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Multiplicative *ABC*, *GA* and *AG* Neighborhood Dakshayani Indices of Dendrimers

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Abstract. Connectivity indices are applied to measure the chemical characteristics of chemical compounds in Chemical Sciences, Medical Sciences. In this study, we introduce the multiplicative *ABC* neighborhood Dakshayani index, multiplicative *GA* neighborhood Dakshayani index and multiplicative *AG* neighborhood Dakshayani index of a molecular graph. We compute these multiplicative connectivity neighborhood Dakshayani indices of *POPAM* dendrimers. Also we determine the multiplicative sum connectivity neighborhood Dakshayani index and multiplicative product connectivity neighborhood Dakshayani index of *POPAM* dendrimers.

Keywords: Multiplicative connectivity neighborhood Dakshayani indices, dendrimer

AMS Mathematics Subject Classification (2010): 05*C*05, 05*C*07, 05*C*35, 92E10

1. Introduction

Chemical Graph Theory is a branch of Mathematical Chemistry which has an important effect on the development of Chemical Sciences, Medical Sciences. Numerous topological indices or graph indices have been considered in Theoretical Chemistry, especially in QSPR/QSAR study, see [1, 2].

Let G be a finite, simple, connected graph with vertex V(G) and edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The set of all vertices which adjacent to a vertex v is called open neighborhood of v and denoted by $N_G(v)$. The closed neighborhood set of a vertex v is the set $N_G[v] = N_G(v) \cup \{v\}$. The set $N_G[v]$ is the set of closed neighborhood vertices of v. Let $D_G(v) = d_G(v) + \sum_{u \in N_G(v)} D_G(u)$ be the degree sum of closed neighborhood vertices of v. We refer [3] for undefined definitions and notations.

The first and second neighborhood Dakshayani indices of a graph were introduced by Kulli in [4], defined as

$$ND_1(G) = \sum_{uv \in E(v)} \left[D_G(u) + D_G(v)\right], \qquad ND_2(G) = \sum_{uv \in E(v)} D_G(u) D_G(v).$$

Recently, some novel variants of neighborhood Dakshayani indices were introduced and studied such as F-neighborhood Dakshayani index [5], square neighborhood Dakshayani index [6], sum and product connectivity neighborhood Dakshayani indices [7].

In [8], Kulli introduced the first and second multiplicative neighborhood Dakshayani indices, first and second multiplicative hyper Dakshayani indices, multiplicative sum and product connectivity neighborhood Dakshayani indices, reciprocal multiplicative neighborhood Dakshayani index, general first and second multiplicative neighborhood Dakshayani indices of a graph, defined as

$$ND_1II(G) = \prod_{uv\in E(G)} \left[D_G(u) + D_G(v)\right].$$

$$ND_2II(G) = \prod_{uv\in E(G)} D_G(u) D_G(v).$$

$$HND_1II(G) = \prod_{uv\in E(G)} \left[D_G(u) + D_G(v)\right]^2.$$

$$HND_2II(G) = \prod_{uv\in E(G)} \left[D_G(u) D_G(v)\right]^2.$$

$$SNDII(G) = \prod_{uv\in E(G)} \frac{1}{\sqrt{D_G(u) + D_G(v)}}.$$

$$PND(G) = \prod_{uv\in E(G)} \frac{1}{\sqrt{D_G(u) D_G(v)}}.$$

$$RNDII(G) = \prod_{uv\in E(G)} \sqrt{D_G(u) D_G(v)}.$$

$$ND_1^aII(G) = \prod_{uv\in E(G)} \left[D_G(u) + D_G(v)\right]^a, \tag{1}$$

$$ND_2^aII(G) = \prod_{uv\in E(G)} \left[D_G(u) D_G(v)\right]^a, \tag{2}$$

where a is a real number.

We introduce the multiplicative atom bond connectivity (*ABC*) neighborhood Dakshayani index of a graph G, defined as

$$ABCNDII(G) = \prod_{uv\in E(G)} \sqrt{\frac{D_G(u) + D_G(v) - 2}{D_G(u) D_G(v)}}. \tag{3}$$

We now propose the multiplicative geometric-arithmetic (*GA*) neighborhood Dakshayani index, defined as

$$GANDII(G) = \prod_{uv\in E(G)} \frac{2\sqrt{D_G(u) D_G(v)}}{D_G(u) + D_G(v)}. \tag{4}$$

Also we introduce the multiplicative arithmetic-geometric (*AG*) neighborhood Dakshayani index of a graph, defined as

$$AGNDII(G) = \prod_{uv\in E(G)} \frac{D_G(u) + D_G(v)}{2\sqrt{D_G(u) D_G(v)}}. \tag{5}$$

Recently some new multiplicative indices were studied for example, in [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 15, 26].

In this study, the multiplicative sum and product connectivity neighborhood Dakshayani indices, *ABC*, *GA*, *AG* neighborhood Dakshayani indices of *POPAM* dendrimers were determined.

2. Results for *POPAM* Dendrimers

We consider the family *POPAM* dendrimers, which is symbolized by $POD_2[n]$, where n is the steps of growth in *POPAM* dendrimers. A graph of $POD_2[2]$ is presented in Figure 1.

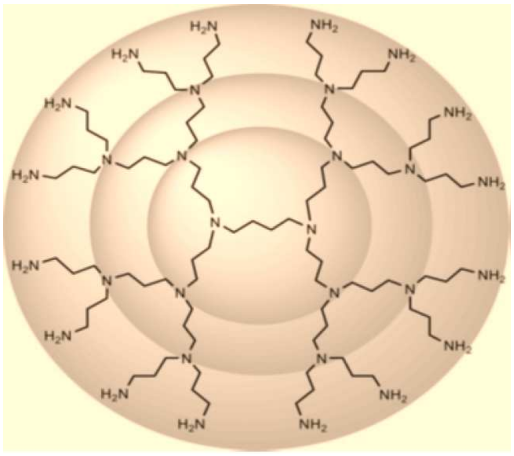

Figure 1: Graph of $POD_2[2]$.

Let G be the graph of a $POD_2[n]$ dendrimer. By calculation, G has $2^{n+5}- 11$ edges. The edge partition of G based on the degree sum of closed neighborhood vertices of each vertex is obtained in given in Table 1.

Table 1: Edge partition of $POD_2[n]$

$D_G(u), D_G(v) \setminus uv\in E(G)$	(3, 5)	(5, 6)	(6, 6)	(6, 7)	(7, 9)
Number of edges	2^{n+2}	2^{n+2}	1	$3\times2^{n+2} - 6$	$3\times2^{n+2} - 6$

Theorem 1. The general first multiplicative neighborhood Dakshayani index of a $POD_2[n]$ is

$$ND_1^a II\left[POD_2(n)\right] = 8^{a2^{n+2}} \times 11^{a2^{n+2}} \times 12^a \times 8^{a\left(3\times2^{n+2}-6\right)} \times 16^{a\left(3\times2^{n+2}-6\right)}. \quad (6)$$

Proof: Let $G = POD_2[n]$. By using equation (1) and Table 1, we obtain

$$ND_1^a II\left(POD_2[n]\right) = \prod_{uv\in E(G)} \left[D_G(u) + D_G(v)\right]^a$$

$$= (3+5)^{a2^{n+2}} \times (5+6)^{a2^{n+2}} \times (6+6)^a \times (6+7)^{a\left(3\times2^{n+2}-6\right)} + (7+9)^{a\left(3\times2^{n+2}-6\right)}$$

$$= 8^{a2^{n+2}} \times 11^{a2^{n+2}} \times 12^a \times 13^{a\left(3\times2^{n+2}-6\right)} \times 16^{a\left(3\times2^{n+2}-6\right)}.$$

We obtain the following results by using Theorem 1.

Corollary 1.1. If $POD_2[n]$ is a graph of *POPAM* dendrimer, then

i) $ND_1II\left(POD_2[n]\right)=8^{2^{n+2}}\times 11^{2^{n+2}}\times 12\times 13^{3\times 2^{n+2}-6}\times 16^{3\times 2^{n+2}-6}$.

ii) $HND_1II\left(POD_2[n]\right)=8^{2^{n+3}}\times 11^{2^{n+3}}\times 12^2\times 13^{6\times 2^{n+2}-12}\times 16^{6\times 2^{n+2}-12}$.

iii) $SNDII\left(POD_2[n]\right)=\left(\frac{1}{\sqrt{8}}\right)^{2^{n+2}}\times\left(\frac{1}{\sqrt{11}}\right)^{2^{n+2}}\times\left(\frac{1}{\sqrt{12}}\right)\times\left(\frac{1}{\sqrt{13}}\right)^{3\times 2^{n+2}-6}\times\left(\frac{1}{4}\right)^{3\times 2^{n+2}-6}$.

Proof: Put a = 1, 2, – ½ in equation (6), we get the required results.

Theorem 2. The general second multiplicative neighborhood Dakshayani index of $POD_2[n]$ is

$$ND_2^aII\left[POD_2(n)\right]=15^{a2^{n+2}}\times 30^{a2^{n+2}}\times 36^a\times 42^{a\left(3\times 2^{n+2}-6\right)}\times 63^{a\left(3\times 2^{n+2}-6\right)} \tag{7}$$

Proof: Let G be a graph of $POD_2[n]$. By using equation (2) and Table 1, we deduce

$$ND_1^aII\left(POD_2[n]\right)=\prod_{uv\in E(G)}\left[D_G(u)D_G(v)\right]^a$$

$$=(3\times 5)^{a2^{n+2}}\times(5\times 6)^{a2^{n+2}}\times(6\times 6)^a+(6\times 7)^{a\left(3\times 2^{n+2}-6\right)}\times(7\times 9)^{a\left(3\times 2^{n+2}-6\right)}$$

$$=15^{a2^{n+2}}\times 30^{a2^{n+2}}\times 36^a\times 42^{a\left(3\times 2^{n+2}-6\right)}\times 63^{a\left(3\times 2^{n+2}-6\right)}.$$

Corollary 2.1. Let $POD_2[n]$ be a graph of *POPAM* dendrimer. Then

i) $ND_2II\left(POD_2[n]\right)=15^{2^{n+2}}\times 30^{2^{n+2}}\times 36\times 42^{3\times 2^{n+2}-6}\times 63^{3\times 2^{n+2}-6}$.

ii) $HND_2II\left(POD_2[n]\right)=15^{2^{n+3}}\times 30^{2^{n+3}}\times 36^2\times 42^{6\times 2^{n+2}-12}\times 63^{6\times 2^{n+2}-12}$.

iii) $PNDII\left(POD_2[n]\right)=\left(\frac{1}{\sqrt{15}}\right)^{2^{n+2}}\times\left(\frac{1}{\sqrt{30}}\right)^{2^{n+2}}\times\left(\frac{1}{6}\right)\times\left(\frac{1}{\sqrt{42}}\right)^{3\times 2^{n+2}-6}\times\left(\frac{1}{\sqrt{63}}\right)^{3\times 2^{n+2}-6}$.

iv) $RNDII\left(POD_2[n]\right)=\left(\sqrt{15}\right)^{2^{n+2}}\times\left(\sqrt{30}\right)^{2^{n+2}}\times\left(\sqrt{42}\right)^{3\times 2^{n+2}-2}\times\left(\sqrt{63}\right)^{3\times 2^{n+2}-6}$.

Proof: Put a = 1, 2, – ½, ½ in equation (7), we obtain the desired results.

Theorem 3. The multiplicative atom bond connectivity neighborhood Dakshayani index of $POD_2[n]$ is given by

$$ABCNDII\left(POD_2[n]\right)=\left(\frac{2}{5}\right)^{2^{n+2}}\times\left(\sqrt{\frac{3}{10}}\right)^{2^{n+2}}\times\sqrt{\frac{5}{18}}\times\left(\sqrt{\frac{11}{42}}\right)^{3\times 2^{n+2}-6}\times\left(\sqrt{\frac{2}{9}}\right)^{3\times 2^{n+2}-6}.$$

Proof: Let G be a graph of $POD_2[n]$. From equation (3) and by using Table 1, we deduce

$$ABCND\left(POD_2[n]\right)=\prod_{uv\in E(G)}\sqrt{\frac{D_G(u)+D_G(v)-2}{D_G(u)D_G(v)}}$$

$$=\left(\sqrt{\frac{3+5-2}{3\times 5}}\right)^{2^{n+2}}\times\left(\sqrt{\frac{5+6-2}{5\times 6}}\right)^{2^{n+2}}\times\left(\sqrt{\frac{6+6-2}{6\times 6}}\right)^{1}$$

$$\times\left(\sqrt{\frac{6+7-2}{6\times 7}}\right)^{3\times 2^{n+2}-6}\times\left(\sqrt{\frac{7+9-2}{7\times 9}}\right)^{3\times 2^{n+2}-6}$$

$$=\left(\sqrt{\frac{2}{5}}\right)^{2^{n+2}}\times\left(\sqrt{\frac{3}{10}}\right)^{2^{n+2}}\times\sqrt{\frac{5}{18}}\times\left(\sqrt{\frac{11}{42}}\right)^{3\times2^{n+2}-6}\times\left(\sqrt{\frac{2}{9}}\right)^{3\times2^{n+2}-6}.$$

Theorem 4. The multiplicative geometric-arithmetic neighborhood Dakshayani index of $POD_2[n]$ is given by

$$GANDII\left(POD_2[n]\right)=\left(\frac{\sqrt{15}}{4}\right)^{2^{n+2}}\times\left(\frac{2\sqrt{30}}{11}\right)^{2^{n+2}}\times\left(\frac{2\sqrt{42}}{13}\right)^{3\times2^{n+2}-6}\times\left(\frac{3\sqrt{7}}{8}\right)^{3\times2^{n+2}-6}.$$

Proof: Let G be a graph of $POD_2[n]$. From equation (4) and by using Table 1, we derive

$$GANDII\left(POD_2[n]\right)=\prod_{uv\in E(G)}\frac{2\sqrt{D_G(u)D_G(v)}}{D_G(u)+D_G(v)}$$

$$=\left(\frac{2\sqrt{3\times5}}{3+5}\right)^{2^{n+2}}\times\left(\frac{2\sqrt{5\times6}}{5+6}\right)^{2^{n+2}}\times\left(\frac{2\sqrt{6\times6}}{6+6}\right)\times\left(\frac{2\sqrt{6\times7}}{6+7}\right)^{3\times2^{n+2}-6}\times\left(\frac{2\sqrt{7\times9}}{7+9}\right)^{3\times2^{n+2}-6}$$

$$=\left(\frac{\sqrt{15}}{4}\right)^{2^{n+2}}\times\left(\frac{2\sqrt{30}}{11}\right)^{2^{n+2}}\times\left(\frac{2\sqrt{42}}{13}\right)^{3\times2^{n+2}-6}\times\left(\frac{3\sqrt{7}}{8}\right)^{3\times2^{n+2}-6}$$

Theorem 5. The multiplicative arithmetic-geometric neighborhood Dakshayani index of $POD_2[n]$ is

$$AGNDII\left(POD_2[n]\right)=\left(\frac{4}{\sqrt{15}}\right)^{2^{n+2}}\times\left(\frac{11}{2\sqrt{30}}\right)^{2^{n+2}}\times\left(\frac{13}{2\sqrt{42}}\right)^{3\times2^{n+2}-6}\times\left(\frac{8}{3\sqrt{7}}\right)^{3\times2^{n+2}-6}.$$

Proof: Let $G = POD_2[n]$. By using equation (5) and Table 1, we obtain

$$AGNDII\left(POD_2[n]\right)=\prod_{uv\in E(G)}\frac{D_G(u)+D_G(v)}{2\sqrt{D_G(u)D_G(v)}}$$

$$=\left(\frac{3+5}{2\sqrt{3\times5}}\right)^{2^{n+2}}\times\left(\frac{5+6}{2\sqrt{5\times6}}\right)^{2^{n+2}}\times\left(\frac{6+6}{2\sqrt{6\times6}}\right)\times\left(\frac{6+6}{2\sqrt{6\times7}}\right)^{3\times2^{n+2}-6}\times\left(\frac{7+9}{2\sqrt{7\times9}}\right)^{3\times2^{n+2}-6}$$

$$=\left(\frac{4}{\sqrt{15}}\right)^{2^{n+2}}\times\left(\frac{11}{2\sqrt{30}}\right)^{2^{n+2}}\times\left(\frac{13}{2\sqrt{42}}\right)^{3\times2^{n+2}-6}\times\left(\frac{8}{3\sqrt{7}}\right)^{3\times2^{n+2}-6}$$

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