of μ and θ, respectively.

We define the binary operations "$\vee$" and "$\wedge$" on the set of all fuzzy subsets of L as:

$$(\mu \vee \theta)(x) = Sup\{\mu(y) \wedge \theta(z): y, z \in L,\ y \vee z = x\} \text{ and}$$
$$(\mu \wedge \theta)(x) = Sup\{\mu(y) \wedge \theta(z): y, z \in L,\ y \wedge z = x\}.$$

If μ and θ are fuzzy ideals of L, then $\mu \wedge \theta = \mu \cap \theta$ and $\mu \vee \theta$ is a fuzzy ideal generated by $\mu \cup \theta$.

For any collection, $\{\mu_i: i \in I\}$ of fuzzy subsets of X, where I is a nonempty index set, the least upper bound $\bigcup_{i \in I} \mu_i$ and the greatest lower bound $\bigcap_{i \in I} \mu_i$ of the μ_i's are given by for each $x \in X$,

$$(\textstyle\bigcup_{i \in I} \mu_i)(x) = \bigvee_{i \in I} \mu_i(x) \text{ and } (\bigcap_{i \in I} \mu_i)(x) = \bigwedge_{i \in I} \mu_i(x),$$

respectively.

For each $t \in [0,1]$ the set

$$\mu_t = \{x \in A: \mu(x) \geq t\}$$

is called the level subset of μ at t [10].

Definition 2.3. *[8] A fuzzy subset μ of a bounded lattice L is said to be a fuzzy ideal of L, if for all $x, y \in L$*

1. $\mu(0) = 1$
2. $\mu(x \vee y) \geq \mu(x) \wedge \mu(y)$

3. $\mu(x \wedge y) \geq \mu(x) \vee \mu(y)$

Remark 2.4. *In [8], Swamy and Raju observed that, a fuzzy subset μ of a lattice L is a fuzzy ideal of L if and only if*

$$\mu(0) = 1 \text{ and } \mu(x \vee y) = \mu(x) \wedge \mu(y) \text{ for all } x, y \in L.$$

Definition 2.5. *[8] A proper fuzzy ideal μ of L is called prime fuzzy ideal of L if for any two fuzzy ideals θ, η of L, $\theta \cap \eta \subseteq \mu \Rightarrow \theta \subseteq \mu$ or $\eta \subseteq \mu$.*

Note that a fuzzy subset μ of L is nonempty if there exists $x \in L$ such that $\mu(x) \neq 0$. The set of all fuzzy ideals of L is denoted by $FI(L)$.

3. Closure fuzzy ideals

In this section, we introduce the concept of closure fuzzy ideals and obtained an isomorphism between the lattice of closure fuzzy ideals and fuzzy ideals of the lattice of dominator ideals.

Now we introduce a closure operation on $FI(L)$.

Definition 3.1. *For any fuzzy ideal μ of L, define a fuzzy subset $\alpha(\mu)$ of $A^e(L)$ as:*

$$\alpha(\mu)((x)^e) = Sup\{\mu(y) : (y)^e = (x)^e,\ y \in L\}.$$

Definition 3.2. *For any fuzzy ideal θ of $A^e(L)$, define a fuzzy subset $\beta(\theta)$ of L as:*

$$\beta(\theta)(x) = \theta((x)^e).$$

Lemma 3.3. *For any fuzzy ideal μ of L, $\alpha(\mu)$ is a fuzzy ideal of $A^e(L)$.*

Proof: Let μ be a fuzzy ideal of L. Clearly $\alpha(\mu)((0)^e) = 1$. For any $(x)^e, (y)^e \in A^e(L)$,

$$\begin{aligned} \alpha(\mu)((x)^e) \wedge \alpha(\mu)((y)^e) &= Sup\{\mu(a) : (a)^e = (x)^e,\ a \in L\} \\ &\quad \wedge Sup\{\mu(b) : (b)^e = (y)^e,\ b \in L\} \\ &= Sup\{\mu(a) \wedge \mu(b) : (a)^e = (x)^e,\ (b)^e = (y)^e\} \\ &\leq Sup\{\mu(a \vee b) : (a \vee b)^e = (x \vee y)^e\} \\ &\leq Sup\{\mu(c) : (c)^e = (x \vee y)^e\} \\ &= \alpha(\mu)((x)^e \underline{\vee} (y)^e) \end{aligned}$$

Again,

$$\begin{aligned} \alpha(\mu)((x)^e) &= Sup\{\mu(a) : (a)^e = (x)^e\} \leq Sup\{\mu(a \wedge y) : (a)^e \wedge (y)^e = (x)^e \wedge (y)^e\} \\ &\leq Sup\{\mu(c) : (c)^e = (x \wedge y)^e\} = \alpha(\mu)((x)^e \wedge (y)^e) \end{aligned}$$

Similarly, $\alpha(\mu)((y)^e) \leq \alpha(\mu)((x)^e \wedge (y)^e$.

Thus $\alpha(\mu)((x)^e \wedge (y)^e) \geq \alpha(\mu)((x)^e) \vee \alpha(\mu)((y)^e)$. Hence $\alpha(\mu)$ is a fuzzy ideal of $A^e(L)$.

Lemma 3.4. *For any fuzzy ideal θ of $A^e(L)$, $\beta(\theta)$ is a fuzzy ideal of L.*

Proof: Let θ be a fuzzy ideal of $A^e(L)$. Since $(0)^e$ is the least element of $A^e(L)$, $\beta(\theta)(0) = 1$. Again,

$$\begin{aligned} \beta(\theta)(x \vee y) &= \theta((x)^e \underline{\vee} (y)^e) \\ &= \theta((x)^e) \wedge \theta((y)^e) \quad since\ \theta\ isafuzzyidealof A^e(L) \\ &= \beta(\theta)(x) \wedge \beta(\theta)(y) \end{aligned}$$

Thus, $\beta(\theta)$ is a fuzzy ideal of L.

Lemma 3.5. *If μ and θ are fuzzy ideals of L, then $\mu \subseteq \theta$ implies $\alpha(\mu) \subseteq \alpha(\theta)$.*

Lemma 3.6. *If $\mu,\ \theta$ are fuzzy ideals of $A^e(L)$, then $\mu \subseteq \theta$ implies $\beta(\mu) \subseteq \beta(\theta)$.*

Theorem 3.7. *The set $FI(A^e(L))$ of all fuzzy ideals of $A^e(L)$ forms a complete distributive lattice, where the infimum and supremum of any family $\{\mu_\gamma : \gamma \in I\}$ of fuzzy ideals is given by:* $\vee$

$$\wedge\, \mu_\gamma = \cap \quad \mu_\gamma \text{ and } \vee\, \mu_\gamma = \langle \cup \quad \mu_\gamma \rangle.$$

Theorem 3.8. *The mapping α is a homomorphism of $FI(L)$ into $FI(A^e(L))$.*

Proof: Let $\mu,\ \theta$ be two fuzzy ideals of L. It is enough to prove that $\alpha(\mu \cap \theta) = \alpha(\mu) \cap \alpha(\theta)$ and $\alpha(\mu \vee \theta) = \alpha(\mu) \underline{\vee}\, \alpha(\theta)$. Since α is isotone, we get $\alpha(\mu \cap \theta) \subseteq \alpha(\mu) \cap \alpha(\theta)$. For any $(x)^e \in A^e(L)$,

$\alpha(\mu)((x)^e) \wedge \alpha(\theta)((y)^e) = Sup\{\mu(a) : (a)^e = (x)^e\} \wedge Sup\{\mu(b) : (b)^e = (y)^e\}$

Since $(a)^e = (x)^e$ and $(b)^e = (y)^e$, we get $(a \wedge b)^e = (x)^e$. Based on this we have,

$$\begin{aligned}
\alpha(\mu)((x)^e) \wedge \alpha(\theta)((x)^e) &\le Sup\{\mu(a \wedge b) : (a \wedge b)^e = (x)^e\} \\
&\quad \wedge\ Sup\{\mu(a \wedge b) : (a \wedge b)^e = (x)^e\} \\
&= Sup\{\mu(a \wedge b) \wedge \theta(a \wedge b) : (a \wedge b)^e = (x)^e\} \\
&= Sup\{(\mu \cap \theta)(a \wedge b) : (a \wedge b)^e = (x)^e\} \\
&\le Sup\{(\mu \cap \theta)(c) : (c)^e = (x)^e\} \\
&= \alpha(\mu \cap \theta)((x)^e)
\end{aligned}$$

Thus $\alpha(\mu) \cap \alpha(\theta) \subseteq \alpha(\mu \cap \theta)$. Hence, $\alpha(\mu) \cap \alpha(\theta) = \alpha(\mu \cap \theta)$. Again, clearly $\alpha(\mu) \underline{\vee}\, \alpha(\theta) \subseteq \alpha(\mu \vee \theta)$. For any $(x)^e \in A^e(L)$,

$$\begin{aligned}
\alpha(\mu \vee \theta)((x)^e) &= Sup\{(\mu \vee \theta)(a) : (a)^e = (x)^e\} \\
&= Sup\{sup\{\mu(y) \wedge \theta(z) : a = y \vee z\},\ (y \vee z)^e = (x)^e\} \\
&\le Sup\{sup\{\mu(b_1) \wedge \theta(b_2) : (b_1)^e = (y)^e,\ (b_2)^e = (z)^e\}, \\
&\qquad (y \vee z)^e = (x)^e\} \\
&= Sup\{sup\{\mu(b_1) : (b_1)^e = (y)^e\} \\
&\qquad \wedge\ sup\{\theta(b_2) : (b_2)^e = (z)^e\},\ (y \vee z)^e = (x)^e\} \\
&= Sup\{\alpha(\mu)(y)^e \wedge \alpha(\theta)(z)^e : (y \vee z)^e = (x)^e\} \\
&= Sup\{\alpha(\mu)(y)^e \wedge \alpha(\theta)(z)^e : (y)^e \underline{\vee}\, (z)^e = (x)^e\} \\
&= (\alpha(\mu) \underline{\vee}\, \alpha(\theta))((x)^e)
\end{aligned}$$

Thus $\alpha(\mu \vee \theta) \subseteq \alpha(\mu) \underline{\vee}\, \alpha(\theta)$. Hence $\alpha(\mu \vee \theta) = \alpha(\mu) \underline{\vee}\, \alpha(\theta)$. Therefore, α is a homomorphism.

Corollary 3.9. *For any two fuzzy ideals μ and θ of L, we have*

$$\beta\alpha(\mu \cap \theta) = \beta\alpha(\mu) \cap \beta\alpha(\theta).$$

Proof: For any $x \in L$, $\beta\alpha(\mu \cap \theta)(x) = \alpha(\mu \cap \theta)((x)^e)$. Since $\alpha(\mu \cap \theta) = \alpha(\mu) \cap \alpha(\theta)$, we have $\beta\alpha(\mu \cap \theta)(x) = \beta\alpha(\mu)(x) \wedge \beta\alpha(\theta)(x)$. Thus $\beta\alpha(\mu \cap \theta) = \beta\alpha(\mu) \cap \beta\alpha(\theta)$.

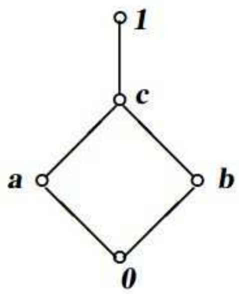
The following lemma can be verified easily.

Lemma 3.10. *For any fuzzy ideal μ of L, the map $\mu \to \beta\alpha(\mu)$ is a closure operator on $FI(L)$. That is.*

1. $\mu \subseteq \beta\alpha(\mu)$
2. $\beta\alpha(\beta\alpha(\mu)) = \beta\alpha(\mu)$
3. $\mu \subseteq \theta \Rightarrow \beta\alpha(\mu) \subseteq \beta\alpha(\theta)$

Definition 3.11. *A fuzzy ideal μ of L is called a closure fuzzy ideal if $\mu = \beta\alpha(\mu)$.*

Example 3.12. *Consider the distributive lattice $L = \{0, a, b, c, 1\}$ whose Hasse diagram is given below.*



Define a fuzzy subset μ of L as follows: $\mu(0) = 1$, $\mu(a) = 0.5$ and $\mu(b) = \mu(c) = \mu(1) = 0.4$. Then it can be easily verified that μ is a closure fuzzy ideal of L.

Theorem 3.13. *For a nonempty fuzzy subset μ of L , μ is a closure fuzzy ideal if and only if each level subset of μ is a closure ideal of L.*

Proof: Let μ be a closure fuzzy ideal of L. Then $\mu_t = (\beta\alpha(\mu))_t$. To prove each level subset of μ is a closure ideal of L, it is enough to show $\beta\alpha(\mu_t) = (\beta\alpha(\mu))_t$. Clearly $(\beta\alpha(\mu))_t \subseteq \beta\alpha(\mu_t)$. Let $x \in \beta\alpha(\mu_t)$. Then $(x)^e \in \alpha(\mu_t)$ and there is $y \in \mu_t$ such that $(x)^e = (y)^e$. Which implies $Sup\{\mu(a): (x)^e = (y)^e\} \geq t$. This shows that $x \in (\beta\alpha(\mu))_t$. Thus $\mu_t = \beta\alpha(\mu_t)$ and hence each level subset of μ is a closure ideal of L.

Conversely, assume that each level subset of μ is a closure ideal. Clearly $\mu \subseteq \beta\alpha(\mu)$. Let $t = \beta\alpha(\mu)(x) = Sup\{\mu(y): (y)^e = (x)^e\}$. Then for each $\varepsilon > 0$, there is $a \in L$, $(a)^e = (x)^e$ such that $\mu(a) > t - \varepsilon$. Which implies $a \in \mu_{t-\varepsilon}$, $(a)^e = (x)^e$, $(a)^e = (x)^e$ and $\beta\alpha(\mu_{t-\varepsilon}) = \mu_{t-\varepsilon}$. This shows that $x \in \bigcap_{\varepsilon>0} \mu_{t-\varepsilon} = \mu_t$. Thus $\beta\alpha(\mu) \subseteq \mu$. Hence, μ is a closure fuzzy ideal of L.

Corollary 3.14. *For a nonempty subset I of L, I is a closure ideal if and only if χ_I is a closure fuzzy ideal of L.*

Proof: Take a closure ideal I of L. Then $\beta\alpha(I) = \{x \in L: (x)^e \in \alpha(I)\} = I$. Let $x \in L$. If $x \in I$, then $\beta\alpha(\chi_I)(x) = 1 = \chi_I(x)$. Let $x \notin I$. Assume that $\beta\alpha(\chi_I)(x) = 1$, there is $y \in I$ such that $(y)^e = (x)^e \in \alpha(I)$. Since I is a closure ideal, $x \in I$. Which is contradiction. Thus, $\beta\alpha(\chi_I)(x) = 0$. Hence, χ_I is a closure fuzzy ideal.

Conversely, assume that χ_I is a closure fuzzy ideal of L. Clearly, $I \subseteq \beta\alpha(I)$. Let $x \in \beta\alpha(I)$. Since χ_I is a closure fuzzy ideal, $\beta\alpha(\chi_I)(x) = 1 = \chi_I(x)$. Thus, $x \in I$ and hence I is a closure ideal of L.

Theorem 3.15. *Let μ be a fuzzy ideal of L. Then μ is a closure fuzzy ideal if and only if for each $x, y \in L$, $(x)^e = (y)^e$ imply $\mu(x) = \mu(y)$.*
Proof: Let μ be a closure fuzzy ideal of L and $x, y \in L$ such that $(x)^e = (y)^e$. Then

$$\begin{aligned}\mu(x) &= Sup\{\mu(a): (a)^e = (x)^e,\ a \in L\}\\ &= Sup\{\mu(a): (a)^e = (y)^e,\ a \in L\} = \mu(y)\end{aligned}$$

Conversely, suppose that for each $x, y \in L$, $(x)^e = (y)^e$ imply $\mu(x) = \mu(y)$. For any $x \in L$, $\beta\alpha(\mu)(x) = Sup\{\mu(a): (a)^e = (x)^e,\ a \in L\} = \mu(x)$. Thus μ is a closure fuzzy ideal of L.

Lemma 3.16. *For any fuzzy ideal θ of $A^e(L)$, $\alpha\beta(\theta) = \theta$.*
Proof: Since θ is a fuzzy ideal of $A^e(L)$, $\beta(\theta)$ is a fuzzy ideal of L and $\alpha\beta(\theta)$ is a fuzzy ideal of $A^e(L)$. Now,
$\alpha\beta(\theta)((x)^e) = Sup\{\beta(\theta)(a): (a)^e = (x)^e\} = Sup\{\theta((a)^e): (a)^e = (x)^e\} = \theta((x)^e)$.
Hence the result.

Let us denote the set of all closure fuzzy ideals of L by $FI_c(L)$.

Theorem 3.17. *The class $FI_c(L)$ of all closure fuzzy ideals of L forms a complete distributive lattice with respect to set inclusion.*
Proof: Clearly $(FI_c(L), \subseteq)$ is a partially ordered set. For $\mu, \theta \in FI_c(L)$, define $\mu \wedge \theta = \mu \cap \theta$ and $\mu \underline{\vee} \theta = \beta\alpha(\mu \vee \theta)$.

Clearly $\mu \wedge \theta,\ \mu \underline{\vee} \theta \in FI_c(L)$. We need to show $\mu \underline{\vee} \theta$ is the least upper bound of μ and θ. Since $\theta,\ \mu \subseteq \mu \vee \theta \subseteq \mu \underline{\vee} \theta$, $\mu \underline{\vee} \theta$ is an upper bound of μ and θ. Let η be any upper bound for $\mu,\ \theta$ in $FI_c(L)$. Then $\mu \vee \theta \subseteq \eta$. Which implies that $\beta\alpha(\mu \vee \theta) \subseteq \beta\alpha(\eta) = \eta$. Therefore, $\beta\alpha(\mu \vee \theta)$ is the supremum of both μ and θ in $FI_c(L)$. Hence $(FI_c(L), \wedge, \underline{\vee})$ is a lattice. We now prove the distributivity. Let $\mu,\ \theta \in FI_c(L)$. Then

$$\begin{aligned}\mu \underline{\vee} (\theta \cap \eta) &= \beta\alpha((\mu \vee \theta) \cap (\mu \vee \eta))\\ &= \beta\alpha(\mu \vee \theta) \cap \beta\alpha(\mu \vee \eta)\\ &= (\mu \underline{\vee} \theta) \cap (\mu \underline{\vee} \eta)\end{aligned}$$

Thus, $FI_c(L)$ is a distributive lattice. Next we prove the completeness. Since $\{0\}$ and L are closure ideals, $\chi_{\{0\}}$ and χ_L are least and greatest elements of $FI_c(L)$ respectively. Let $\{\mu_i: i \in I\} \subseteq FI_c(L)$. Then $\bigcap_{i\in I} \mu_i$ is a fuzzy ideal of L and $\bigcap_{i\in I} \mu_i \subseteq \beta\alpha(\bigcap_{i\in I} \mu_i)$.

$$\begin{aligned}\textstyle\bigcap_{i\in I} \mu_i \subseteq \mu_i,\ \forall i \in I &\Rightarrow \beta\alpha(\textstyle\bigcap_{i\in I} \mu_i) \subseteq \mu_i,\ \forall i \in I\\ &\Rightarrow \beta\alpha(\textstyle\bigcap_{i\in I} \mu_i) \subseteq \textstyle\bigcap_{i\in I} \mu_i\end{aligned}$$

Thus $\beta\alpha(\bigcap_{i\in I}) = \bigcap_{i\in I} \mu_i$ and hence $(FI_c(L), \wedge, \underline{\vee})$ is a complete distributive lattice.

Theorem 3.18. *The class $FI_c(L)$ of all closure fuzzy ideals of L are isomorphic to the class $FI(A^e(L))$ of all fuzzy ideals of $A^e(L)$.*
Proof: Define $f: FI_c(L) \to FI(A^e(L))$, $f(\mu) = \alpha(\mu)$, $\forall \mu \in FI_c(L)$. Let $\mu, \theta \in FI_c(L)$ and $f(\mu) = f(\theta)$. Then $\alpha(\mu) = \alpha(\theta)$. This implies $\beta\alpha(\mu) = \beta\alpha(\theta)$. This shows that $\mu = \theta$. Therefore f is one to one. Let $\eta \in FI(A^e(L))$. Then $\beta(\eta)$ is a fuzzy ideal of L. We show that $\beta(\eta)$ is a closure fuzzy ideal of L. Let $x \in L$. Then $\beta\alpha(\beta(\eta))(x) =$

$\alpha\beta(\eta)((x)^e)$. Then we get $\alpha\beta(\eta)((x)^e) = \eta((x)^e) = \beta(\eta)(x)$ by lemma(3.15). Hence $\beta(\eta) = \beta\alpha(\beta(\eta))$. Thus for each $\eta \in FI(A^e(L))$, $f(\beta(\eta)) = \eta$. Therefore, f is onto. Now for any $\mu, \theta \in FI_c(L)$, $f(\mu \underline{\vee} \theta) = f(\beta\alpha(\mu \vee \theta)) = \alpha(\beta\alpha(\mu \vee \theta)) = \alpha(\mu \vee \theta) = \alpha(\mu) \vee \alpha(\theta) = f(\mu) \vee f(\theta)$. Similarly $f(\mu \cap \theta) = f(\mu) \cap f(\theta)$. Therefore, f is an isomorphism of $FI_c(L)$ onto the lattice of fuzzy ideals of $A^e(L)$.

Corollary 3.19. *The prime closure fuzzy ideals are in correspondence with the prime fuzzy ideals $A^e(L)$.*

Proof: By the above theorem the map f is an isomorphism from $FI_c(L)$ into $FI(A^e(L))$. Let μ be a prime closure fuzzy ideal of L. Then $\alpha(\mu) \in FI(A^e(L))$. Now we prove $\alpha(\mu)$ is a prime fuzzy ideal of $FI(A^e(L))$. Let $\theta, \eta \in FI(A^e(L))$ such that $\theta \wedge \eta \subseteq \alpha(\mu)$. Since f is onto, there exist $\lambda, \gamma \in FI_c(L)$ such that $f(\lambda) = \theta$ and $f(\gamma) = \eta$. Thus $\alpha(\lambda \wedge \gamma) \subseteq \alpha(\mu)$. Since β is isotone, we have $\beta\alpha(\lambda \wedge \gamma) \subseteq \beta\alpha(\mu)$. Which implies $\lambda \wedge \gamma \subseteq \mu$. Since μ is a prime fuzzy ideal, either $\lambda \subseteq \mu$ or $\gamma \subseteq \mu$. This shows that either $\alpha(\lambda) \subseteq \alpha(\mu)$ or $\alpha(\gamma) \subseteq \alpha(\mu)$. Thus $\theta \subseteq \alpha(\mu)$ or $\eta \subseteq \alpha(\mu)$. Hence $\alpha(\mu)$ is a prime fuzzy ideal of $A^e(L)$.

Conversely, suppose that θ is a fuzzy ideal in $A^e(L)$. Since f is onto, there exists a closure fuzzy ideal μ in $FI_c(L)$ such that $\theta = \alpha(\mu)$. Let $\eta, \lambda \in FI(L)$ such that $\eta \wedge \lambda \subseteq \mu$. Since α is isotone, we get $\alpha(\eta \wedge \lambda) \subseteq \alpha(\mu) = \theta$. Which implies $\alpha(\eta) \wedge \alpha(\lambda) \subseteq \alpha(\mu)$. Since $\alpha(\mu)$ is a prime fuzzy ideal of $A^e(L)$, either $\alpha(\eta) \subseteq \alpha(\mu)$ or $\alpha(\lambda) \subseteq \alpha(\mu)$. This implies $\eta \subseteq \beta\alpha(\eta) \subseteq \beta\alpha(\mu)$ or $\lambda \subseteq \beta\alpha(\lambda) \subseteq \beta\alpha(\mu)$. Since μ is a closure fuzzy ideal, we get $\eta \subseteq \mu$ or $\lambda \subseteq \mu$. This shows that μ is prime fuzzy ideal in $FI(L)$. Hence the prime closure fuzzy ideals corresponds to prime fuzzy ideals of $A^e(L)$.

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