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Fuzzy γ-Baire Spaces and Fuzzy γ-D-Baire Spaces

Dipankar De

Tripura University, Agartala
Tripura, India
Email: dipankardee@yahoo.co.in

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Abstract. In this paper, we introduce the concept of fuzzy γ-Baire space and fuzzy γ-D-Baire space. Some basic theorems and results of these spaces are introduced and also studied. Several examples are given to explain the concepts of these spaces.

Keywords: Fuzzy γ-open set, fuzzy γ- nowhere set, fuzzy γ-first category, fuzzy γ-second category, fuzzy γ-Baire space, fuzzy γ-D Baire space, fuzzy γ-D'-Baire space.

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1. Introduction

The concept of fuzzy sets which formed the backbone of fuzzy mathematics was first introduced by Zadah [14] in his classical paper in the year of 1965. There after Chang in 1968 was introduced the concepts of fuzzy topology as generalization of topological space. Since then much attention has been imply to develop the fundamental concepts of it by many authors. Thus a modern theory of fuzzy topology has been developed.

The concept of fuzzy γ-open set was introduced by Hanfy et al. [8] in 1999 as a union of notation of fuzzy semi-open sets and fuzzy preopen sets. Hanafy introduced the fuzzy γ-open sets which are weaker than each of them.Using this notation, he studied fuzzy γ-continious mapping on fuzzy topological space. Thangaraj et al. []9-13] discussed fuzzy α −Baire space in fuzzy topological space and studied its properties.

We shall introduce and study the concepts of fuzzy γ-Baire space and fuzzy γ-D Baire space by using fuzzy γ- open sets in fuzzy topological space and relations of fuzzy Baire, fuzzy γ-Baire and fuzzy γ-D-Baire spaces are discussed. Also we discuss some theorems and results on this spaces. Many works have been done by Balasubramanian [2-3], Caldas et al. [4] and De [5-7]. Jana, Senapati at al. have investigated on BCK/BCI algebras and related algebras, see [15-17].

2. Preliminaries

Some basic definitions and properties which will be needed are recalled in this section. In this paper we use the notion of a fuzzy topology in the original sense of Chang (1968). Throughout this paper by (X,τ) we mean a fuzzy topological space(fts,shortly).

For a fuzzy set λ in a fts (X,τ), cl λ, intλ, intclintλ will respectively stand for the fuzzy closure,fuzzy interior and interior closure interior of λ in (X,τ).

Definition 2.1. [8]A fuzzy subset λ of fuzzy topological space (X,τ) is said to be fuzzy-γ open set (res. fuzzy $\gamma -$ closed set) if

$$\lambda \leq clint\lambda \vee \text{intcl } \lambda \ (\text{res.} \lambda \ \geq \text{clint}\, \lambda \wedge \text{intcl}\, \lambda\,)$$

Also, it may be write that the complement of fuzzy $\gamma -$open set is fuzzy $\gamma -$-closed set. Any union (resp. intersection) of Fuzzy of $\gamma -$open (resp. fuzzy γ-closed) set is fuzzy $\gamma -$open(resp. fuzzy $\gamma -$closed.).

But intersection of two fuzzy $\gamma -$open sets need not be fuzzy $\gamma -$open

The intersection of a fuzzy open set which is a crisp subset and a fuzzy γopen set is fuzzy γ open.

The union of fuzzy closed set which is a crisp subset and fuzzy $\gamma -$closed set is fuzzy$\gamma -$closed.

Remark 2.2. It is obvious that every fuzzy open (resp. fuzzy closed) set is a fuzzy $\gamma -$open set(resp. fuzzy $\gamma -$closed set).But The converse need not be true in general.

Proposition 2.3.[1] If μ and λ are two fuzzy subsets of a fts(X,τ) then
(i) $cl\mu \vee \text{cl}\lambda = \text{cl}(\mu \vee \lambda)$ and $\text{cl}\, \mu \wedge \text{cl}\, \lambda \geq \text{cl}(\mu \wedge \lambda)$
(ii) $int\mu \wedge \text{int}\, \lambda = \text{int}(\mu \wedge \lambda)$ and $\text{int}\mu \vee \text{int}\lambda \leq \text{int}(\mu \vee \lambda) \wedge$
(iii) $1 - int\lambda = \text{cl}(1 - \lambda)$
(iv) $1 - cl\lambda = \text{int}(1 - \lambda)$

Definition 2.4. [8] The $\gamma -$interior and $\gamma -$closure of a fuzzy set A in (X,τ) are denoted by $\gamma -$int(A) and γ-cl(A) respectively and are defined as
$\gamma -$ int(A) =∨{B: B≤A, B is fuzzy $\gamma -$open set in X}
$\gamma -$ cl(A)=∧{C: C ≥A, C is fuzzy $\gamma -$closed set in X}.

Here, let $\mu_A(x)$and $\mu_B(x)$ be the membership function of every x in A and B respectively.Then a member of A is contained in a member of B which is denoted by A≤B iff $\mu_A(x) \leq \mu_B(x)$.

Proposition 2.5. [8] If λ is a subset of (X,τ) and $\lambda'$is its complement then
(i) $\gamma - \text{cl}\lambda' = \ (\gamma - int\lambda)'$
(ii) $\gamma - \text{int}\lambda' = \ (\gamma - cl\lambda)'$

Definition 2.6. [10]
i) A fuzzy set λ in a fuzzy topological space (X,τ) is called fuzzy dense if there exists no fuzzy closed set μ in (X,τ) such that $\lambda < \mu < 1.$ That is, $\text{cl}(\lambda) = 1.$
(ii)A fuzzy set λ in a fts (X,τ) is called fuzzy nowhere dense if there exists no non-zero fuzzy open set μ in (X,τ) such that $\mu < \text{cl}(\lambda),$ that is $\text{intcl}(\lambda) = 0.$

Definition 2.7. [3] A fuzzy set λ in a fts (X,τ) is called a fuzzy G_δ-set in (X,τ) if $\lambda = \bigwedge^{\infty}_{i=1}(\lambda_i)$ where $\lambda_i \in \tau$ for i∈ Λ.

Proposition 2.8. [1] For a family A of $\{\lambda_\alpha\}$ of fuzzy sets of a fts, (X, τ),

$$\vee(cl(\lambda_\alpha) \leq \mathrm{cl}(\vee\lambda_\alpha).$$

In case A is finite set, $\vee(cl(\lambda_\alpha) = \mathrm{cl}(\vee\lambda_\alpha)$
Also $\vee(int(\lambda_\alpha) \leq \mathrm{int}(\vee\lambda_\alpha)$. in (X,τ).

Definition 2.9. [12] A fts (X,τ) is called a fuzzy Baire space if $int\left(\vee_{\mathrm{i}=1}^{\infty}(\lambda_i)\right) = 0$ where (λ_i)'s are fuzzy nowhere dense sets in (X,τ).

Definition 2.10. [12] A fuzzy set λ in fts(X,τ) is called a fuzzy first category set if $\lambda = \vee_{\mathrm{i}=1}^{\infty}(\lambda_i)$ where (λ_i)'s are nowhere dense sets in (X,τ).Any other fuzzy set in (X,τ) is said to be of fuzzy second category.

Definition 2.11. [10]A fts(X,τ) is called a fuzzy first category set if $\vee_{\mathrm{i}=1}^{\infty}(\lambda_i) = 1_X$, where (λ_i)'s are nowhere dense sets in (X,τ).
Any other fts (X,τ) is said to be of fuzzy second category.

Theorem 2.12.[12] If λ is a fuzzy dense set and fuzzy G_δ set in a fuzzy topological space(X,τ), then 1-λ is a fuzzy first category set in (X,τ).

Definition 2.13.[9]A fts(X,τ) is called fuzzy P-space if every non zero fuzzy G_δ-set in (X,τ) is fuzzy open in (X,τ). That is,

if $\lambda = \wedge_{\mathrm{i}=1}^{\infty}(\lambda_i)$ where $\lambda_i \in \tau$ for $\mathrm{i} \in \Lambda$.

Definition 2.14.[13] A fts(X,τ) is called a fuzzy Volterra space if $cl\wedge_{\mathrm{i}=1}^{\mathrm{N}}(\lambda_i) = 1$, where (λ_i)'s are fuzzy dense and fuzzy G_δ-sets in (X,τ) .

Definition 2.15.[3]A fuzzy topological space(X,τ) is called a fuzzy submaximal space if for each fuzzy open set λ in (X,τ) such that $cl(\lambda) = 1$.

3. Fuzzy γ −Baire Space

Definition 3.1. Fuzzy γ −Baire Space: Let (X,τ) be a fuzzy topological space. Then(X,τ) is called fuzzy γ −Baire Space if
$\gamma - \mathrm{int}(\vee_{\mathrm{i}=1}^{\infty}(\lambda_i)) = 0$, where $\lambda_{i'}$'s are fuzzy γ −no where dense sets in (X,τ).

Proposition3.2. Let (X,τ) be a fuzzy topological space. Then the following are equivalent:
i) (X,τ) is a fuzzy γ −Baire space.
ii)$\gamma - \mathrm{intcl}(\vee(\lambda_i)) = 0$,for every fuzzy γ-first category set λ in (X,τ)
iii) $\gamma - \mathrm{int}\ \alpha = 1$ for every fuzzy γ −residual setα in (X,τ)
Proof: From i)⇒ii)

$$Let\ \lambda\ be\ a\ fuzzy\ \gamma - \ first\ category\ set\ in\ (X,\tau).$$

Then $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$,where λ_i are fuzzy γ −no where dense sets in (X,τ)

$\therefore \gamma - \mathrm{int}\,\lambda = \vee_{i=1}^{\infty}(\lambda_i) = 0$ [since (X,τ) is γ −Baire space.]

$$\therefore \gamma - \mathrm{int}\,\lambda = 0$$

(ii)⇒(iii) :

Letα be a fuzzy γ −residual set in (X,τ).

Then 1-α is fuzzy γ −first category set in (X,τ).

$$\therefore \gamma - \mathrm{int}(1 - \alpha) = 0$$

$\Rightarrow 1 - (\gamma - \mathrm{int}\,\alpha) = 0$

$$\therefore \gamma - \mathrm{int}\,\alpha = 1$$

(ii)⇒(i) :

λ be a fuzzy γ −first category set in(X,τ)

Then $\lambda = V_{i=1}^{\infty}\lambda_i$,where $\lambda_i$are fuzzyγ −no where dense sets in (X,τ).

Now,λ isγ −first category set.

1-λ is a γ −residual in (X,τ)

Now we have,

$$\gamma - cl(1-\lambda) = 1$$
$$1 - (\gamma - int\lambda) = 1$$

$\therefore \gamma - int\lambda) = 0$

$\Rightarrow \gamma - \mathrm{int}\vee_{i=1}^{\infty}(\lambda_i) = 0$, where$\lambda_i$is no where dense set

$\Rightarrow (X,\tau)$ is a fuzzyγ − Baire set.

Theorem 3.3. Let(X,τ) is a fuzzy γ −Baire space and λ is fuzzy set in (X,τ). Prove that $\gamma - int(\gamma - cl\mathrm{int}\vee\lambda_i) \geq 0$.

Proof: Let$\{\lambda_i, \mathrm{i} \in \Lambda{:}\}$ be a collection of fuzzy γ − open sets in (X,τ).

Now, we have $\lambda_i \leq intcl\lambda_i \vee \mathrm{clint}\lambda_i$

Let (X,τ) is a fuzzy γ −Baire space.

By definition ,for no where dense set if

$\gamma - int(\gamma - cl(\lambda)) = 0$

And for$(X,\tau)$is fuzzy γ −open set

We have $\lambda \leq intcl\lambda \vee \mathrm{clint}\lambda$ and arbitrary union of γ −open set is γ −open, let $\lambda = \vee\lambda_i$

And γ −int(γ −cl(λ)) mean that intclonγ −open sets

Now taking union on both sides

$$\vee\lambda \leq \vee(intcl\lambda \vee \mathrm{clint}\lambda) \leq intcl\vee\lambda_i \vee \mathrm{clint}\vee\lambda_i), \mathrm{i} = 1 \text{ to } \infty$$

Taking γ −int $(\gamma - cl)$ on both sides,

γ −int $(\gamma - cl)\vee\lambda_i \leq \gamma - int(\gamma - cl)(intcl\vee\lambda_i \vee \mathrm{clint}\vee\lambda_i))$

$$\leq (\gamma - int(\gamma - cl)\vee\lambda_i) \vee ((\gamma - int(\gamma - cl\mathrm{nt}\vee\lambda_i))$$

$\leq ((\gamma - int(\gamma - cl\mathrm{nt}\vee\lambda_i)))$, [since λ_i is γ −no where dense set,

$$(\gamma - int(\gamma - cl)\vee\lambda_i) = 0]$$

$\therefore 0 \leq \gamma - int(\gamma - cl\mathrm{int}(\vee\lambda_i))$

$\gamma - int(\gamma - cl\text{int}(\vee\lambda_i)) \geq 0$, proved.

Definition 3.4. Fuzzy γ-Baire Space: Let (X, τ) be a fuzzy topological space. Then(X, τ) is called fuzzy γ −Baire space if $\gamma - int(\gamma - cl\text{nt}(\vee\lambda_i)) \geq 0$.

Example 3.5. Let μ_1, μ_2, μ_3 be fuzzy sets on X=[0,1] defined as

$\mu_1(x) = 0$ if $0 \leq x \leq \frac{1}{2}$ $\quad$ $\mu_2(x) = 1, if\ 0 \leq x \leq \frac{1}{4}$

$= 2x - 1$ if $\frac{1}{2} \leq x \leq 1$, $\quad$ $= -4x + 2$, if $\frac{1}{4} \leq x \leq \frac{1}{2}$

$= 0$, if $\frac{1}{2} \leq x \leq 1$

$$\mu_3(x) = 0, \text{if } 0 \leq x \leq \frac{1}{4}$$

$$= \frac{4x-1}{3} \text{ if } \frac{1}{4} \leq x \leq 1$$

Consider τ={0,1, μ_1,μ_2,μ_3,$\mu_1\vee_2$} on X. In $(X, \tau)\mu_2$,μ_3 and $\mu_1\wedge\mu_2$are fuzzy γ −open sets but $\mu_2\wedge\mu_3$is not fuzzy γ −open set.

Indeed $\mu_2\wedge\mu_3 \nleq$ clint $(\mu_2\wedge\mu_3)\vee$intcl$(((\mu_2\wedge\mu_3)$, $\mu_2\wedge\mu_3$is not fuzzy γ −open set.

But $\mu_1\wedge\mu_2 \leq$ clint $(\mu_1\wedge\mu_2)\vee$intcl$(\mu_1\wedge\mu_2) \therefore \mu_1\wedge\mu_2$ is fuzzy γ −open set

Now, verify whether μ_2,μ_3 and $\mu_1\wedge\mu_2$ are fuzzy γ −no where dense sets.

$intcl(\mu_1\wedge\mu_2) = 0$, incl $intcl\mu_2 = intcl\mu_3 \neq 0$

so, $\mu_1\wedge\mu_2$is a fuzzy γ −Baire space in(X, τ).

Definition 3.6. A fuzzy γ −open set λ in fts(X, τ) is called fuzzy γ −dense if there exists no fuzzy γ −closed set μ in(X, τ) such that $\lambda < \mu < 1$; That is $cl(\lambda) = 1$.

Definition 3.7. A fuzzy γ −open set λ in fts(X, τ) is said fuzzy nowhere γ −dense if there exists no non zero γ −open setμ in(X, τ) such that $\mu < cl(\lambda)$.. That is, $(\lambda) = 0$.

Definition 3.8. A fuzzy γ −open set λ in fts(X, τ) is called fuzzy $\gamma - F_\sigma$set in (X,τ) if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $1 - \lambda_i \in \tau, for\ i\ \in I$.

Definition 3.9. A fuzzy γ −open set λ in fts(X, τ) is called fuzzy $\gamma - G_\delta$set in (X,τ) if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where $\lambda_i \in \tau, for\ i\ \in I$.

Definition 3.10. A fuzzy γ −open set λ in a fts (X, τ) is fuzzy first categoryγ −set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere γ −dense sets in(X, τ). Any other fuzzy γ −set in (X, τ) is said to be fuzzy second category.

Definition 3.11. A fts (X, τ) is called γ −P space if countable intersection of fuzzy γ −open sets in(X, τ) is fuzzy γ −open. That is, every non zero fuzzy $\gamma - G_\delta$ set in (X,τ) is fuzzyγ −open in(X, τ).

Definition 3.12. Fuzzy γ −hyper connected space: A fuzzy topological space X is called fuzzy γ − hyper connected if there is no proper fuzzy set of X which is both fuzzyγ − open and fuzzy γ −closed.

Definition 3.13. A fuzzy topological space (X, τ) is called a fuzzy γ − submaximal space if for each fuzzy γ −open set λ in (X,τ) such that $cl(\lambda) = 1$.

Theorem 3.14. If a fuzzy γ −P space (X, τ) is a fuzzy γ −hyper connected space, then (X, τ) is a fuzzy γ −Baire space

Proof: Here, let λ be a fuzzy $\gamma - G_\delta$ space in fuzzy γ −P space(X, τ).
Since, fts (X,τ) is fuzzy γ −hyper connected space, the fuzzy γ −open set λ is dense set in (X, τ)
i.e.$cl(\lambda) = 1$. Also, we know, since λ is fuzzy γ −P space and γ −dense set,
Therefore, (1-λ) is first category set in (X, τ) .
$\therefore 1 - \lambda = \vee_{i=1}^{\infty}(\lambda_i)$ where (λ_i)'s are fuzzy nowhere dense sets in (X, τ)

$$int(\vee_{i=1}^{\infty}(\lambda_i) = int(1 - \lambda_i) = 1 - cl\lambda_i = 1 - 1 = 0$$

$\Rightarrow (X, \tau)$ is fuzzy γ −Baire Space. Proved

Remark 3.15. the converse of the above proposition need not be true. A fuzzy γ −second category space need not be a fuzzy γ − Baire space.
An example are given support of it:

Example 3.16. Let μ_1, μ_2, μ_3 be fuzzy sets on X=[0,1] defined as

$$\mu_1(x) = 0 \text{ if } 0 \le x \le \tfrac{1}{2} \qquad \mu_2(x) = 1, \text{if } 0 \le x \le \tfrac{1}{4}$$

$$= 2x - 1 \text{ if } \tfrac{1}{2} \le x \le 1, \qquad = -4x + 2, \text{ if } \tfrac{1}{4} \le x \le \tfrac{1}{2}$$

$$= 0 \text{ if} \tfrac{1}{2} \le x \le 1$$

$$\mu_3(x) = 0 \ , if\ 0 \le x \le \tfrac{1}{4}$$

$$= \frac{4x - 1}{3} \text{ if } \frac{1}{4} \le x \le 1$$

Consider τ={0,1, μ_1,μ_2,μ_3,$\mu_1 \vee_2$} on X.
In (X, τ), μ_2,μ_3 and $\mu_1 \wedge \mu_2$ are fuzzy γ −open sets but $\mu_2 \wedge \mu_3$is not fuzzy γ −open set.
For Fuzzy γ −no where dense set: Verify $\gamma - intcl\lambda_i = 0$, where λ_i are fuzzy γ −open set.
Here $\mu_1 \wedge \mu_2, \mu_2$ are fuzzy γ −no where dense set.
Now For γ −second category set: To verify $\vee\lambda_i \neq 1$
Here($(\mu_1 \wedge \mu_2) \vee \mu_2 = \mu_2 \neq 1$
$\therefore$(X,τ) is a fuzzy γ −second category space.
Now to test, whether is it a fuzzy γ −Baire space i.e. $\gamma -$ int$\vee\lambda_i = 0$
$\gamma - int\{(\mu_1 \wedge \mu_2) \vee \mu_2\} = \mu_1 \neq 0$
Here (X,τ) is not fuzzy γ −Baire space.

Shows if (X,τ) is a fuzzy γ −second category need not be fuzzy γ −Baire space.

Proposition 3.17. If the fuzzy topological space (X,τ) is fuzzy γ-Baire space, then (X,τ) is fuzzy γ −second category space.
Proof: Let (X,τ) be a fuzzy γ −Baire space.
We have, γ −int($\vee_{i=1}^{\infty}(\lambda_i)$) = 0, where λ_i,s are fuzzy γ −nowhere dense sets in (X,τ).
If possible let $\vee_{i=1}^{\infty}(\lambda_i) = 1_X$
Taking γ −int, γ −int $\vee_{i=1}^{\infty}(\lambda_i) = (\gamma - \text{int})(1_X) = 1$
0 = 1, it is absurd.

$$\vee_{i=1}^{\infty}(\lambda_i) \neq 1$$

⇒(X,τ) is not a first category.
Hence (X,τ) is a fuzzy γ −second category space.

Theorem 3.18. If λ is a fuzzy $\gamma - G_\delta$-set in a fuzzy γ −Baire space and fuzzy P-space(X,τ), then (X,τ) is a fuzzy γ −second category set in (X,τ).
Proof: Let λ be a fuzzy $\gamma - G_\delta$ set in fuzzy γ −Baire space and fuzzy P-space(X,τ).
Here, (X,τ) is a fuzzy P-space, The fuzzy set $\gamma - G_\delta$-set λ is a fuzzy γ −open in(X,τ). Since (X,τ) is a γ −Baire space, we know the open set λ is not a first category set in (X,τ). Hence λ is a fuzzy γ −second category set in (X,τ).

4. Fuzzyγ −D Baire space

Definition 4.1. Fuzzy γ − D-Baire Space: A fuzzy topological space (X,τ) is called a fuzzy γ −D Baire space in (X,τ) if every fuzzy γ −first category set in (X,τ) is a fuzzy γ − nowhere dense set in (X,τ)'
That is, (X,τ) is a fuzzy γ −D Baire space if
γ −intcl(λ) = 0 for each fuzzy γ −first category set in (X,τ).

Proposition 4.2. If (X,τ) is a γ −P space, then (X,τ) is not a fuzzy γ −D- Baire space.
Proof. Let λ be a fuzzyγ first category set in fuzzy γ −P space (X,τ). We know if the γ −first category set λ in fuzzyγ −P space (X,τ) , then λ is not a γ − nowhere dense set in (X,τ). That is, Fuzzy γ-first category set in (X,τ) is not a fuzzyγ −no where dense set in (X,τ). Hence (X,τ) is not a Fuzzy γ −D Baire space

Theorem 4.3: If a fuzzy γ −P-Space (X,τ) is a fuzzy γ −submaximal and fuzzy γ −Baire space, then (X,τ) is fuzzyγ −D-Baire space.
Proof: Let (X,τ) is fuzzyγ −P-Space (X,τ) and be a fuzzy γ −submaximal and fuzzyγ −Baire space. Let λ is γ −first category in (X,τ).
So, we have ,$\gamma - int\lambda = 0$.[since (X,τ) is a fuzzy γ −Baire space]
$\therefore 1 - (\gamma - int\lambda) = 1$
⇒(1-λ) is fuzzy γ −dense set in (X,τ)
Since (X,τ) is fuzzy γ −submaximal,so every (1-λ) is fuzzy γ −open set in(X,τ)
∴λ is fuzzy γ −closed set in (X,τ) ie in P-space.

$$\Rightarrow cl(\lambda) = \lambda$$

⇒int$cl(\lambda) = int\lambda = 0$ [since λ is in fuzzy γ −Baire space]
$\therefore \text{int}cl(\lambda) = 0$
⇒λ is fuzzyγ − no where dense set.

By definition, if each λ fuzzy $\gamma -$ ist category be fuzzy$\gamma -$nowhere dense set then (X, τ) is fuzzy $\gamma -$D Baire space.
$\therefore (X, \tau)$ is fuzzy $\gamma -$D Baire space.

Example 4.4. Let μ_1, μ_2, μ_3 be fuzzy sets on X = [0,1] defined as

$$\mu_1(x) = 0 \text{ if } 0 \leq x \leq \frac{1}{2}$$
$$= 2x - 1 \text{ if } \tfrac{1}{2} \leq x \leq 1,$$
$$\mu_2(x) = 1, \text{if } 0 \leq x \leq \frac{1}{4}$$
$$= -4x + 2, \text{ if } \tfrac{1}{4} \leq x \leq \tfrac{1}{2}$$
$$= 0 \ \ if \tfrac{1}{2} \leq x \leq 1$$
$$\mu_3(x) = 0, \text{if } 0 \leq x \leq \tfrac{1}{4}$$
$$= \frac{4x - 1}{3} \text{ if } \frac{1}{4} \leq x \leq 1$$

Consider τ = {0,1, μ_1, μ_2, μ_3 } on X.
Here, in (X, τ), μ_1, μ_2, μ_3 and $\mu_1 \wedge \mu_2$ are fuzzy $\gamma -$open sets.
and $1 - \mu_2$, $1 - (\mu_1 \wedge \mu_2)$ are fuzzy $\gamma -$nowhere dense sets.
Therefore, $\text{int}((1 - \mu_2) \vee (1 - \mu_1 \wedge \mu_2)) = 0$.
Hence (X, τ) is fuzzy $\gamma -$Baire space.
Here $1 - \mu_1$ is fuzzy γ-first category.
γ-intcl$(1 - \mu_1) = 0 \Rightarrow$ fuzzy $\gamma -$nowhere dense set.
$\therefore (X, \tau)$ is fuzzy $\gamma -$D Baire Space.

Definition 4.5. A fuzzy topological space $(X, \tau)$is fuzzy$\gamma -$ Baire space. Then (X, τ) is fuzzy $\gamma - D' -$ Baire Spaceif every fuzzy setwith empty $\gamma -$nowhere dense sets in(X, τ).

5. Conclusion

In this paper, we introduce and study a new concept fuzzy $\gamma -$Baire space and Fuzzy $\gamma - D -$Baire space in fuzzy topological space. This new concept and the characteristic properties of the discussed fuzzy $\gamma -$first category, fuzzy $\gamma -$second category, fuzzy $\gamma -$Baire space and fuzzy $\gamma - D -$Baire space will play an important part in the studying the theory of fuzzy $\gamma - D' -$Baire space and fuzzy $\gamma -$Baire space. This work may be extended to fuzzy $\gamma -$ Volterra space and fuzzy $\gamma - D -$ Volterra space in fuzzy topological space.It may have applications in quantum particle physics.

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