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Generalized Closed Set in Intuitionistic Fuzzy Topology

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Abstract. The goal of this paper is to introduce new sets called ($g_{ɭ}$-closed set and $g_{ɭ}$-open set) defined on a intuitionistic topological space. A sufficient condition is presented to be a g_I-closed set and $g_{ɭ}$-open set. And we also discuss union and intersection around these groups. Finally, we introduce a new intuitionistic separation axiom called $\mathfrak{I}_{ɭ_{1/2}}$-space and verify its basic properties.

Keywords: Intuitionisticset; $g_{ɭ}$-closed; $g_{ɭ}$-open; $\mathfrak{I}_{ɭ_{1/2}}$-space

AMS Mathematics Subject Classification (2010): 34A25

1. Introduction

In 1965 the mathematician Zadeh [7] introduced the concept of fuzzy sets that have important applications in many areas of life. The fuzzy set was used to tackle life problems that are ambiguous and which cannot be addressed using classical sets. In 1983 Atanassov [1] introduced the concept of intuitionistic fuzzy combinations that incorporate the gradient level called the Stuttering Edge. Information and semantic imaging of an intuitionistic fuzzy set became important in an intelligent and materialistic way, because it includes the property level and the level of non-related and the troubled edge then again, Coker [3] introduced the separate type of intuitionistic fuzzy set to be specific intuitionistic sets in 1996, where each of sets are completely new sets. In any case, it has a score and 3 degrees of non-participation, so this idea gives us increasingly adaptable methodologies in speaking to ambiguity in scientific articles integrating those in building fields with the traditional rationale. In 2000, Coker [4] additionally introduced the idea of intuitionistic topological spaces with an intuitionistic set and explored their properties.Talukder et al. [5] Tamilmani [6] later began clarifying the concept of opaque intuitionistic fuzzy topological spaces by using intuitionistic fuzzy sets. In 2018, Mahbub, Hossain and Hossain [7], Used intuitionistic fuzzy topological spaces to introduced and study the compactness in. Sahoo and Pal [8], used intuitionistic fuzzy to competition graphs, Bhowmik and Pal [9], conclude some results on intuitionistic fuzzy matrices and intuitionistic circulant fuzzy matrices. Adak et al. [10], defend some properties of generalized intuitionistic fuzzy nilpotent matrices over distributive lattice. Sahoo and Pal [11], investigated different types of products on intuitionistic fuzzy graphs. Senapati et al. [12] used ideals to in different algebras. In this paper, we use the concept of (intuitionistic fuzzy topology) to modify (g_I-closed) and link it with the separation axioms of space (intuitionistic fuzzy topological space).

2. Preliminaries

In this section, we remember some basic concepts related to intuitionistic sets.

Definition 2.1. [2] Intuitionistic set (ɭS for short) $\ddot{A}$ of a non-void set Ӽ is $\ddot{A} = \langle A', A'' \rangle$ where $A', A'' \subseteq$ Ӽ and $A' \cap A'' = \emptyset$.

Definition 2.2. [2] If$\ddot{A} = \langle A', A'' \rangle$, $\ddot{B} = \langle B', B'' \rangle$ and $\{\ddot{A}_i : i \in I\}$ are intuitionistic sets in Ӽ, where $\ddot{A}_i = \langle A'_i, A''_i \rangle$. Then

1. $\ddot{A} \subseteq_{ɭ} \ddot{B} \Leftrightarrow A' \subseteq B' \& B'' \subseteq A''$.
2. $\ddot{A} = \ddot{B} \Leftrightarrow \ddot{A} \subseteq_{ɭ} \ddot{B} \& \ddot{B} \subseteq_{ɭ} \ddot{A}$.
3. $\cup_{ɭ} \ddot{A}_i = \langle \cup A'_i, \cap A''_i \rangle$.
4. $\cap_{ɭ} \ddot{A}_i = \langle \cap A'_i, \cup A''_i \rangle$.
5. $\ddot{Ӽ}_{ɭ} = \langle Ӽ, \emptyset \rangle$.
6. $\ddot{\emptyset}_{ɭ} = \langle \emptyset, Ӽ \rangle$.
7. $cop.\ddot{A} = \langle A'', A' \rangle$.
8. $\ddot{A} \setminus \ddot{B} = \ddot{A} \cap_{ɭ} \ddot{B}$.

Definition 2.3. [2] Intuitionistic point of a non-void set Ӽ is $\ddot{p}_{ɭ} = \langle \{p\}, \{p\}^c \rangle$, where $p \in$ Ӽ.

Definition 2.4. [5] A collection $\mathfrak{I}_{ɭ}$ of intuitionistic sets of Ӽ called intuitionistic topology (ɭT) if is satisfying the following axioms:

1. $\ddot{Ӽ}_{ɭ}, \ddot{\emptyset}_{ɭ} \in \mathfrak{I}_{ɭ}$,
2. $\ddot{A} \cap_{ɭ} \ddot{B} \in \mathfrak{I}_{ɭ}, for\ each \ddot{A}, \ddot{B} \in \mathfrak{I}_{ɭ}$,
3. $\cup_{ɭ} \ddot{A}_i \in \mathfrak{I}_{ɭ}\ for\ each\ \ddot{A}_i \in \mathfrak{I}_{ɭ}$.

the pair $(Ӽ, \mathfrak{I}_{ɭ})$is called a intuitionistic topological space (ɭTS) and any intuitionistic set belong to ɭT called ɭ-open set. $cop.(\ddot{A})$ is ɭ-closed if and only if $\ddot{A}$ is ɭ-open.

Definition 2.5. [5] Let $(Ӽ, \mathfrak{I}_{ɭ})$ be an ɭTS and $\ddot{A}$ is intuitionistic set. Then
$int_{ɭ}(\ddot{A}) = \cup_{ɭ} \{\ddot{G} : \ddot{G} \in \mathfrak{I}_{ɭ}\ and\ \ddot{G} \subseteq_{ɭ} \ddot{A}\}$.
$cl_{ɭ}(\ddot{A}) = \cap_{ɭ} \{\ddot{F} : \ddot{F}\ is\ ɭ - closed\ and\ \ddot{A} \subseteq_{ɭ} \ddot{F}\}$.

Obvious

1. $int_{ɭ}(\ddot{A})$ is ɭ-open and $cl_{ɭ}(\ddot{A})$ is ɭ- closed.
2. $\ddot{A}$ is ɭ-open if and only if $int_{ɭ}(\ddot{A}) = \ddot{A}$.
3. $\ddot{A}$ is ɭ-closed if and only if $cl_{ɭ}(\ddot{A}) = \ddot{A}$.

Definition 2.6. Let $(Ӽ, \mathfrak{I}_{ɭ})$ be ɭTS. A collection $\mathfrak{O}_{ɭ}$ of ɭ-open sets called ɭ-open cover of ɭTS if and only if $\cup_{ɭ} \{\ddot{G} : \ddot{G} \in \mathfrak{O}_{ɭ}\} = \ddot{Ӽ}_{ɭ}$ and called ɭ-open cover of intuitionistic set $\ddot{A}$ if and only if $\ddot{A} \subseteq_{ɭ} \cup_{ɭ} \{\ddot{G} : \ddot{G} \in \mathfrak{O}_{ɭ}\}$.

Definition 2.7. Let $(\mathfrak{X}, \Im_{\ell})$ be ℓTS. $(\mathfrak{X}, \Im_{\ell})$ called ℓ-compact space if and only if every ℓ-open cover of $\mathfrak{X}$, has finite ℓ-open subcover.

3. g_{ℓ}-Closed and properties

In this section, we will know the concept (g_{ℓ}-closed) and study the most important properties of this concept.

Definition 3.1. Let $(\mathfrak{X}, \Im_{\ell})$ be ℓTS and $\ddot{A}$is intuitionistic set of $(\mathfrak{X}, \Im_{\ell})$. $\ddot{A}$ is g_{ℓ}-closed if and only if $cl_{\ell}(\ddot{A}) \subseteq_{\ell} \ddot{U}$ where $\ddot{A} \subseteq_{\ell} \ddot{U}$ and $\ddot{U}$ is ℓ-open.

Theorem 3.2. Let $(\mathfrak{X}, \Im_{\ell})$ be ℓTS and $\ddot{A}$is intuitionistic set of $(\mathfrak{X}, \Im_{\ell})$. $\ddot{A}$ is g_{ℓ}-closed if and only if $\ddot{F} \subseteq_{\ell} cl_{\ell}(\ddot{A}) - \ddot{A}$ and $\ddot{F}$ $is$$\ell$-closed set, then $\ddot{F} = \ddot{\emptyset}_{\ell}$.

Proof: Assume that $\ddot{F} \subseteq_{\ell} cl_{\ell}(\ddot{A}) - \ddot{A}$. Then $\ddot{A} \subseteq_{\ell}$ cop. $\ddot{F}$ and we have $cl_{\ell}(\ddot{A}) \subseteq_{\ell}$ cop. $\ddot{F}$ or $\ddot{U} \subseteq_{\ell}$ cop. $cl_{\ell}(\ddot{A})$ (because $\ddot{A}$ is g_{ℓ}-closed).

conversely.

Assume that $\ddot{A} \subseteq_{\ell} \ddot{U}$ and $\ddot{U}$ is ℓ-open. If $cl_{\ell}(\ddot{A}) \not\subseteq_{\ell} \ddot{U}$., then $\ddot{\emptyset}_{\ell} \neq cl_{\ell}(\ddot{A}) \cap_{\ell} \ddot{U} \subseteq_{\ell} cl_{\ell}(\ddot{A}) - \ddot{A}$. Those, $cl_{\ell}(\ddot{A}) \subseteq_{\ell} \ddot{U}$. Hence $\ddot{A}$ is g_{ℓ}-closed.

Corollary 3.3. Intuitionistic set $\ddot{A}$ of ℓTS $(\mathfrak{X}, \Im_{\ell})$ is g_{ℓ}-closed if and only if $cl_{\ell}(\ddot{A}) - \ddot{A}$ is ℓ-closed.

Proof: Assume that $\ddot{A}$ is ℓ-closed. clearly $cl_{\ell}(\ddot{A}) - \ddot{A} = \ddot{\emptyset}_{\ell}$.

Conversely. Assume that $cl_{\ell}(\ddot{A}) - \ddot{A}$ is ℓ-closed. Since $\ddot{A}$ is g_{ℓ}-closed and $cl_{\ell}(\ddot{A}) - \ddot{A}$ is a ℓ-closed intuitionistic subset of itself, then by using (Theorem 2.2), we have $cl_{\ell}(\ddot{A}) - \ddot{A} = \ddot{\emptyset}_{\ell}$. So $cl_{\ell}(\ddot{A}) = \ddot{A}$. Hence

Theorem 3.4. $\ddot{A} \cup_{\ell} \ddot{B}$ is g_{ℓ}-closed for any two g_{ℓ}-closed sets $\ddot{A}$ and $\ddot{B}$ of $(\mathfrak{X}, \Im_{\ell})$.

Proof: Assume that $\ddot{A} \cup_{\ell} \ddot{B} \subseteq_{\ell} \ddot{U}$ where $\ddot{U}$ is ℓ-open. So that $cl_{\ell}(\ddot{A} \cup_{\ell} \ddot{B}) = cl_{\ell}(\ddot{A}) \cup_{\ell} cl_{\ell}(\ddot{B}) \subseteq_{\ell} \ddot{U}$. Hence $\ddot{A} \cup_{\ell} \ddot{B}$ is g_{ℓ}-closed.

Remark 3.5. Not necessary $\ddot{A} \cap_{\ell} \ddot{B}$ is g_{ℓ}-closed where $\ddot{A}$ and $\ddot{B}$ are g_{ℓ}-closed e.g. If $\mathfrak{X} = \{x_1, x_2, x_3\}$ and $\Im_{\ell} = \{\ddot{\emptyset}_{\ell}, \langle\{x_1\}, \{x_2, x_3\}\rangle, \ddot{\mathfrak{X}}_{\ell}\}$. let $\ddot{A} = \langle\{x_1, x_2\}, \{x_3\}\rangle$ and $\ddot{B} = \langle\{x_1, x_3\}, \{x_2\}\rangle$.Obvious $\ddot{A}$ and $\ddot{B}$ are g_{ℓ}-closed, but $\ddot{A} \cap_{\ell} \ddot{B}$ is not g_{ℓ}-closed.

Theorem 3.6. Let $(\mathfrak{X}, \Im_{\ell})$ ba a ℓTS and $\ddot{A}_{\ell} = \langle A, \emptyset\rangle$ sub set of $\mathfrak{X}$, then $\Im_{\ell A} = \{\ddot{A}_{\ell} \cap_{\ell} \ddot{U} : \ddot{U} \in \Im_{\ell}\}$ is intuitionistic topology called ℓ-relative topology and $(A, \Im_{\ell\ddot{A}})$ called ℓ-subspace of $(\mathfrak{X}, \Im_{\ell})$.

Proof: Since $\ddot{\mathfrak{X}}_{\ell} \in \Im_{\ell}$, then $\ddot{A}_{\ell} \cap_{\ell} \ddot{\mathfrak{X}}_{\ell} = \ddot{A}_{\ell} \in \Im_{\ell A}$ and since $\ddot{\emptyset}_{\ell} \in \Im_{\ell}$, then $\langle A, \emptyset\rangle \cap_{\ell} \langle\emptyset, \mathfrak{X}\rangle = \ddot{A}_{\ell} \cap_{\ell} \ddot{\emptyset}_{\ell} = \langle\emptyset, A\rangle \in \Im_{\ell A}$

Theorem 3.7. If$\ddot{A}$ is g_{ℓ}-closed set and $\ddot{B}$ is ℓ-closed set. Then $\ddot{A} \cap_{\ell} \ddot{B}$ is g_{ℓ}-closed set.

Proof: Obvious.

Theorem 3.8. Every $g_{ɭ}$-closed set Ä of ɭ-compact topological space $(Ӽ, \mathfrak{J}_{ɭ})$ is ɭ-compact.
Proof: Assume that $\mathfrak{Q}$ is ɭ-open cover of Ä. $cl_{ɭ}(Ä) \subseteq_{ɭ} \cup_{ɭ} \mathfrak{Q}$ since Ä is $g_{ɭ}$- closed. Since $cl_{ɭ}(Ä)$ is ɭ- compact, then $Ä \subseteq_{ɭ} cl_{ɭ}(Ä) \subseteq_{ɭ} Ü_1 \cup_{ɭ} \ldots \cup_{ɭ} Ü_n$ for some $Ü_i \in \mathfrak{Q}$. Hence Äis ɭ-compact.

4. $g_{ɭ}$-Open and properties

In this section, we will know the concept ($g_{ɭ}$-closed) and study the most important properties of this concept.

Definition 4.1. Let $(Ӽ, \mathfrak{J}_{ɭ})$ be ɭTS and Äis intuitionistic set of $(Ӽ, \mathfrak{J}_{ɭ})$. Ä *is* $g_{ɭ}$-open set if and only if $cop.$Ä is $g_{ɭ}$- closed.

Theorem 4.2. Ä is $g_{ɭ}$- open if and only if $F̈ \subseteq_{ɭ} int_{ɭ}(Ä)$ where F̈ is ɭ-closed and $F̈ \subseteq_{ɭ} Ä$.

Theorem 4.3. $Ä \cup_{ɭ} B̈$ is $g_{ɭ}$-open for any two ɭ-separated $g_{ɭ}$-open sets Ä and B̈
Proof: Assume that F̈ is a ɭ-closed subset of $Ä \cup_{ɭ} B̈$. So $F̈ \cap_{ɭ} cl_{ɭ}(Ä) \subseteq_{ɭ} Ä$ and by using (Theorem 4.2) we have $F̈ \cap_{ɭ} cl_{ɭ}(Ä) \subseteq_{ɭ} int_{ɭ}(Ä)$. Similarly, $F̈ \cap_{ɭ} cl_{ɭ}(B̈) \subseteq_{ɭ} int_{ɭ}(B̈)$. Now

$$F̈ = F̈ \cap_{ɭ} (Ä \cup_{ɭ} Ä) \subseteq_{ɭ} \left(F̈ \cap_{ɭ} cl_{ɭ}(Ä)\right) \cap_{ɭ} \left(F̈ \cap_{ɭ} cl_{ɭ}(B̈)\right) \subseteq_{ɭ}$$

$int_{ɭ}(Ä) \cup_{ɭ} int_{ɭ}(Ä) \subseteq_{ɭ} cl_{ɭ}(Ä \cup_{ɭ} B̈)$. Therefore $F̈ \subseteq_{ɭ} int_{ɭ}(Ä \cup_{ɭ} B̈)$ and by using (Theorem 4.2) we have $Ä \cup_{ɭ} B̈$ is $g_{ɭ}$-open.

Remark 4.4. Not necessary $Ä \cap_{ɭ} B̈$ is $g_{ɭ}$-open where Ä and B̈ are $g_{ɭ}$-closed (See remark 2.5).

Corollary 4.5. If Ä, B̈ are $g_{ɭ}$-closed sets and $cop.$Ä, $cop.$B̈ are ɭ-separated, then we have $Ä \cap_{ɭ} B̈$ is $g_{ɭ}$-closed.

Theorem 4.6. Ä is $g_{ɭ}$-open if and only if $Ü = Ӽ̈_{ɭ}$ where Ü is ɭ-open and

$$int_{ɭ}(Ä) \cup_{ɭ} cop.Ä \subseteq_{ɭ} Ü.$$

An intuitionistic set Ä is $g_{ɭ}$-open in $(Ӽ, \mathfrak{J}_{ɭ})$ iff $Ü = Ӽ̈_{ɭ}$ whenever Ü is ɭ-open and $int_{ɭ}(Ä) \cup_{ɭ} cop.Ä \subseteq_{ɭ} Ü$.
Proof: Assume that Ü is a ɭ-open and $int_{ɭ}(Ä) \cup_{ɭ} cop.Ä \subseteq_{ɭ} Ü$. So $cop.Ü \subseteq_{ɭ} cl_{ɭ}(cop.Ä) \cap_{ɭ} Ä = cl_{ɭ}(cop.Ä) - cop.Ä$. Hence $cop.Ü = \ddot{\emptyset}_{ɭ}$ or $Ü = Ӽ̈_{ɭ}$ because $cop.$Ü is ɭ-closed set and $cop.$Ä is $g_{ɭ}$-closed set, by (Theorem 2.2)
Conversely.
Assume that F̈ɭ-closed set and $F̈ \subseteq_{ɭ} Ä$. By using (Theorem 4.2), it's enough to show that $F̈ \subseteq_{ɭ} int_{ɭ}(Ä)$.Clearly $int_{ɭ}(Ä) \cup_{ɭ} cop.Ä \subseteq_{ɭ} int_{ɭ}(Ä) \cup_{ɭ} cop.F̈$ and hence $int_{ɭ}(Ä) \cup_{ɭ} cop.F̈ = Ӽ̈_{ɭ}$. So $F̈ \subseteq_{ɭ} int_{ɭ}(Ä)$.

Theorem 4.7. Let $int_{ɭ}(A) \subseteq_{ɭ} B̈$ and Ä$g_{ɭ}$-open. Then B̈ is $g_{ɭ}$-open.
Proof:
$cop.Ä \subseteq_{ɭ} cop.B̈ \subseteq_{ɭ} cl_{ɭ}(cop.Ä)$ and since $cop.$Ä is $g_{ɭ}$-closed, then $cop.$B̈ is $g_{ɭ}$-closed by

using (Theorem 2.8). So $\ddot{\mathrm{B}}$ is $\mathrm{g}_{ɫ}$-open.

Theorem 4.8. $\ddot{\mathrm{A}}$ is $\mathrm{g}_{ɫ}$-closed if and only if $cl_{ɫ}(\ddot{\mathrm{A}}) - \ddot{\mathrm{A}}$ is $\mathrm{g}_{ɫ}$-open.

Proof: Assume that $\ddot{\mathrm{A}}$ is $\mathrm{g}_{ɫ}$-closed and $\ddot{\mathrm{F}} \subseteq_{ɫ} cl_{ɫ}(\ddot{\mathrm{A}}) - \ddot{\mathrm{A}}$, $\ddot{\mathrm{F}}$ is ɫ-closed. By using (Theorem 2.2) $\ddot{\mathrm{F}} = \ddot{\emptyset}_{ɫ}$ and we have $\ddot{\mathrm{F}} \subseteq_{ɫ} int_{ɫ}(cl_{ɫ}(\ddot{\mathrm{A}}) - \ddot{\mathrm{A}})$. So, by using (Theorem 4.2), $cl_{ɫ}(\ddot{\mathrm{A}}) - \ddot{\mathrm{A}}$ is $\mathrm{g}_{ɫ}$-open set.

Conversely. Assume that $\ddot{\mathrm{A}} \subseteq_{ɫ} \ddot{\mathrm{U}}$ and $\ddot{\mathrm{U}}$ is an ɫ-open set.
So $cl_{ɫ}(\ddot{\mathrm{A}}) \cap_{ɫ} cop.\ddot{\mathrm{U}} \subseteq_{ɫ} cl_{ɫ}(\ddot{\mathrm{A}}) \cap_{ɫ} cop.\ddot{\mathrm{A}} = cl_{ɫ}(\ddot{\mathrm{A}}) - \ddot{\mathrm{A}}$ and since $cl_{ɫ}(\ddot{\mathrm{A}}) \cap_{ɫ} cop.\ddot{\mathrm{U}}$ɫ -closed and $cl_{ɫ}(\ddot{\mathrm{A}}) - \ddot{\mathrm{A}}$ is $\mathrm{g}_{ɫ}$-open, then $cl_{ɫ}(\ddot{\mathrm{A}}) \cap_{ɫ} cop.\ddot{\mathrm{U}} \subseteq_{ɫ} int_{ɫ}(cl_{ɫ}(\ddot{\mathrm{A}}) - \ddot{\mathrm{A}}) = \ddot{\emptyset}_{ɫ}$. Therefore $cl_{ɫ}(\ddot{\mathrm{A}}) \cap_{ɫ} cop.\ddot{\mathrm{U}} = \ddot{\emptyset}_{ɫ}$ or $cl_{ɫ}(\ddot{\mathrm{A}}) \subseteq_{ɫ} \ddot{\mathrm{U}}$. Hence $\ddot{\mathrm{A}}$ is $\mathrm{g}_{ɫ}$-closed.

5. $\mathfrak{I}_{ɫ_{1/2}}$-space

In this section, we will know the concept of ($\mathfrak{I}_{ɫ_{1/2}}$-space) and clarify the relationship between it and ($\mathfrak{I}_{ɫ_0}$-space) and ($\mathfrak{I}_{ɫ_1}$-space).

Definition 5.1. ɫ-topological space $(\mathfrak{X}, \mathfrak{I}_{ɫ})$ called $\mathfrak{I}_{ɫ_{1/2}}$-space if and only if for each $\mathrm{g}_{ɫ}$-closed set is ɫ-closed.

Theorem 5.2. Every $\mathfrak{I}_{ɫ_{1/2}}$-space is $\mathfrak{I}_{ɫ_0}$-space.

Proof: Assume that $(\mathfrak{X}, \mathfrak{I}_{ɫ})$ is not $\mathfrak{I}_{ɫ_0}$-space. Then there exist $\ddot{p}_{ɫ} \neq \ddot{q}_{ɫ}$ two intuitionistic pointssuch that $cl_{ɫ}(\ddot{p}_{ɫ}) = cl_{ɫ}(\ddot{q}_{ɫ})$. Put $\ddot{\mathrm{A}} = cl_{ɫ}\left(\underline{\ddot{p}_{ɫ}}\right) \cap_{ɫ} cop.\ddot{p}_{ɫ}$. To prove that $\ddot{\mathrm{A}}$ is $\mathrm{g}_{ɫ}$-closed, not ɫ-closed. Let $\ddot{p}_{ɫ} \in \ddot{\mathrm{U}} \in \mathfrak{I}_{ɫ}$, then $\ddot{\emptyset}_{ɫ} \neq \underline{\ddot{q}_{ɫ}} \subseteq_{ɫ} \ddot{\mathrm{U}} \cap_{ɫ} \ddot{\mathrm{A}}$ and so $\ddot{p}_{ɫ} \in cl_{ɫ}(\ddot{\mathrm{A}})$. Plainly $\ddot{p}_{ɫ} \notin \ddot{\mathrm{A}}$ and so $\ddot{\mathrm{A}}$ is not ɫ-closed. Now assume that $\ddot{\mathrm{A}} \subseteq_{ɫ} \ddot{\mathrm{U}}^*$. To prove that $cl_{ɫ}(\ddot{\mathrm{A}}) \subseteq_{ɫ} \ddot{\mathrm{U}}^*$, it's enough to prove that $cl_{ɫ}(\ddot{p}_{ɫ}) \subseteq_{ɫ} \ddot{\mathrm{U}}^*$. Since $cl_{ɫ}(\ddot{p}_{ɫ}) \cap_{ɫ} cop.\ddot{p}_{ɫ} = \ddot{\mathrm{A}} \subseteq_{ɫ} \ddot{\mathrm{U}}^*$ and those we just need to prove it $\ddot{p}_{ɫ} \in \ddot{\mathrm{U}}^*$. If $\ddot{p}_{ɫ} \in cop.\ddot{\mathrm{U}}^*$, then $\ddot{q}_{ɫ} \in cl_{ɫ}(\ddot{p}_{ɫ}) \subseteq_{ɫ} cop.\ddot{\mathrm{U}}^*$. It is, obviously, obvious that $\ddot{q}_{ɫ} \in \ddot{\mathrm{A}} \subseteq_{ɫ} \ddot{\mathrm{U}}^*$ and those $\underline{\ddot{p}_{ɫ}} \in \ddot{\mathrm{U}}^* \cap_{ɫ} cop.\ddot{\mathrm{U}}^*$.

Theorem 5.3. Every $\mathfrak{I}_{ɫ_1}$-space is $\mathfrak{I}_{ɫ_{1/2}}$-space.

Proof: Assume that $\ddot{\mathrm{A}}$ not ɫ-closed. Let $\ddot{p}_{ɫ} \in cl_{ɫ}(\ddot{\mathrm{A}}) - \ddot{\mathrm{A}}$. Then $\ddot{p}_{ɫ} \subseteq_{ɫ} cl_{ɫ}(\ddot{\mathrm{A}}) - \ddot{\mathrm{A}}$ and $\ddot{p}_{ɫ}$ is ɫ-closed because we are in a $\mathfrak{I}_{ɫ_1}$-space. By using (Theorem 2.2) we have $\ddot{\mathrm{A}}$ is not $\mathrm{g}_{ɫ}$-closed.

Example 5.4. Let $\mathfrak{X} = \{x_1, x_2\}$ and suppose that $\mathfrak{I}_{ɫ} = \{\ddot{\emptyset}_{ɫ}, \langle\{x_1\},\{x_2\}\rangle, \ddot{\mathfrak{X}}_{ɫ}\}$. Then $(\mathfrak{X}, \mathfrak{I}_{ɫ})$ is a $\mathfrak{I}_{ɫ_{1/2}}$ space which is not $\mathfrak{I}_{ɫ_1}$.

Example 5.5. Let $\mathfrak{X} = \{x_1, x_2, x_3\}$ and suppose that
.$\mathfrak{I}_{ɫ} = \{\ddot{\mathfrak{X}}_{ɫ}, \langle\{x_1\},\{x_2, x_3\}\rangle, \langle\{x_1, x_2\},\{x_3\}\rangle, \ddot{\mathfrak{X}}_{ɫ}\}$. Then $(\mathfrak{X}, \mathfrak{I}_{ɫ})$ is a $\mathfrak{I}_{ɫ_0}$ space, but not a $\mathfrak{I}_{ɫ_{1/2}}$-space since $\langle\{x_1, x_3\},\{x_2\}\rangle$ is $\mathrm{g}_{ɫ}$-closed, but not ɫ-closed.

Corollary 5.6. The property of $\mathfrak{I}_{ɫ_{1/2}}$is carefully between $\mathfrak{I}_{ɫ_0}$ and $\mathfrak{I}_{ɫ_1}$.

5. Conclusion

Science (intuitionistic set) is a new field that contributes greatly to applications in the natural sciences, engineering, computer and information systems. In this paper a new concept called (g_ι- (Closed, Open) has been studied and linked with separate axioms of intuitionistic topological space and there is a great field for researchers to use this concept such as linking it with the ideal concept and the concept of soft.

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