Fuzzy $\gamma$-Baire Spaces and Fuzzy $\gamma$-D-Baire Spaces

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Abstract. In this paper, we introduce the concept of fuzzy $\gamma$-Baire space and fuzzy $\gamma$-D-Baire space. Some basic theorems and results of these spaces are introduced and also studied. Several examples are given to explain the concepts of these spaces.

Keywords: Fuzzy $\gamma$-open set, fuzzy $\gamma$-nowhere set, fuzzy $\gamma$-first category, fuzzy $\gamma$-second category, fuzzy $\gamma$-Baire space, fuzzy $\gamma$-D Baire space, fuzzy $\gamma$-D’-Baire space.

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1. Introduction

The concept of fuzzy sets which formed the backbone of fuzzy mathematics was first introduced by Zadah [14] in his classical paper in the year of 1965. There after Chang in 1968 was introduced the concepts of fuzzy topology as generalization of topological space. Since then much attention has been imply to develop the fundamental concepts of it by many authors. Thus a modern theory of fuzzy topology has been developed.

The concept of fuzzy $\gamma$-open set was introduced by Hanfy et al. [8] in 1999 as a union of notation of fuzzy semi-open sets and fuzzy preopen sets. Hanafy introduced the fuzzy $\gamma$-open sets which are weaker than each of them. Using this notation, he studied fuzzy $\gamma$-continious mapping on fuzzy topological space. Thangaraj et al. [9-13] discussed fuzzy $\alpha$ – Baire space in fuzzy topological space and studied its properties.

We shall introduce and study the concepts of fuzzy $\gamma$-Baire space and fuzzy $\gamma$-D Baire space by using fuzzy $\gamma$-open sets in fuzzy topological space and relations of fuzzy Baire, fuzzy $\gamma$-Baire and fuzzy $\gamma$-D-Baire spaces are discussed. Also we discuss some theorems and results on this spaces. Many works have been done by Balasubramanian [2-3], Caldas et al. [4] and De [5-7]. Jana, Senapati at al. have investigated on BCK/BCI algebras and related algebras, see [15-17].

2. Preliminaries

Some basic definitions and properties which will be needed are recalled in this section. In this paper we use the notion of a fuzzy topology in the original sense of Chang (1968). Throughout this paper by $(X, \tau)$ we mean a fuzzy topological space(ts, shortly).

For a fuzzy set $\lambda$ in a fts $(X, \tau)$, $cl \lambda$, $int \lambda$, $intcl \lambda$. will respectively stand for the fuzzy closure, fuzzy interior and interior closure interior of $\lambda$ in $(X, \tau)$. 

75
Definition 2.1. [8] A fuzzy subset $\lambda$ of fuzzy topological space $(X, \tau)$ is said to be fuzzy-$\gamma$ open set (resp. fuzzy-$\gamma$ closed set) if
\[ \lambda \leq \text{cl}\text{int}\lambda \vee \text{int}\text{cl}\lambda \quad (\text{res. } \lambda \geq \text{cl}\text{int}\lambda \wedge \text{intcl}\lambda) \]
Also, it may be written that the complement of fuzzy $\gamma$-open set is fuzzy $\gamma$-closed set. Any union (resp. intersection) of fuzzy $\gamma$-open (resp. fuzzy $\gamma$-closed) set is fuzzy $\gamma$-open (resp. fuzzy $\gamma$-closed).

Remark 2.2. It is obvious that every fuzzy open (resp. fuzzy closed) set is a fuzzy $\gamma$-open (resp. fuzzy $\gamma$-closed set). But the converse need not be true in general.

Proposition 2.3. [1] If $\mu$ and $\lambda$ are two fuzzy subsets of a fts $(X, \tau)$ then
(i) $\text{cl}\mu \vee \text{cl}\lambda = \text{cl}(\mu \lor \lambda)$ and $\text{cl}\mu \wedge \text{cl}\lambda \geq \text{cl}(\mu \land \lambda)$
(ii) $\text{int}\mu \land \text{int}\lambda = \text{int}(\mu \land \lambda)$ and $\text{int}\mu \lor \text{int}\lambda \leq \text{int}(\mu \lor \lambda)$
(iii) $1 - \text{int}\lambda = \text{cl}(1 - \lambda)$
(iv) $1 - \text{cl}\lambda = \text{int}(1 - \lambda)$

Definition 2.4. [8] The $\gamma$-interior and $\gamma$-closure of a fuzzy set $A$ in $(X, \tau)$ are denoted by $\gamma\text{-int}(A)$ and $\gamma\text{-cl}(A)$ respectively and are defined as
\[ \gamma\text{-int}(A) = \bigvee\{B: B \leq A, B \text{ is fuzzy } \gamma\text{-open set in } X\} \]
\[ \gamma\text{-cl}(A) = \bigwedge\{C: C \geq A, C \text{ is fuzzy } \gamma\text{-closed set in } X\} \]
Here, let $\mu_A(x)$ and $\mu_B(x)$ be the membership function of every $x \in A$ and $B$ respectively. Then a member of $A$ is contained in a member of $B$ which is denoted by $A \leq B$ iff $\mu_A(x) \leq \mu_B(x)$.

Proposition 2.5. [8] If $\lambda$ is a subset of $(X, \tau)$ and $\lambda'$ is its complement then
(i) $\gamma - \text{cl}\lambda' = (\gamma - \text{int}\lambda)'$
(ii) $\gamma - \text{int}\lambda' = (\gamma - \text{cl}\lambda)'$

Definition 2.6. [10]
i) A fuzzy set $\lambda$ in a fuzzy topological space $(X, \tau)$ is called fuzzy dense if there exists no fuzzy closed set $\mu$ in $(X, \tau)$ such that $\lambda < \mu < 1$. That is, $\text{cl}(\lambda) = 1$.
(ii) A fuzzy set $\lambda$ in a fts $(X, \tau)$ is called fuzzy nowhere dense if there exists no non-zero fuzzy open set $\mu$ in $(X, \tau)$ such that $\mu < d(\lambda)$, that is $\text{intd}(\lambda) = 0$.

Definition 2.7. [3] A fuzzy set $\lambda$ in a fts $(X, \tau)$ is called a fuzzy $G_\delta$-set in $(X, \tau)$ if $\lambda =$ $\bigwedge_{i \in \Lambda}(\lambda_i)$ where $\lambda_i \in \tau$ for $i \in \Lambda$.

Proposition 2.8. [1] For a family $A$ of $\{\lambda_\alpha\}$ of fuzzy sets of a fts $(X, \tau)$,
Fuzzy $\gamma$-Baire Spaces and Fuzzy $\gamma$-D-Baire Spaces

\[ \forall (\lambda_a) \leq \text{cl}(\forall \lambda_a). \]

In case $A$ is finite set, \[ \forall (\text{cl}(\lambda_a)) = \text{cl}(\forall \lambda_a). \]

Also \[ \forall (\text{int}(\lambda_a)) \leq \text{int}(\forall \lambda_a). \]

**Definition 2.9.** [12] A fts $(X, \tau)$ is called a fuzzy Baire space if \[ \forall (\bigvee_{i=1}^{\infty} (\lambda_i)) = 0 \] where $(\lambda_i)$’s are fuzzy nowhere dense sets in $(X, \tau)$.

**Definition 2.10.** [12] A fuzzy set $\lambda$ in fts $(X, \tau)$ is called a fuzzy first category set if \[ \forall (\bigvee_{i=1}^{\infty} \lambda_i) = 1_X, \] where $(\lambda_i)$’s are nowhere dense sets in $(X, \tau)$. Any other fts in $(X, \tau)$ is said to be of fuzzy second category.

**Theorem 2.12.** [12] If $\lambda$ is a fuzzy dense set and fuzzy $\gamma$ set in a fuzzy topological space $(X, \tau)$, then $1-\lambda$ is a fuzzy first category set in $(X, \tau)$.

**Definition 2.13.** [9] A fts $(X, \tau)$ is called fuzzy P-space if every non zero fuzzy $G_\delta$-set in $(X, \tau)$ is fuzzy open in $(X, \tau)$. That is, if \[ \forall (\bigwedge_{i=1}^{\infty} \lambda_i) \in \tau \] for $i \in \Lambda$.

**Definition 2.14.** [13] A fts $(X, \tau)$ is called a fuzzy Volterra space if \[ \forall (\bigvee_{i=1}^{\infty} \lambda_i) = 1_X, \] where $(\lambda_i)$’s are fuzzy dense and fuzzy $G_\delta$-sets in $(X, \tau)$.

**Definition 2.15.** [3] A fuzzy topological space $(X, \tau)$ is called a fuzzy submaximal space if for each fuzzy open set $\lambda$ in $(X, \tau)$ such that $\text{cl}(\lambda) = 1$.

3. Fuzzy $\gamma$ – Baire Space

**Definition 3.1.** Fuzzy $\gamma$ – Baire Space: Let $(X, \tau)$ be a fuzzy topological space. Then $(X, \tau)$ is called fuzzy $\gamma$ – Baire Space if \[ \forall (\bigvee_{i=1}^{\infty} (\lambda_i)) = 0, \] where $(\lambda_i)$’s are fuzzy $\gamma$ – nowhere dense sets in $(X, \tau)$.

**Proposition 3.2.** Let $(X, \tau)$ be a fuzzy topological space. Then the following are equivalent:

i) $(X, \tau)$ is a fuzzy $\gamma$ – Baire space.

ii) $\forall (\bigvee_{i=1}^{\infty} (\lambda_i)) = 0$, for every fuzzy $\gamma$-first category set $\lambda$ in $(X, \tau)$.

iii) $\forall (\bigvee_{i=1}^{\infty} (\lambda_i)) = 1$ for every fuzzy $\gamma$ – residual set $\alpha$ in $(X, \tau)$

**Proof:** From i)⇒ii) 

Let $\lambda$ be a fuzzy $\gamma$ – first category set in $(X, \tau)$.  

77
Then $\lambda = \bigvee_{i=1}^{\infty} (\lambda_i)$, where $\lambda_i$ are fuzzy $\gamma$- nowhere dense sets in $(X, \tau)$.

$\therefore \gamma - \text{int} \lambda = \bigvee_{i=1}^{\infty} (\lambda_i) = 0$ [since $(X, \tau)$ is $\gamma$-Baire space.]

$\therefore \gamma - \text{int} \lambda = 0$

(ii)$\Rightarrow$(iii):

Let $\alpha$ be a fuzzy $\gamma$- residual set in $(X, \tau)$.

Then $1-\alpha$ is a fuzzy $\gamma$- first category set in $(X, \tau)$.

(iii)$\Rightarrow$(ii):

Let $\lambda$ be a fuzzy $\gamma$- first category set in $(X, \tau)$.

Then $\lambda = \bigvee_{i=1}^{\infty} (\lambda_i)$, where $\lambda_i$ are fuzzy $\gamma$- nowhere dense sets in $(X, \tau)$.

Now, $\lambda$ is $\gamma$- first category set.

$1-\lambda$ is a $\gamma$- residual in $(X, \tau)$.

Now we have,

$\gamma - \text{cl}(1 - \lambda) = 1$

$1 - (\gamma - \text{int} \lambda) = 1$

$\therefore \gamma - \text{int} \lambda = 0$

$\Rightarrow \gamma - \bigvee_{i=1}^{\infty} (\lambda_i) = 0$, where $\lambda_i$ is nowhere dense set

$\Rightarrow (X, \tau)$ is a fuzzy $\gamma$- Baire set.

**Theorem 3.3.** Let $(X, \tau)$ is a fuzzy $\gamma$- Baire space and $\lambda$ is fuzzy set in $(X, \tau)$. Prove that $\gamma - \text{int}(\gamma - \text{cl}\text{nt}\lambda) \geq 0$.

**Proof:** Let $\{\lambda_i, i \in A: \}$ be a collection of fuzzy $\gamma$- open sets in $(X, \tau)$.

Now, we have $\lambda_i \leq \text{int}cl\lambda_i \text{nt} \lambda_i$.

Let $(X, \tau)$ is a fuzzy $\gamma$- Baire space.

By definition, for no where dense set if

$\gamma - \text{int}(\gamma - \text{cl}\lambda) = 0$

And for $(X, \tau)$ is fuzzy $\gamma$- open set

We have $\lambda \leq \text{int}cl\lambda \text{nt} \lambda$ and arbitrary union of $\gamma$- open set is $\gamma$- open, let $\lambda = \bigvee \lambda_i$

And $\gamma - \text{int}(\gamma - \text{cl}\lambda)$ mean that intcl$\gamma$- open sets

Now taking union on both sides

$\bigvee \lambda \leq \bigvee (\text{int}cl\lambda \text{nt} \lambda) \leq \text{int}cl(\bigvee \lambda_i \text{nt} \lambda_i)$, $i = 1$ to $\infty$

Taking $\gamma - \text{int}(\gamma - \text{cl})$ on both sides,

$\gamma - \text{int}(\gamma - \text{cl})\lambda \leq \gamma - \text{int}(\gamma - \text{cl})(\text{int}cl\bigvee \lambda_i \text{nt} \lambda_i))$

$\leq (\gamma - \text{int}(\gamma - \text{cl})\lambda_i) \text{nt}((\gamma - \text{int}(\gamma - \text{cl})\lambda_i))$

$\leq ((\gamma - \text{int}(\gamma - \text{cl})\lambda_i))$, [since $\lambda_i$ is $\gamma$- nowhere dense set,

$\gamma - \text{int}(\gamma - \text{cl})\lambda_i = 0$]

$\therefore 0 \leq \gamma - \text{int}(\gamma - \text{cl}\lambda)$

78
Fuzzy $\gamma$-Baire Spaces and Fuzzy $\gamma$-$D$-Baire Spaces

$\gamma - \text{int}(\gamma - \text{clint}(\lor \lambda_i)) \geq 0$, proved.

**Definition 3.4.** Fuzzy $\gamma$-Baire Space: Let $(X, \tau)$ be a fuzzy topological space. Then $(X, \tau)$ is called fuzzy $\gamma$–Baire space if $\gamma - \text{int}(\gamma - \text{clint}(\lor \lambda_i)) \geq 0$.

**Example 3.5.** Let $\mu_1, \mu_2, \mu_3$ be fuzzy sets on $X=[0,1]$ defined as

$$\mu_1(x) = 0 \text{ if } 0 \leq x \leq \frac{1}{2}, \quad \mu_2(x) = 1 \text{ if } 0 \leq x \leq \frac{1}{4},$$

$$= 2x - 1 \text{ if } \frac{1}{2} \leq x \leq 1, \quad = -4x + 2, \text{ if } \frac{1}{4} \leq x \leq \frac{1}{2}$$

$$= 0, \text{ if } \frac{1}{2} \leq x \leq 1$$

Consider $\tau=\{0,1, \mu_1 \mu_2 \mu_3 \mu_4 \vee 2 \}$ on $X$. In $(X, \tau)\mu_2, \mu_3$ and $\mu_1 \wedge \mu_2$ are fuzzy $\gamma$–open sets but $\mu_2 \wedge \mu_3$ is not fuzzy $\gamma$–open set.

Indeed $\mu_2 \wedge \mu_3 \notin \text{clint}(\mu_2 \wedge \mu_3) \vee \text{intcl}(\mu_2 \wedge \mu_3) \wedge \mu_2 \wedge \mu_3$ is not fuzzy $\gamma$–open set.

But

$$\mu_1 \wedge \mu_2 \leq \text{clint}(\mu_1 \wedge \mu_2) \vee \text{intcl}(\mu_1 \wedge \mu_2) \wedge \mu_1 \wedge \mu_2$$

is fuzzy $\gamma$–open set

Now, verify whether $\mu_2 \mu_3$ and $\mu_1 \wedge \mu_2$ are fuzzy $\gamma$–no where dense sets. $\text{intcl}(\mu_1 \wedge \mu_2) = 0$, $\text{incl} \text{intcl}\mu_2 = \text{intcl}\mu_3 \neq 0$

so, $\mu_1 \wedge \mu_2$ is a fuzzy $\gamma$–Baire space in $(X, \tau)$.

**Definition 3.6.** A fuzzy $\gamma$–open set $\lambda$ in fts$(X, \tau)$ is called fuzzy $\gamma$–dense if there exists no fuzzy $\gamma$–closed set $\mu$ in $(X, \tau)$ such that $\lambda < \mu < 1$; That is $\text{cl}(\lambda) = 1$.

**Definition 3.7.** A fuzzy $\gamma$–open set $\lambda$ in fts$(X, \tau)$ is said fuzzy nowhere $\gamma$–dense if there exists no non zero $\gamma$–open set $\mu$ in $(X, \tau)$ such that $\mu < \text{cl}(\lambda)$.. That is, $\lambda = 0$.

**Definition 3.8.** A fuzzy $\gamma$–open set $\lambda$ in fts$(X, \tau)$ is called fuzzy $\gamma - F_\sigma$ set in $(X, \tau)$ if $\lambda = \bigwedge_{\lambda_i \in \tau, f or \ i \in I} (\lambda_i)$, where $\lambda_i \in \tau, f or \ i \in I$.

**Definition 3.9.** A fuzzy $\gamma$–open set $\lambda$ in fts$(X, \tau)$ is called fuzzy $\gamma - G_\delta$ set in $(X, \tau)$ if $\lambda = \bigwedge_{\lambda_i \in \tau, f or \ i \in I} (\lambda_i)$, where $\lambda_i \in \tau, f or \ i \in I$.

**Definition 3.10.** A fuzzy $\gamma$–open set $\lambda$ in a fts $(X, \tau)$ is fuzzy first category $\gamma$–set if $\lambda = \bigvee_{\lambda_i \in \tau} (\lambda_i)$, where $(\lambda_i)$'s are fuzzy nowhere $\gamma$–dense sets in $(X, \tau)$. Any other fuzzy $\gamma$–set in $(X, \tau)$ is said to be fuzzy second category.

**Definition 3.11.** A fts $(X, \tau)$ is called $\gamma$–P space if countable intersection of fuzzy $\gamma$–open sets in $(X, \tau)$ is fuzzy $\gamma$–open. That is, every non zero fuzzy $\gamma - G_\delta$ set in $(X, \tau)$ is fuzzy $\gamma$–open in $(X, \tau)$.
Definition 3.12. Fuzzy γ −hyper connected space: A fuzzy topological space X is called fuzzy γ − hyper connected if there is no proper fuzzy set of X which is both fuzzy γ − open and fuzzy γ − closed.

Definition 3.13. A fuzzy topological space (X, τ) is called a fuzzy γ − submaximal space if for each fuzzy γ − open set λ in (X, τ) such that int(λ) = 1.

Theorem 3.14. If a fuzzy γ −P space (X, τ) is a fuzzy γ − hyper connected space, then (X, τ) is a fuzzy γ − Baire space.

Proof: Here, let λ be a fuzzy γ − Gδ space in fuzzy γ −P space (X, τ). Since, its (X, τ) is fuzzy γ − hyper connected space, the fuzzy γ − open set λ is dense set in (X, τ) i.e. cl(λ) = 1. Also, we know, since λ is fuzzy γ −P space and γ − dense set, Therefore, (1−λ) is first category set in (X, τ).

\[ 1 − λ = ∨_{i=1}^{α} (λ_i) \] where (λ_i)’s are fuzzy nowhere dense sets in (X, τ)

\[ int(∨_{i=1}^{α} (λ_i)) = int(1 − λ_i) = 1 − clλ_i = 1 − 1 = 0 \]

⇒ (X, τ) is fuzzy γ −Baire Space. Proved

Remark 3.15. The converse of the above proposition need not be true. A fuzzy γ −second category space need not be a fuzzy γ − Baire space.

An example are given support of it:

Example 3.16. Let μ_1, μ_2, μ_3 be fuzzy sets on X=[0,1] defined as

\[ μ_1(x) = 0 \text{ if } 0 \leq x \leq \frac{1}{2} \]

\[ = 2x − 1 \text{ if } \frac{1}{2} \leq x \leq 1, \]

\[ μ_2(x) = 1, \text{if } 0 \leq x \leq \frac{1}{4} \]

\[ = −4x + 2, \text{ if } \frac{1}{4} \leq x \leq \frac{1}{2} \]

\[ = 0 \text{ if } \frac{1}{2} \leq x \leq 1 \]

\[ μ_3(x) = 0, \text{if } 0 \leq x \leq \frac{1}{4} \]

\[ = \frac{4x − 1}{3}, \text{if } \frac{1}{4} \leq x \leq 1 \]

Consider τ={0,1, μ_1, μ_2, μ_3} on X.

In (X, τ), μ_2, μ_3 and μ_1 ∨ μ_2 are fuzzy γ−open sets but μ_2 ∧ μ_3 is not fuzzy γ−open set.

For Fuzzy γ−no where dense set: Verify γ−intclλ_i = 0, where λ_i are fuzzy γ−open set.

Here μ_1 ∧ μ_2, μ_2 are fuzzy γ−no where dense set.

Now For γ−second category set: To verify ∨\λ_i ≠ 1

Here (μ_1 ∧ μ_2) ∨ μ_2 = μ_2 ≠ 1

∴ (X, τ) is a fuzzy γ−second category space.

Now to test, whether is it a fuzzy γ−Baire space i.e. γ− int\∨λ_i = 0

γ− int((μ_1 ∧ μ_2) ∨ μ_2) = μ_1 ≠ 0

Here (X, τ) is not fuzzy γ−Baire space.
Fuzzy $\gamma$-Baire Spaces and Fuzzy $\gamma$-D-Baire Spaces

Shows if $(X, \tau)$ is a fuzzy $\gamma$-second category need not be fuzzy $\gamma$ -Baire space.

**Proposition 3.17.** If the fuzzy topological space $(X, \tau)$ is fuzzy $\gamma$-Baire space, then $(X, \tau)$ is fuzzy $\gamma$ -second category space.

**Proof:** Let $(X, \tau)$ be a fuzzy $\gamma$ -Baire space.

We have, $\gamma -\operatorname{int}(\bigvee_{i=1}^{\infty}(\lambda_i)) = 0$, where $\lambda_i$s are fuzzy $\gamma$ -nowhere dense sets in $(X, \tau)$.

If possible let $\bigvee_{i=1}^{\infty}(\lambda_i) = 1_X$

Taking $\gamma -\operatorname{int}, \gamma -\operatorname{int} \bigvee_{i=1}^{\infty}(\lambda_i) = (\gamma - \operatorname{int})(1_X) = 1$

$0 = 1$, it is absurd.

Hence $(X, \tau)$ is a fuzzy $\gamma$ -second category space.

**Theorem 3.18.** If $\lambda$ is a fuzzy $\gamma -G_\delta$-set in a fuzzy $\gamma$ -Baire space and fuzzy P-space $(X, \tau)$, then $(X, \tau)$ is a fuzzy $\gamma$ -second category set in $(X, \tau)$.

**Proof:** Let $\lambda$ be a fuzzy $\gamma -G_\delta$ set in fuzzy $\gamma$ -Baire space and fuzzy P-space $(X, \tau)$.

Here, $(X, \tau)$ is a fuzzy P-space. The fuzzy set $\gamma -G_\delta$-set $\lambda$ is a fuzzy $\gamma$ -open in $(X, \tau)$.

Since $(X, \tau)$ is a $\gamma -$Baire space, we know the open set $\lambda$ is not a first category set in $(X, \tau)$.

Hence $\lambda$ is a fuzzy $\gamma$ -second category set in $(X, \tau)$.

4. Fuzzy $\gamma$ -D Baire space

**Definition 4.1.** Fuzzy $\gamma$ -D-Baire Space: A fuzzy topological space $(X, \tau)$ is called a fuzzy $\gamma$ -D Baire space in $(X, \tau)$ if every fuzzy $\gamma$ -first category set in $(X, \tau)$ is a fuzzy $\gamma$ -nowhere dense set in $(X, \tau)$.

That is, $(X, \tau)$ is a fuzzy $\gamma$ -D Baire space if $\gamma -\operatorname{intcl}(\lambda) = 0$ for each fuzzy $\gamma$ -first category set in $(X, \tau)$.

**Proposition 4.2.** If $(X, \tau)$ is a $\gamma$ -P space, then $(X, \tau)$ is not a fuzzy $\gamma$ -D-Baire space.

**Proof:** Let $\lambda$ be a fuzzy $\gamma$ -first category set in fuzzy $\gamma$ -P space $(X, \tau)$. We know if the $\gamma$ -first category set $\lambda$ in fuzzy $\gamma$ -P space $(X, \tau)$, then $\lambda$ is not a $\gamma$ -nowhere dense set in $(X, \tau)$. That is, Fuzzy $\gamma$-first category set in $(X, \tau)$ is not a fuzzy $\gamma$ -nowhere dense set in $(X, \tau)$.

Hence $(X, \tau)$ is not a Fuzzy $\gamma$ -D Baire space.

**Theorem 4.3:** If a fuzzy $\gamma$ -P-Space $(X, \tau)$ is a fuzzy $\gamma$ -submaximal and fuzzy $\gamma$ -Baire space, then $(X, \tau)$ is fuzzy $\gamma$ -D-Baire space.

**Proof:** Let $(X, \tau)$ is fuzzy $\gamma$ -P-Space $(X, \tau)$ and be a fuzzy $\gamma$ -submaximal and fuzzy $\gamma$ -Baire space. Let $\lambda$ is $\gamma$ -first category in $(X, \tau)$.

So, we have $\gamma -\operatorname{int}\lambda = 0$, [since $(X, \tau)$ is a fuzzy $\gamma$ -Baire space]

$\therefore 1 - (\gamma - \operatorname{int}\lambda) = 1$

$\Rightarrow (1-\lambda)$ is fuzzy $\gamma$ -dense set in $(X, \tau)$

Since $(X, \tau)$ is fuzzy $\gamma$ -submaximal, so every $(1-\lambda)$ is fuzzy $\gamma$ -open set in $(X, \tau)$

$\therefore \lambda$ is fuzzy $\gamma$ -closed set in $(X, \tau)$ ie in P-space.

$\therefore \operatorname{cl}(\lambda) = \lambda$

$\Rightarrow \operatorname{intcl}(\lambda) = \operatorname{int}\lambda = 0$ [since $\lambda$ is in fuzzy $\gamma$ -Baire space]

$\therefore \operatorname{intcl}(\lambda) = 0$

$\Rightarrow \lambda$ is fuzzy $\gamma$ - no where dense set.
By definition, if each $\lambda$ fuzzy $\gamma$-ist category be fuzzy nowhere dense set then $(X, \tau)$ is fuzzy $\gamma$-D Baire space.

\[\therefore (X, \tau) \text{ is fuzzy } \gamma\text{ -D Baire space.}\]

**Example 4.4.** Let $\mu_1, \mu_2, \mu_3$ be fuzzy sets on $X = [0,1]$ defined as

\[\mu_1(x) = 0 \text{ if } 0 \leq x \leq \frac{1}{2}, \quad = 2x - 1 \text{ if } \frac{1}{2} \leq x \leq 1,\]

\[\mu_2(x) = 1, \text{ if } 0 \leq x \leq \frac{1}{4}, \quad = -4x + 2, \text{ if } \frac{1}{4} \leq x \leq \frac{1}{2},\]

\[= 0 \text{ if } \frac{1}{2} \leq x \leq 1\]

\[\mu_3(x) = 0, \text{ if } 0 \leq x \leq \frac{1}{4}, \quad = \frac{4x - 1}{3} \text{ if } \frac{1}{4} \leq x \leq 1\]

Consider $\tau = \{0, 1, \mu_1, \mu_2, \mu_3\}$ on $X$.

Here, in $(X, \tau)$, $\mu_1, \mu_2, \mu_3$ and $\mu_1 \wedge \mu_2$ are fuzzy $\gamma$-open sets.

and $1 - \mu_2, 1 - (\mu_1 \wedge \mu_2)$ are fuzzy $\gamma$-nowhere dense sets.

Therefore, $\text{int}((1 - \mu_2) \vee (1 - \mu_1 \wedge \mu_2)) = 0$.

Hence $(X, \tau)$ is fuzzy $\gamma$-Baire space.

Here $1 - \mu_1$ is fuzzy $\gamma$-first category.

$\gamma\text{-intcl}(1 - \mu_1) = 0 \Rightarrow$ fuzzy $\gamma$-nowhere dense set.

\[\therefore (X, \tau) \text{ is fuzzy } \gamma\text{ -D Baire Space.}\]

**Definition 4.5.** A fuzzy topological space $(X, \tau)$ is fuzzy $\gamma$-Baire space. Then $(X, \tau)$ is fuzzy $\gamma - D'$-Baire Space if every fuzzy set with empty $\gamma$-nowhere dense sets in $(X, \tau)$.

5. **Conclusion**

In this paper, we introduce and study a new concept fuzzy $\gamma$-Baire space and Fuzzy $\gamma - D$-Baire space in fuzzy topological space. This new concept and the characteristic properties of the discussed fuzzy $\gamma$-first category, fuzzy $\gamma$-second category, fuzzy $\gamma$-Baire space and fuzzy $\gamma - D$-Baire space will play an important part in the studying the theory of fuzzy $\gamma - D'$-Baire space and fuzzy $\gamma$-Baire space. This work may be extended to fuzzy $\gamma$-Volterra space and fuzzy $\gamma - D$-Volterra space in fuzzy topological space. It may have applications in quantum particle physics.

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**REFERENCES**
Fuzzy $\gamma$-Baire Spaces and Fuzzy $\gamma$-D-Baire Spaces