On Multiplicative Minus Indices of Titania Nanotubes

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Received 27 October 2018; accepted 24 November 2018

Abstract. In this paper, we introduce the multiplicative minus index, multiplicative modified minus index, multiplicative minus connectivity index, multiplicative reciprocal minus connectivity index and general multiplicative minus index of a graph and compute exact formulas for titania nanotubes.

Keywords: multiplicative minus indices, titania nanotube.

AMS Mathematics Subject Classification (2010): 05C05, 05C07, 05C90

1. Introduction

Let $G$ be a finite, simple connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex $v$ is the number of vertices adjacent to $v$. For additional definitions and notations, the reader may refer to [1].

A molecular graph is a graph whose vertices correspond to the atoms and the edges to the bonds. Chemical graph theory has an important effect on the development of chemical sciences. A single number that can be used to characterize some property of the graph of molecular is called a topological index. Several topological indices have been considered in Theoretical Chemistry, see [2].

In [3], Albertson introduced the irregularity index as

$$Alb(G) = \sum_{u \in V(G)} |d_G(u) - d_G(v)|.$$  

In [4], this index is referred to as the minus index. In this paper, we introduce the following multiplicative minus topological indices:

The multiplicative minus index of a graph $G$ is defined as

$$\text{MiII}(G) = \prod_{u \in E(G)} |d_G(u) - d_G(v)|.$$  

(1)

The multiplicative square minus index of a graph $G$ is defined as

$$\text{SMiII}(G) = \prod_{u \in E(G)} \left[|d_G(u) - d_G(v)|^2\right].$$  

(2)

The multiplicative modified minus index of a graph $G$ is defined as

$$\text{MiII}(G) = \prod_{u \in E(G)} \left[|d_G(u) - d_G(v)|^2\right].$$  

(3)
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\[ M_{II}(G) = \prod_{uv \in E(G)} \frac{1}{d_G(u) - d_G(v)} \] \tag{3}

The multiplicative minus connectivity index of a graph \( G \) is defined as

\[ \text{MicII}(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{d_G(u) - d_G(v)}} \] \tag{4}

The multiplicative reciprocal minus connectivity index of a graph \( G \) is defined as

\[ \text{RMicII}(G) = \prod_{uv \in E(G)} \sqrt{d_G(u) - d_G(v)} \] \tag{5}

The general multiplicative minus index of a graph \( G \) is defined as

\[ M^*(G) = \prod_{uv \in E(G)} \left[ d_G(u) - d_G(v) \right]^2 \] \tag{6}

Recently, some new multiplicative topological indices were studied, for example, in [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20].

A study of titania nanotubes has received much attention in Mathematical and Chemical literature (see 21, 22, 23). In this paper, the multiplicative minus topological indices for titania nanotubes are determined.

2. Titania nanotubes

Titania nanotubes denoted by \( \text{TiO}_2[m, n] \) for \( m, n \in \mathbb{N} \), in which \( m \) is the number of octagons \( C_8 \) in a row and \( n \) is the number of octagons \( C_8 \) in a column. The graph of \( \text{TiO}_2[m, n] \) is shown in Figure 1.

![Figure 1: The graph of TiO\(_2\)[m, n] nanotube](image)

Let \( G \) be the graph of a titania nanotube \( \text{TiO}_2[m, n] \) with \( 6n(m+1) \) vertices and \( 10mn + 8n \) edges. In \( G \), by calculation, there are four types of edges based on the degree of end vertices of each edge as given in Table 1.

<table>
<thead>
<tr>
<th>( d_G(u), d_G(v) )</th>
<th>( uv \in E(G) )</th>
<th>(2, 4)</th>
<th>(2, 5)</th>
<th>(3, 4)</th>
<th>(3, 5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of edges</td>
<td>6n</td>
<td>4mn+2n</td>
<td>2n</td>
<td>6mn – 2n</td>
<td></td>
</tr>
</tbody>
</table>

**Table 1:** Edge partition of \( \text{TiO}_2[m, n] \)
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In the following theorem, we compute the multiplicative minus index of $\text{TiO}_2 \left[ m,n \right]$. 

**Theorem 1.** The multiplicative minus index of a titania nanotube $\text{TiO}_2 \left[ m, n \right]$ is 

$$ \text{Mill}(\text{TiO}_2) = 2^{6m+4n} \times 3^{4m+2n}.$$ 

**Proof:** Let $G = \text{TiO}_2 \left[ m, n \right]$ be the graph of a titania nanotube. By using equation (1) and Table 1, we derive 

$$ \text{Mill}(\text{TiO}_2) = \prod_{u \in E(G)} \left| d_G(u) - d_G(v) \right| $$ 

$$ = (|2 - 4|)^{6n} \times (|2 - 5|)^{4m+2n} \times (|3 - 4|)^{2n} \times (|3 - 5|)^{6m-2n} $$ 

$$ = 2^{6m+4n} \times 3^{4m+2n} $$

In the following theorem, we compute the multiplicative square minus index of $\text{TiO}_2 \left[ m,n \right]$. 

**Theorem 2.** The multiplicative square minus index of a titania nanotube $\text{TiO}_2$ is 

$$ \text{SMill}(\text{TiO}_2) = 2^{12m+8n} \times 3^{4m+4n}.$$ 

**Proof:** Let $G = \text{TiO}_2 \left[ m, n \right]$ be the graph of a titania nanotube. By using equation (2) and Table 1, we deduce 

$$ \text{SMill}(\text{TiO}_2) = \prod_{uv \in E(G)} \left[ d_G(u) - d_G(v) \right]^2 $$ 

$$ = (|2 - 4|)^{26n} \times (|2 - 5|)^{2(4m+2n)} \times (|3 - 4|)^{2c2n} \times (|3 - 5|)^{2(6m-2n)} $$ 

$$ = 2^{12m+8n} \times 3^{4m+4n} $$

In the following theorem, we compute the multiplicative modified minus index of $\text{TiO}_2 \left[ m,n \right]$. 

**Theorem 3.** The multiplicative modified minus index of a titania nanotube $\text{TiO}_2$ is 

$$ ^mM,II(\text{TiO}_2) = \left( \frac{1}{2} \right)^{6m+4n} \times \left( \frac{1}{3} \right)^{6m-2n}.$$ 

**Proof:** Let $G=\text{TiO}_2 \left[ m, n \right]$ be the graph of a titania nanotube. By using equation (3) and Table 1, we obtain 

$$ ^mM,II(\text{TiO}_2) = \prod_{uv \in E(G)} \frac{1}{\left| d_G(u) - d_G(v) \right|} $$ 

$$ = \left( \frac{1}{|2 - 4|} \right)^{6n} \times \left( \frac{1}{|2 - 5|} \right)^{4m+2n} \times \left( \frac{1}{|3 - 4|} \right)^{2n} \times \left( \frac{1}{|3 - 5|} \right)^{6m-2n} $$ 

$$ = \left( \frac{1}{2} \right)^{6m+4n} \times \left( \frac{1}{3} \right)^{6m+2n}. $$

In the following theorem, we compute the multiplicative minus connectivity index of $\text{TiO}_2 \left[ m, n \right]$. 

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Theorem 4. The multiplicative minus connectivity index of $TiO_2[m,n]$ nanotubes is

$$\text{Mic}II(TiO_2) = \left(\frac{1}{2}\right)^{3mn+4n} \times \left(\frac{1}{3}\right)^{2mn+n}.$$ 

Proof: Let $G=TiO_2[m,n]$ be the graph of a titania nanotube. By using equation (4) and Table 1, we have

$$\text{Mic}II(TiO_2) = \prod_{u \in E(G)} \frac{1}{\sqrt{d_G(u) - d_G(v)}} = \left(\frac{1}{\sqrt{|2-4|}}\right) \times \left(\frac{1}{\sqrt{|2-5|}}\right) \times \left(\frac{1}{\sqrt{|3-4|}}\right) \times \left(\frac{1}{\sqrt{|3-5|}}\right)^{6mn-2n} = \left(\frac{1}{2}\right)^{3mn+4n} \times \left(\frac{1}{3}\right)^{2mn+n}. $$

In the following theorem, we determine the multiplicative reciprocal minus connectivity index of a titania nanotube $TiO_2[m,n]$.

Theorem 5. The multiplicative reciprocal minus connectivity index of $TiO_2[m,n]$ is

$$\text{RMic}II(TiO_2) = 2^{3mn+2n} \times 3^{2mn+n}.$$

Proof: Let $G=TiO_2[m,n]$ be the graph of a titania nanotube. By using equation (5) and Table 1, we deduce

$$\text{RMic}II(TiO_2) = \prod_{u \in E(G)} \sqrt{d_G(u) - d_G(v)} = \left(\sqrt{|2-4|}\right)^{2mn+2n} \times \left(\sqrt{|2-5|}\right)^{2n} \times \left(\sqrt{|3-4|}\right)^{6mn-2n} = 2^{3mn+2n} \times 3^{2mn+n}.$$

In the following, we compute the general multiplicative minus index of $TiO_2[m,n]$.

Theorem 6. The general multiplicative minus index of $TiO_2[m,n]$ is

$$M^*_{-II}(G) = (2)^{(6mn+4n)} \times (3)^{(4mn+2n)}.$$

Proof: Let $G=TiO_2[m,n]$ be the graph of a titania nanotube. By using equation (6) and Table 1, we obtain

$$M^*_{-II}(G) = \prod_{u \in E(G)} \left|d_G(u) - d_G(v)\right| = (|2-4|)^{6n} \times (|2-5|)^{4mn+2n} \times (|3-4|)^{2n} \times (|3-5|)^{6mn-2n} = (2)^{(6mn+4n)} \times (3)^{(4mn+2n)}.$$

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