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Multiplicative *KV* and Multiplicative Hyper -*KV* Indices of Certain Dendrimers

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Abstract. In this study, we define the multiplicative *KV* indices and multiplicative hyper- *KV* indices of a graph. Also we introduce the general multiplicative *KV* indices of a graph. Furthermore, we compute these indices for POPAM and tetrathiafulvalene dendrimers.

Keywords: Multiplicative *KV* indices, multiplicative hyper *KV* indices, general multiplicative *KV* indices, dendrimer.

AMS Mathematics Subject Classification (2010): 05*C*05, 05*C*07, 05*C*12, 05*C*35

1. Introduction

We consider only finite, connected, undirected graphs without loops and multiple edges. The degree of a vertex v, denoted by $d_G(v)$, is the number of vertices adjacent to v. Let $M_G(v)$ denote the product of the degrees of all vertices adjacent to v. We refer to [1] for undefined terminology not given here.

A topological index is a numerical parameter mathematically derived from the graph structure. Several topological indices have been considered in Mathematical Chemistry.

In [2], Kulli put forward the first and second *KV* indices:

$$KV_1(G) = \sum_{uv \in E(G)} \left[M_G(u) + M_G(v)\right],$$

$$KV_2(G) = \sum_{uv \in E(G)} \left[M_G(u) M_G(v)\right].$$

Recently, some new variants of *KV* indices were proposed and studied such as hyper *KV* indices [3], square *KV* index [3], connectivity *KV* indices [4].

We now define first and second multiplicative *KV* indices as

$$KV_1II(G) = \prod_{uv \in E(G)} \left[M_G(u) + M_G(v)\right],$$

$$KV_2II(G) = \prod_{uv \in E(G)} M_G(u) M_G(v).$$

Also we introduce the first and second multiplicative hyper *KV* indices and they are defined as

$$HKV_1II(G) = \prod_{uv \in E(G)} \left[M_G(u) + M_G(v)\right]^2,$$

$$HKV_2II(G)=\prod_{uv\in E(G)}\left[M_G(u)M_G(v)\right]^2.$$

We propose the general first and second multiplicative *KV* indices of a graph *G*, defined as

$$KV_1^aII(G)=\prod_{uv\in E(G)}\left[M_G(u)+M_G(v)\right]^a, \tag{1}$$

$$KV_2^aII(G)=\prod_{uv\in E(G)}\left[M_G(u)M_G(v)\right]^a. \tag{2}$$

where *a* is a real number.

Recently some new multiplicative indices were studied, see [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15].

In this paper, the first and second multiplicative *KV* indices, first and second multiplicative hyper *KV* indices, and general first and second multiplicative *KV* indices of POPAM and tetrathiafulvalene dendrimers are determined.

2. POPAM dendrimers

The family of POPAM dendrimers is symbolized by $POD_2[n]$, where *n* is the steps of growth in this type of dendrimers. The graph of $POD_2[2]$ is shown in Figure 1.

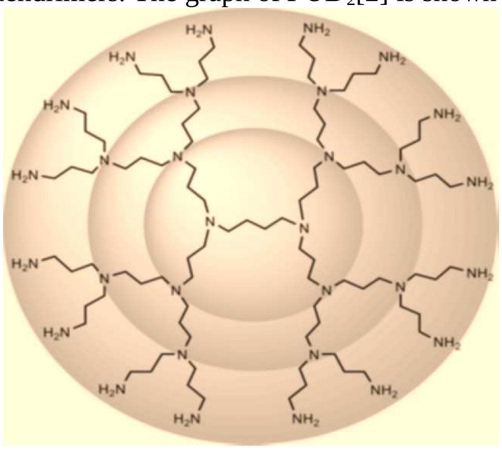

Figure 1: Graph of $POD_2[2]$

Let $G = POD_2[n]$. By calculation, we have $|V(G)| = 2^{n+5} – 10$ and $|E(G)| = 2^{n+5} – 11$.

Lemma 1. Let $G = POD_2[n]$ be a POPAM dendrimer with $2^{n+5} – 11$ edges. The edge partition of $POD_2[n]$ based on the degree product of neighbors of end vertices of each edge is as follows:

$E_1 = \{ uv \in E(G) \mid M_G(u) = M_G(v) = 2 \}$, $|E_1| = 2^{n+2}$.
$E_2 = \{ uv \in E(G) \mid M_G(u) = 2, M_G(v) = 4 \}$, $|E_2| = 2^{n+2}$.
$E_3 = \{ uv \in E(G) \mid M_G(u) = M_G(v) = 4\}$, $|E_3| = 1$.
$E_4 = \{ uv \in E(G) \mid M_G(u) = 4, M_G(v) = 6 \}$, $|E_4| = 3\times2^n – 6$.
$E_5 = \{ uv \in E(G) \mid M_G(u) = 6, M_G(v) = 8 \}$, $|E_5| = 3\times2^n – 6$.

Theorem 2. The general first multiplicative *KV* index of a POPAM dendrimer $POD_2[n]$ is

$$KV_1^a II\left(POD_2[n]\right) = 4^{a2^{n+2}} \times 6^{a2^{n+2}} \times 8^a \times 10^{a\left(3\times 2^n - 6\right)} \times 14^{a\left(3\times 2^n - 6\right)}. \quad (3)$$

Proof: Let $G = POD_2[n]$. From equation (1) and by Lemma 1, we deduce

$$KV_1^a II\left(POD_2[n]\right) = \prod_{uv\in E(G)} \left[M_G(u) + M_G(v)\right]^a$$
$$= \left[(2+2)^a\right]^{2^{n+2}} \times \left[(2+4)^a\right]^{2^{n+2}} \times \left[(4+4)^a\right]^1 \times \left[(4+6)^a\right]^{3\times 2^n - 6} \times \left[(6+8)^a\right]^{3\times 2^n - 6}$$
$$= 4^{a2^{n+2}} \times 6^{a2^{n+2}} \times 8^a \times 10^{a\left(3\times 2^n - 6\right)} \times 14^{a\left(3\times 2^n - 6\right)}.$$

The following results are obtained by using Theorem 2.

Corollary 2.1. The first multiplicative *KV* index of $POD_2[n]$ is

$$KV_1II(POD_2[n]) = 4^{2^{n+2}} \times 6^{2^{n+2}} \times 8 \times 10^{3\times 2^n - 6} \times 14^{3\times 2^n - 6}.$$

Proof: Put $a = 1$ in equation (3), we get the required result.

Corollary 2.2. The first multiplicative hyper *KV* index of $POD_2[n]$ is

$$HKV_1II(POD_2[n]) = 4^{2^{n+3}} \times 6^{2^{n+3}} \times 8^2 \times 10^{6\times 2^n - 12} \times 14^{6\times 2^n - 12}.$$

Proof: Put $a = 2$ in equation (3), we obtain the desired result.

Theorem 3. The general second multiplicative *KV* index of a POPAM dendrimer $POD_2[n]$ is

$$KV_2^a II\left(POD_2[n]\right) = 4^{a2^{n+2}} \times 8^{a2^{n+2}} \times 16^a \times 24^{a\left(3\times 2^n - 6\right)} \times 48^{a\left(3\times 2^n - 6\right)}. \quad (4)$$

Proof: Let $G = POD_2[n]$. By using equation (2) and by Lemma 1, we obtain

$$KV_2^a II\left(POD_2[n]\right) = \prod_{uv\in E(G)} \left[M_G(u) M_G(v)\right]^a$$
$$= \left[(2\times 2)^a\right]^{2^{n+2}} \times \left[(2\times 4)^a\right]^{2^{n+2}} \times \left[(4\times 4)^a\right]^1 \times \left[(4\times 6)^a\right]^{3\times 2^n - 6} \times \left[(6\times 8)^a\right]^{3\times 2^n - 6}$$
$$= 4^{a2^{n+2}} \times 8^{a2^{n+2}} \times 16^a \times 24^{a\left(3\times 2^n - 6\right)} \times 48^{a\left(3\times 2^n - 6\right)}.$$

We obtain the following results by using Theorem 3.

Corollary 3.1. The second multiplicative *KV* index of $POD_2[n]$ is

$$KV_2II(POD_2[n]) = 4^{2^{n+2}} \times 8^{2^{n+2}} \times 16 \times 24^{3\times 2^n - 6} \times 48^{3\times 2^n - 6}.$$

Proof: Put $a = 1$ in equation (4), we get the required result.

Corollary 3.2. The second multiplicative hyper *KV* index of $POD_2[n]$ is

$$HKV_2II(POD_2[n]) = 4^{2^{n+3}} \times 8^{2^{n+3}} \times 16^2 \times 24^{3\times 2^{n+1} - 12} \times 48^{3\times 2^{n+1} - 12}.$$

Proof: Put $a = 2$ in equation (4), we obtain the desired result.

2. Tetrathiafulvalene dendrimers $TD_2[n]$

The family of tetrathiafulvalene dendrimers is denoted by $TD_2[n]$, where n is the steps of growth in this type of dendrimers. The molecular graph of $TD_2[2]$ is shown in Figure 2.

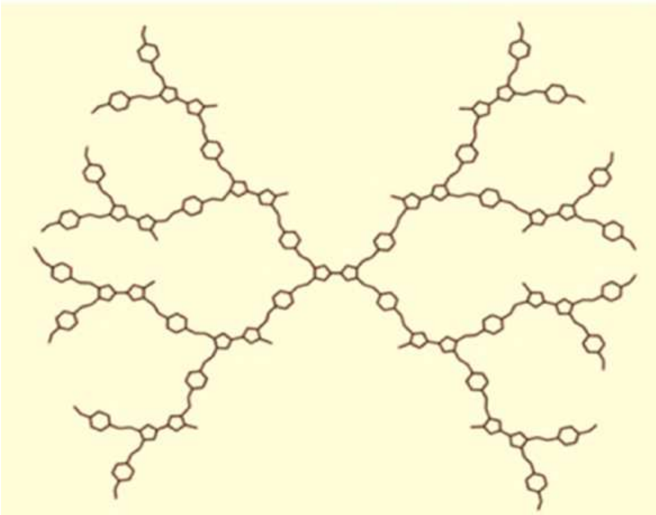

Figure 2: The molecular graph of $TD_2[2]$

Let $G = TD_2[n]$. By calculation, we obtain that G has $31\times2^{n+2} - 74$ vertices and $35\times2^{n+2} - 85$ edges. The edge partition of $TD_2[n]$ based on the degree product of neighbors of end vertices of each edge is obtained as given in Table 1.

$M_G(u), M_G(v)\backslash uv\in E(G)$	Number of edges
(2, 3)	2^{n+2}
(3, 6)	$2^{n+2} - 4$
(3, 8)	2^{n+2}
(6, 6)	$7\times2^{n+2} - 16$
(6, 8)	$11\times2^{n+2} - 24$
(6, 9)	$2^{n+2} - 4$
(6, 12)	$3\times2^{n+2} - 8$
(9, 12)	$8\times2^{n+2} - 24$
(12, 12)	$2\times2^{n+2} - 5$

Table 1: Edge partition of $TD_2[n]$

Theorem 4. The general first multiplicative *KV* index of a tetrathiafulvalene dendrimer $TD_2[n]$ is

$$KV_1^a II(TD_2[n]) = 5^{a2^{n+2}} \times 9^{a(2^{n+2}-4)} \times 11^{a2^{n+2}} \times 12^{a(7\times 2^{n+2}-16)} \times 14^{a(11\times 2^{n+2}-24)}$$

$$\times 15^{a(2^{n+2}-4)} \times 18^{a(3\times 2^{n+2}-8)} \times 21^{a(8\times 2^{n+2}-24)} \times 24^{a(2\times 2^{n+2}-5)} \quad (5)$$

Proof: Let $G = TD_2[n]$. By using equation (1) and Table 1, we obtain

$$KV_1^a II(TD_2[n]) = \prod_{uv\in E(G)} \left[M_G(u) + M_G(v)\right]^a$$

$$= \left[(2+3)^a\right]^{2^{n+2}} \times \left[(3+6)^a\right]^{2^{n+2}} \times \left[(3+8)^a\right]^{2^{n+2}} \times \left[(6+6)^a\right]^{7\times 2^{n+2}-16}$$

$$\times \left[(6+8)^a\right]^{11\times 2^{n+2}-24} \times \left[(6+9)^a\right]^{2^{n+2}-4} \times \left[(6+12)^a\right]^{3\times 2^{n+2}-4}$$

$$\times \left[(9+12)^a\right]^{8\times 2^{n+2}-24} \times \left[(12+12)^a\right]^{2\times 2^{n+2}-5}$$

$$= 5^{a2^{n+2}} \times 9^{a(2^{n+2}-4)} \times 11^{a2^{n+2}} \times 12^{a(7\times 2^{n+2}-16)} \times 14^{a(11\times 2^{n+2}-24)}$$

$$\times 15^{a(2^{n+2}-4)} \times 18^{a(3\times 2^{n+2}-8)} \times 21^{a(8\times 2^{n+2}-24)} \times 24^{a(2\times 2^{n+2}-5)}$$

We establish the following results by using Theorem 4.

Corollary 4.1. The first multiplicative *KV* index of $TD_2[n]$ is

$$KV_1 II(TD_2[n]) = 5^{2^{n+2}} \times 9^{2^{n+2}-4} \times 11^{2^{n+2}} \times 12^{7\times 2^{n+2}-16} \times 14^{11\times 2^{n+2}-24}$$

$$\times 15^{2^{n+2}-4} \times 18^{3\times 2^{n+2}-8} \times 21^{8\times 2^{n+2}-24} \times 24^{2\times 2^{n+2}-5}.$$

Proof: Put $a = 1$ in equation (5), we obtain the desired result.

Corollary 4.2. The first multiplicative hyper *KV* index of $TD_2[n]$ is

$$HKV_1 II(TD_2[n]) = 5^{2\times 2^{n+2}} \times 9^{2(2^{n+2}-4)} \times 11^{2\times 2^{n+2}} \times 12^{2(7\times 2^{n+2}-16)} \times 14^{2(11\times 2^{n+2}-24)}$$

$$\times 15^{2(2^{n+2}-4)} \times 18^{2(3\times 2^{n+2}-8)} \times 21^{2(8\times 2^{n+2}-24)} \times 24^{2(2\times 2^{n+2}-5)}.$$

Proof: Put $a = 2$ in equation (5), we obtain the desired result.

Theorem 5. The general second multiplicative *KV* index of a tetrathiafulvalene dendrimer $TD_2[n]$ is

$$KV_2^a II(TD_2[n]) = 6^{a2^{n+2}} \times 18^{a(2^{n+2}-4)} \times 24^{a2^{n+2}} \times 36^{a(7\times 2^{n+2}-16)} \times 48^{a(11\times 2^{n+2}-24)}$$

$$\times 54^{a(2^{n+2}-4)} \times 72^{a(3\times 2^{n+2}-8)} \times 108^{a(8\times 2^{n+2}-24)} \times 144^{a(2\times 2^{n+2}-5)} \quad (6)$$

Proof: Let $G = TD_2[n]$. By using equation (2) and Table 1, we deduce

$$KV_2^a II\left(TD_2[n]\right) = \prod_{uv\in E(G)} \left[M_G(u)M_G(v)\right]^a$$

$$= \left[(2\times 3)^a\right]^{2^{n+2}} \times \left[(3\times 6)^a\right]^{2^{n+2}-4} \times \left[(3\times 8)^a\right]^{2^{n+2}} \times \left[(6\times 6)^a\right]^{7\times 2^{n+2}-16}$$

$$\times \left[(6\times 8)^a\right]^{11\times 2^{n+2}-24} \times \left[(6\times 9)^a\right]^{2^{n+2}-4} \times \left[(6\times 12)^a\right]^{3\times 2^{n+2}-8}$$

$$\times \left[(9\times 12)^a\right]^{8\times 2^{n+2}-24} \times \left[(12\times 12)^a\right]^{2\times 2^{n+2}-5}$$

$$= 6^{a2^{n+2}} \times 18^{a\left(2^{n+2}-4\right)} \times 24^{a2^{n+2}} \times 36^{a\left(7\times 2^{n+2}-16\right)} \times 48^{a\left(11\times 2^{n+2}-24\right)}$$

$$\times 54^{a\left(2^{n+2}-4\right)} \times 72^{a\left(3\times 2^{n+2}-8\right)} \times 108^{a\left(8\times 2^{n+2}-24\right)} \times 144^{a\left(2\times 2^{n+2}-5\right)}$$

We establish the following results by using Theorem 5.

Corollary 5.1. The second multiplicative *KV* index of $TD_2[n]$ is

$$KV_2 II\left(TD_2[n]\right) = 6^{2^{n+2}} \times 18^{2^{n+2}-4} \times 24^{2^{n+2}} \times 36^{7\times 2^{n+2}-16} \times 48^{11\times 2^{n+2}-24}$$

$$\times 54^{2^{n+2}-4} \times 72^{3\times 2^{n+2}-8} \times 108^{8\times 2^{n+2}-24} \times 144^{2\times 2^{n+2}-5}.$$

Proof: Put $a = 1$ in equation (6), we obtain the desired result.

Corollary 5.2. The second multiplicative hyper *KV* index of $TD_2[n]$ is

$$HKV_2 II\left(TD_2[n]\right) = 6^{2\times 2^{n+2}} \times 18^{2\left(2^{n+2}-4\right)} \times 24^{2\times 2^{n+2}} \times 36^{2\left(7\times 2^{n+2}-16\right)} \times 48^{2\left(11\times 2^{n+2}-24\right)}$$

$$\times 54^{2\left(2^{n+2}-4\right)} \times 72^{2\left(3\times 2^{n+2}-8\right)} \times 108^{2\left(8\times 2^{n+2}-24\right)} \times 144^{2\left(2\times 2^{n+2}-5\right)}.$$

Proof: Put $a = 2$ in equation (6), we obtain the desired result.

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