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On Solving Neutrosophic Unconstrained Optimization Problems by Steepest Descent Method and Fletcher Reeves Method

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Abstract. In this paper, we proposed a method for solving unconstrained optimization problems by Newton's method with single-valued neutrosophic triangular fuzzy number coefficients. Also, some numerical examples demonstrate the effectiveness of the proposed algorithm. MATLAB programs are also developed for the proposed method.

Keywords: Single-valued neutrosophic number, neutrosophic set, steepest descent method, Fletchr reeves method, unconstrained optimization.

AMS Mathematics Subject Classification (2010): 65Y04, 65K10, 65F10, 90C70

1. Introduction

The concept of neutrosophic set (NS) was first introduced by Smarandache, which is a generalization of fuzzy sets. Zadeh's classical concept of a fuzzy set is a strong mathematics tool to deal with the complexity generally arising from uncertainty in the form of ambiguity in the recent scenario. In 1965, Zadeh invented the "Fuzzy sets", which play a significant role in dealing with ambiguity and impreciseness. In 1970, Bellman and Zadeh developed "a method for making decisions in a fuzzy environment". In 1983, Atanassov introduced his intuitionistic fuzzy set. Hepzibah et al. (2017) investigated neutrosophic multi-objective linear programming problems [17]. This paper, deals with the fuzzy Steepest Descent method and Fletcher Reeves method with single-valued neutrosophic triangular coefficient to solve unconstrained optimization problems. This paper is organised as follows. The second section provides some background information on this research topic. Several strategies for solving unconstrained optimization problems in a Neutrosophic fuzzy environment are proposed in section three. In section 4, some illustrative cases are offered to demonstrate this method.

2. Preliminaries

This section provided an introduction to fuzzy unconstrained optimization models and stressed the importance to considering the topics like linear and nonlinear optimization problems in the fuzzy environment using arithmetic operations, and provided certain definitions which are related to this research work. In this section, the concept of single-valued neutrosophic numbers, single-valued trapezoidal neutrosophic numbers and single-valued triangular neutrosophic number with operations are introduced.

Definition 1. [7] Let $\tilde{A} = \{x, \mu_{\tilde{A}}(x): x \in \mu_{\tilde{A}}(x) \in [0,1]\}$ is a fuzzy set. The first element $\mathbb{X}$ in the pair $(x, \mu_{\tilde{A}}(x))$ belongs to the classical set A, whereas the second element $\mu_{\tilde{A}}(x)$, belongs to the interval [0,1] known as membership function, indicated by $\tilde{A} = \{\mu_{\tilde{A}}(x) \setminus x: x \in A, \mu_{\tilde{A}}(x) \in [0,1]\}$.

Definition 2. [7] Let $\Re$ be the set of real numbers and $\tilde{A}: \Re \to [0,1]$ be a fuzzy set then we say that $\tilde{A}$ is a fuzzy number that contains the following properties:

(i) 0 is normal, i.e., there exist $x_0 \in \Re$ such that $\tilde{A}(x_0)$=1;

(ii) $\tilde{A}$ is convex, i.e., $\tilde{A}(tx + (1-t)y \geq \min\{\tilde{A}(x), \tilde{A}(y)\}$, where $x, y \in \Re$ and $t \in [0,1]$.

(iii) $\tilde{A}(x)$ is upper semi-continuous on $\Re$, i.e., $\left\{\frac{x}{\tilde{A}(x)} \geq \alpha\right\}$ is a closed subset of $\Re$ for each $\alpha \in [0,1]$.

Definition 3. [7] Let us take a fuzzy number $\tilde{\mathbb{A}}$ on $\Re$ is said to be a triangular fuzzy number (TFN) or linear fuzzy number if its membership function $\tilde{\mathbb{A}}: \Re \to [0,1]$ meets the following features. It is a fuzzy number represents with three points as follows $\tilde{\mathbb{A}} = (a_1, a_2, a_3)$. The following conditions apply to this representative, which is regarded as membership functions:

(i) $a_1\ to\ a_2$ is increasing function

(ii) $a_2\ to\ a_3$ is decreasing function.

(iii) $a_1 \leq a_2 \leq a_3$

$$\mu_{\tilde{\mathbb{A}}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & \text{for } a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & \text{for } a_2 \leq x \leq a_3 \\ 0 & \text{Otherwise} \end{cases}$$

The TFN is diagrammatically shown below.

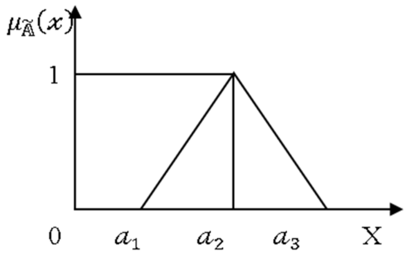

Figure 1.1 Triangular Fuzzy number

Let $F(\mathfrak{R})$ to denote the set of all TFNs. The α level set of $\tilde{\mathbb{A}}$ is defined as $\tilde{\mathbb{A}}_\alpha = [(a_2 - a_1)\alpha + a_1, a_3 - (a_3 - a_2)\alpha]$.

Definition 5. [18 Let G be a universe. A Neutrosophic Set(NS) A in G is characterised by a truth-membership function T_A, a indeterminacy-membership function I_A, and a falsity membership function F_A. $T_A(\mathrm{x}); I_A(\mathrm{x})$ and $F_A(x)$ are real standard elements of [0,1]. It can be written as $A = \{x, (T_A(\mathrm{x}); I_A(\mathrm{x}) F_A(x)) >: x \in G, T_A(\mathrm{x}), I_A(x), F_A(x) \in]^{-}0,1[^{+}\}$. There is no restriction on the sum of $T_A(x): I_A(x)\ and\ F_A(\mathrm{x})$, so on $0^{-} \leq supT_A(x) + supI_A(x) + supF_A(x) \leq 3^{+}$.

Definition 6. [18] Let G be a universe. A single valued Neutrosophic Set (SVNS) A, which can be used I a real scientific and engineering applications, in G is characterised by a truth membership function T_A, an indeterminancy- membership function I_A and a falsity membership function F_A. $T_A(x); I_A(x)\ and\ F_A(x)$, are real standard elements of [0,1] . It can be written as $\mathrm{A} = \{< \mathrm{x}, (T_A(x), I_A(x), F_A(x)) >: x \in E, T_A(x), I_A(x)\ and\ F_A \in [0,1]$. Thereis no restriction on the sum of $T_A(x), F_A(x) and\ I_A(x)$, so $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 7. [18] Let $j_{\tilde{a}}, k_{\tilde{a}}, q_{\tilde{a}} \in [0,1]$ be any real numbers. A single valued neutrosophic number $\tilde{a} = (s_1, d_1, f_1, g_1); j_{\tilde{a}}), (s_2, d_2, f_2, g_2); k_{\tilde{a}}), (s_3, d_3, f_3, g_3); q_{\tilde{a}}))$, is defined as a special single valued neutrosophic set on the set of real numbers R, whose truth-membership function $\mu_{\tilde{a}}: R \to [0, j_{\tilde{a}}]$, a determinancy-membership function $\nu_{\tilde{a}}: R \to [0, k_{\tilde{a}}]$ and a falsity-membership function $\lambda_{\tilde{a}}: R \to [0, q_{\tilde{a}}]$ as given by

$$\mu_{\tilde{a}}(x) = \begin{cases} f_{\mu l}(x), & (s_1 \le x \le d_1) \\ j_{\tilde{a}}, & (d_1 \le x \le f_1) \\ f_{\mu r}(x), & f_1 \le x \le g_1) \\ 0, & \text{otherwise} \end{cases}$$

$$\mu_{\tilde{a}}(x) = \begin{cases} f_{\lambda l}(x), & (s_3 \le x \le d_3) \\ k_{\tilde{a}}, & (d_3 \le x \le f_3) \\ f_{\lambda r}(x), & f_3 \le x \le g_3) \\ 1, & \text{otherwise} \end{cases}$$

$$\mu_{\tilde{a}}(x) = \begin{cases} f_{\nu l}(x), & (s_2 \le x \le d_2) \\ q_{\tilde{a}}, & (d_2 \le x \le f_2) \\ f_{\nu r}(x), & f_2 \le x \le g_2) \\ 1, & \text{otherwise} \end{cases}$$

2.2. Arithmetic Operations of single valued triangular neutrosophic numbers

Let $\tilde{a} = ((s_1, d_1, f_1); j_{\tilde{a}}, k_{\tilde{a}}, q_{\tilde{a}})$ *and* $\tilde{b} = ((s_2, d_2, f_2); j_{\tilde{b}}, k_{\tilde{b}}, q_{\tilde{b}}))$ be two single valued triangular neutrosophic numbers . Then

Addition [18]

$$\tilde{a} + \tilde{b} = ((s_1 + s_2, d_1 + d_2, f_1 + f_2); (j_{\tilde{a}} \wedge j_{\tilde{b}}, k_{\tilde{a}} \vee k_{\tilde{b}}, q_{\tilde{a}} \vee q_{\tilde{b}}))$$

Subtraction [18]

$$\tilde{a} - \tilde{b} = ((s_1 - f_3, d_2 - d_1, f_3 - s_1,); (j_{\tilde{a}} \wedge j_{\tilde{b}}, k_{\tilde{a}} \vee k_{\tilde{b}}, q_{\tilde{a}} \vee q_{\tilde{b}}))$$

Multiplication

$\tilde{a}.\tilde{b} = (\left(s_1 . R(\tilde{b}), d_1 . R(\tilde{b}), f_1 . R(\tilde{b})\right); (j_{\tilde{a}} \wedge j_{\tilde{b}}, k_{\tilde{a}} \vee k_{\tilde{b}}, q_{\tilde{a}} \vee q_{\tilde{b}}))$, where $R(\tilde{b}) = ((s_2 + d_2 + f_2) \times (2 + j_{\tilde{b}} + k_{\tilde{b}} - q_{\tilde{b}}))/8$

Division

$\tilde{a}/\tilde{b} = ((s_1/R(\tilde{b}), d_1/R(\tilde{b}), f_1/R(\tilde{b})); (j_{\tilde{a}} \wedge j_{\tilde{b}}, k_{\tilde{a}} \vee k_{\tilde{b}}, q_{\tilde{a}} \vee q_{\tilde{b}}))$ here $R(\tilde{b}) = ((s_2 + d_2 + f_2) \times (2 + j_{\tilde{b}} + k_{\tilde{b}} - q_{\tilde{b}}))/8$

Scalar Multiplication

$$\gamma\tilde{a} = ((\gamma s_1, \gamma d_1, \gamma f_1); j_{\tilde{a}}, k_{\tilde{a}}, q_{\tilde{a}})) \textit{ where } \gamma > 0$$
$$\gamma\tilde{a} = ((\gamma g_1, \gamma f_1, \gamma d_1); j_{\tilde{a}}, k_{\tilde{a}}, q_{\tilde{a}})) \textit{ where } \gamma < 0.$$

Definition 8. [18] Let $\tilde{a} = ((s_1, d_1, f_1); j_{\tilde{a}}, k_{\tilde{a}}, q_{\tilde{a}}))$ be a single valued triangular neutrosophic number. Then $S(\tilde{a}) = ((s_1 + d_1 + f_1) \times (2 + j_{\tilde{a}} - k_{\tilde{a}} - q_{\tilde{a}}))/8$ and

$A(\tilde{a}) = ((s_1 + d_1 + f_1) \times (2 + j_{\tilde{a}} - k_{\tilde{a}} - q_{\tilde{a}}))/8$ are called score and accuracy degrees of $\tilde{a}$ respectively.

3. Neutrosophic fuzzy steepest descent and fuzzy conjugate gradient method

3.1. Method of neutrosophic fuzzy steepest descent

Now we consider an unconstrained $\underset{(x_1,x_{2,}...x_{n,})\in R}{min} f(x_1, x_2, ... x_n)$ has a local minimizer to be referred just as minimizer. The n tuple $x = (x_1, x_{2,} ... x_{n,}) \in R^n$ will called the design vector and $f(x)$ the corresponding objective function. Gradient based techniques are motivated by the fact that f decreased most rapidly at a point P in R^n in the direction of $-\nabla$f(P). consequently, the iteration procedure of the form

$$\bar{x}^{(k+1)} = \bar{x}^k + \tilde{\lambda}^k \bar{d}(\bar{x}^k)$$

where as $\bar{x}^k$ is the current estimate of $\bar{x}^*, \alpha^{(k)}$ is the step length parameter and $\bar{d}^{(k)} = \bar{d}(x^{(k)})$ is the search direction in the space R^n of design variables.

If we take $\bar{d}^{(k)} = -\bar{g}^{(k)} \equiv -\nabla f(x^{(k)})$, we get the method of steepest descent. We have $\bar{x}^{(k+1)} = \bar{x}^k - \alpha^k(\bar{g}^{(k)})$, $\bar{g}^{(k)} \equiv \nabla f(x^{(k)})$

Where $\tilde{\lambda}^{(k)}$ is the minimizer of the function

$$\phi(\tilde{\lambda}) = f(x^{(k)} - \tilde{\lambda}\, \bar{g}^{(k)})$$

We can use any of the 1-dimensional searches to determine the $\alpha^{(k)}$. Initial approximation $\bar{x}^0$ is carefully selected to start the iteration procedure, as it is problem depended.

3.2. Convergence of neutrosophic fuzzy steepest descent method for quadratic function

To Illustrate convergence properties of gradient base methods, a convenient function will be a quadratic function of the form $\tilde{f}(x) = \frac{1}{2}\tilde{x}^T Q\tilde{x} = \frac{1}{2}\langle Q\tilde{x}, \tilde{x}\rangle$ where Q is positive definite. If f has a minimum at $\tilde{x}^* = 0$ with $\tilde{f}(\tilde{x}^*)$=0, $\tilde{g} = \nabla\tilde{f}(\tilde{x}) = Q\tilde{x}$ and $\nabla^2\tilde{f}(\tilde{x}) = Q$. Then

$$\phi(\tilde{\lambda}) = \tilde{f}(\tilde{x} - \tilde{\lambda}\,\tilde{g}) = \frac{1}{2}\langle[\tilde{Q} - 2\tilde{\lambda}\,\widetilde{Q^2} + \tilde{\lambda}^2\widetilde{Q^3}]\tilde{x},\, \tilde{x}\rangle$$

This gives

$$\tilde{\lambda} = \frac{\langle\widetilde{Q^2}\tilde{x},\, \tilde{x}\rangle}{\langle\widetilde{Q^3}\tilde{x},\, \tilde{x}\rangle} = \frac{\langle\tilde{Q}\,\tilde{x}, \tilde{Q}\,\tilde{x}\rangle}{\langle\tilde{Q}\,(\widetilde{Q\,x}), \tilde{Q}\,\tilde{x}\rangle} = \frac{\langle\tilde{g},\, \tilde{g}\rangle}{\langle\tilde{Q}\,\tilde{g},\, \tilde{g}\rangle}$$

Hence the steepest descent iteration procedure for the quadratic function takes the form

$$\tilde{x}^{(k+1)} = \tilde{x}^{(k)} - \frac{\langle\tilde{g}^k, \tilde{g}^k\rangle}{\langle\tilde{Q}\,\tilde{g}^k, \tilde{g}^k\rangle}\tilde{g}^k$$

Using the above formulation and the fact that

$$\tilde{f}(\tilde{x}^{(k+1)}) = \tilde{f}(\tilde{x}^{(k)}) - \frac{1}{2}\frac{[\langle\tilde{g}^k, \tilde{g}^k\rangle]^2}{\langle\tilde{Q}\,\tilde{g}^k, \tilde{g}^k\rangle}$$

This gives

$$\frac{\tilde{f}\left(\tilde{x}^{(k)}\right) - \tilde{f}\left(\tilde{x}^{(k+1)}\right)}{\tilde{f}\left(\tilde{x}^{(k)}\right)} = \frac{[\langle \tilde{g}^k, \tilde{g}^k \rangle]^2}{\langle \tilde{Q}\, \tilde{g}^k, \tilde{g}^k \rangle \langle \tilde{g}^k, Q^{-1}\tilde{g}^k, \rangle}$$

And hence

$$\tilde{f}\left(\tilde{x}^{(k+1)}\right) = (1-\gamma_k)\tilde{f}\left(\tilde{x}^{(k)}\right)$$

where $\gamma_k = \frac{[\langle \tilde{g}^k, \tilde{g}^k \rangle]^2}{\langle \tilde{Q}\, \tilde{g}^k, \tilde{g}^k \rangle \langle \tilde{g}^k, Q^{-1}\tilde{g}^k \rangle}$.

Theorem 3.3. Let $\tilde{f}$ be the Neutrosophic fuzzy quadratic function of the forms gives by $\tilde{f}(x) = \frac{1}{2}\tilde{x}^T Q \tilde{x} = \frac{1}{2}\langle Q\tilde{x}, \tilde{x}\rangle$ and $\left(\tilde{x}^{(k)}\right)$*be the iterates* generated by the procedure given by

$$\tilde{x}^{(k+1)} = \tilde{x}^{(k)} - \frac{\langle \tilde{g}^k, \tilde{g}^k \rangle}{\langle \tilde{Q}\, \tilde{g}^k, \tilde{g}^k \rangle} \tilde{g}^k$$

Starting from any initial approximation $\tilde{x}^0$. Then $\tilde{x}^{(k)}$ converges linearly to the minimizer $\tilde{x}^* = 0$.

Proof:

Let

$$\tilde{f}\left(\tilde{x}^{(k+1)}\right) = (1-\widetilde{\gamma_k})\tilde{f}\left(\tilde{x}^{(k)}\right)$$

This give $\tilde{f}\left(\tilde{x}^{(k)}\right) = \prod_{i=0}^{k-1}(1-\tilde{\gamma_i})\tilde{f}\left(\tilde{x}^0\right)$

$\left(\tilde{x}^{(k)}\right)$ converges to $\tilde{x}^* = \tilde{0}$ if and only if $\tilde{f}\left(\tilde{x}^{(k)}\right) \to 0$ and in the view the above equation, this is possible if and only if $\prod_{i=0}^{\infty}(1-\tilde{\gamma_i}) = 0$, which is true, because $(1-\tilde{\gamma_i}) \le \frac{\tilde{\lambda}_{max} - \tilde{\lambda}_{min}}{\tilde{\lambda}_{max}} \le 1$.

As $\tilde{x}^* = 0$ and $\tilde{f}(\tilde{x}) = \frac{1}{2}\langle \tilde{Q}\,\tilde{x},\ \tilde{x}\rangle$, Rayleigh inequality gives $\frac{\lambda_{min}}{2}\left\|\tilde{x}^{(k+1)} - \tilde{x}^{(k)}\right\|^2 \le \tilde{f}(\tilde{x}^{(k+1)} - \tilde{x}^{(*)} = \tilde{f}(\tilde{x}^{(k+1)})$.

Similarly $\frac{\lambda_{max}}{2}\left\|\tilde{x}^{(k)} - \tilde{x}^{(*)}\right\|^2 \ge \tilde{f}(\tilde{x}^{(k)} - \tilde{x}^{(*)} = \tilde{f}(\tilde{x}^{(k)})$.

Consequently, we get $\frac{\tilde{\lambda}_{min}}{2}\left\|\tilde{x}^{(k+1)} - \tilde{x}^{(k)}\right\|^2 \le \tilde{f}(\tilde{x}^{(k+1)} = (1-\widetilde{\gamma_k})\tilde{f}\left(\tilde{x}^{(k)}\right) \le \frac{\tilde{\lambda}_{max} - \tilde{\lambda}_{min}}{2}\left\|\tilde{x}^{(k)} - \tilde{x}^{(*)}\right\|^2$.

This gives $\frac{\left\|\tilde{x}^{(k+1)} - \tilde{x}^{(*)}\right\|}{\left\|\tilde{x}^{(k)} - \tilde{x}^{(*)}\right\|} \le \sqrt{\frac{\tilde{\lambda}_{max} - \tilde{\lambda}_{min}}{\lambda_{max}}}$.

As $\sqrt{\frac{\tilde{\lambda}_{max} - \tilde{\lambda}_{min}}{\tilde{\lambda}_{max}}} > 0$, *it* implies that the convergence of $\tilde{x}^{(k)}$ to $\tilde{x}^{(*)}$ *is* linear.

3.3. Algorithm for neutrosophic fuzzy steepest descent method

Algorithm 3.3.

Step 1: Consider the unconstrained optimization problem with intuitionistic fuzzy triangular coefficient $\tilde{g}^I\left({\tilde{x}^{(k)}}^N\right)$.

Step 2: Input $\tilde{x}^{(0)^N}, \in$ let $k \leftarrow 0$

Step 3: Calculate $\tilde{S}_i^{\ N}$ at $\tilde{X}_i^{\ N}$ by $\tilde{S}_i^{\ N} = \nabla \tilde{g}_i^{\ N}$ where $\tilde{g}^N = \tilde{Q}^N \tilde{x}^N$

Step 4: (i) Calculate $\tilde{\lambda}_i^{\ N}$ by using $\tilde{\lambda}_i^{\ N} = \frac{\langle \tilde{g}^{i^N}, \tilde{g}^{i^N} \rangle}{\langle \tilde{Q}^N \tilde{g}^{i^N}, \tilde{g}^{i^N} \rangle}$ where $\tilde{Q}^{\ N} \tilde{g}^{i^N}, = \nabla \tilde{g}^{i^N}$.(or)

(ii) Find hessian for $\tilde{g}^{i^N}(x)$ then Calculate $\tilde{\lambda}_i^{\ N}$ by using $\tilde{\lambda}_i^{\ N} = \frac{\tilde{S}_i^{\ N} (\tilde{S}_i^{\ N})^T}{\tilde{S}_i^{\ N} \tilde{H}_i^{\ N} (\tilde{S}_i^{\ N})^T}$

where $\tilde{Q}^{\ N} \tilde{g}^{i^N} = \nabla \tilde{g}^{i^N}$.

Step 5: Find the $(\tilde{x}^{(k+1)})^N = (\tilde{x}^{(k)})^N - (\tilde{\lambda}_i \tilde{g}^k)^N$ then $k \leftarrow k+1$

Step 6: Repeat the process until $\left\| (\tilde{x}^{(k)})^N - (\tilde{x}^{(k+1)})^N \right\| < \in$. Then stop the process or go to step 3.

Step 7: Check the optimum.

3.4. Neutrosophic fuzzy fletcher and reeves method (neutrosophic fuzzy conjugate gradient method)

In Steepest Descent method, the search direction Step 5: Find the $(\tilde{d}^{(k)}) = -\tilde{g}^k)$ then gives rise to iterates $\{\tilde{x}^{(k)}\}$which converge to the minimizer $\tilde{x}^*$ in a zig zag way. So, there is need to generate new search direction $\tilde{d}^{(k)}$ which well make the iterates $\{\tilde{x}^{(k)}\}$ converge faster to $\tilde{x}^*$. We take a quadratic function $\tilde{f}(x) = \frac{1}{2}\tilde{x}^T Q \tilde{x} = \frac{1}{2}\langle Q\tilde{x}, \tilde{x}\rangle$ with Q positive definite. We shall generate $\tilde{d}^{(k)}$ as mutually conjugate direction with respect to Q $\langle Q\tilde{d}^j, \tilde{d}^j \rangle = 0, i \neq j$.

The procedure for conjugate direction generation is

$\tilde{d}^0 = -\tilde{g}^0 = -\nabla(\tilde{f}\ (\tilde{x}^{(0)}) = -Q(\tilde{x}^{(0)})$ with $(\tilde{x}^{(0)})$ being initial guess.

Then $\tilde{x}^{(k+1)} = \tilde{x}^k - \tilde{\alpha}^k(\tilde{d}^{(k)})$, we get $\tilde{\alpha}^k = -\frac{\langle \tilde{d}^k, \tilde{g}^k \rangle}{\langle \tilde{Q}\,\tilde{d}^k, \tilde{d}^k \rangle}$.

This α^k minimizes the function $\tilde{\phi}(\alpha) = \tilde{f}(\tilde{x} - \tilde{\alpha}\tilde{d}^k)$

And hence $\tilde{g}^{k+1}$ is orthogonal to $\tilde{d}^k$, for $\tilde{\alpha}$ that minimizes the equation is given by $\tilde{\phi}'(\alpha) = \langle \nabla \tilde{f}(\tilde{x} - \tilde{\alpha}\tilde{d}^k), \tilde{d}^k \rangle = 0$ which is same as $\langle \tilde{g}^{k+1}, \tilde{d}^k \rangle = 0$. The next conjugate direction $\tilde{d}^{k+1}$ is given by $\tilde{d}^{k+1} = -\tilde{g}^{k+1} + \tilde{\beta}^k \tilde{d}^k$ where $\tilde{\beta}^k$ is so chosen that $\tilde{d}^k$ is conjugate to $\tilde{d}^{k+1}$ with respect to $\tilde{Q}$. This gives $\tilde{\beta}^k = \frac{\langle \tilde{g}^{k+1}, \tilde{Q}\,\tilde{d}^k \rangle}{\langle \tilde{d}^k, \tilde{Q}\,\tilde{d}^k \rangle}$.

To evaluate $\langle \tilde{d}^k, \tilde{g}^k \rangle$

$\langle \tilde{d}^k, \tilde{g}^k \rangle = -\langle \tilde{g}^k, \tilde{g}^k \rangle + \tilde{\beta}^{k-1} \langle \tilde{d}^{k-1}, \tilde{g}^k \rangle = -\langle \tilde{g}^k, \tilde{g}^k \rangle$

As $\tilde{g}^k \perp \tilde{d}^{k-1}$ and hence $\tilde{\alpha}^k = -\frac{\langle \tilde{g}^k, \ \tilde{g}^k \rangle}{\langle \tilde{Q}\,\tilde{d}^k, \tilde{d}^k \rangle}$.

Theorem 3.7. The Neutrosophic fuzzy Gradient Vector $\{\tilde{g}^k\}$ are mutually orthogonal and the direction search fuzzy vectors $\{\tilde{d}^k\}$ are mutually fuzzy Q-Conjugate.

Proof: The Result is true for k = 1, since $\tilde{d}^{(0)}$ and $\tilde{d}^{(1)}$ are Q-conjugate by the choice of $\beta^{(0)}$. Also $\tilde{g}^0 = -\tilde{d}^0$
Is orthogonal to $\tilde{g}^1$. Then prove by induction. In case $\tilde{d}^{(0)}, \tilde{d}^{(1)}, \ldots, \tilde{d}^{(k-1)}$ are mutually Q-conjugate and $\tilde{g}^{(0)}, \tilde{g}^{(1)}, \ldots, \tilde{g}^{(k-1)}$ are mutually orthogonal for some $k \geq 2$.
For $k = k-1$, then $\tilde{g}^{(k)} = \tilde{g}^{(k-1)} + \tilde{\alpha}^{(k-1)} Q\tilde{d}^{(k-1)}$. And hence

$$\langle \tilde{g}^{(k)}, \tilde{g}^{(i)} \rangle = \langle \tilde{g}^{(k-1)}, \tilde{g}^{(i)} \rangle + \langle \tilde{\alpha}^{(k-1)} \rangle \langle Q\tilde{g}^{(k)}, \tilde{g}^{(i)} \rangle = \tilde{\alpha}^{(k-1)} \langle Q\tilde{g}^{(k)}, \tilde{g}^{(i)} \rangle = 0,$$
$$0 \leq i \leq k-2$$

As $\tilde{d}^{(0)}, \tilde{d}^{(1)}, \ldots, \tilde{d}^{(k-1)}$ are Q-conjugate. Taking inner product with $Q\tilde{d}^{(i)}$, then $\langle \tilde{d}^{(k)}, Q\tilde{d}^{(i)} \rangle = -\langle \tilde{g}^{(k)}, Q\tilde{d}^{(k)} \rangle + \langle \tilde{\beta}^{(k-1)} \rangle \langle \tilde{d}^{(k-1)}, Q\tilde{d}^{(k)} \rangle = -\langle \tilde{g}^{(i)}, Q\tilde{g}^{(i)} \rangle = 0, 0 \leq i \leq k-2$.

Then, $Q\tilde{d}^{(i)} = \frac{\tilde{g}^{(i+1)} - \tilde{g}^{(i)}}{\tilde{\alpha}^{(i)}}$.

And hence we have $\langle \tilde{d}^{(k)}, Q\tilde{d}^{(i)} \rangle = \frac{\langle \tilde{g}^{(k)}, \tilde{g}^{(i+1)} - \tilde{g}^{(i)} \rangle}{\tilde{\alpha}^{(i)}}$.

Taking inner product with $\tilde{g}^{(k)}$, then $\langle \tilde{g}^{(k)}, \tilde{g}^{(k-1)} \rangle = \langle \tilde{g}^{(k)}, \tilde{d}^{(k-1)} \rangle - \langle \tilde{\beta}^{(k-2)} \rangle \langle \tilde{g}^{(k)}, \tilde{d}^{(k-2)} \rangle = \langle \tilde{\beta}^{(k-2)} \rangle \langle \tilde{g}^{(k)}, \tilde{d}^{(k-2)} \rangle$.
$\langle \tilde{g}^{(k)}, \tilde{g}^{(k-1)} \rangle = \langle \tilde{\beta}^{(k-2)} \rangle \langle \tilde{g}^{(k)}, \tilde{d}^{(k-2)} \rangle + \tilde{\alpha}^{(k-1)} \langle Q\tilde{d}^{(k-1)}, \tilde{d}^{(k-2)} \rangle = \tilde{0}$.
As $\tilde{g}^{(k-1)}$ is orthogonal to $\tilde{d}^{(k-2)}$, and $\tilde{d}^{(k-1)}$ and $\tilde{d}^{(k-2)}$ are Q-Conjugate Also, $\tilde{d}^k$ is defoned from $\tilde{d}^{(k-1)}$ in such a way that $\langle \tilde{d}^{(k)}, Q\tilde{d}^{(k)} \rangle = 0$. Combining the result we have $\langle \tilde{g}^{(k)}, \tilde{g}^{(i)} \rangle = 0 = \langle \tilde{d}^{(k)}, Q\tilde{d}^{(k)} \rangle, 0 \leq i \leq k-1$.

3.5. Algorithm for neutrosophic fuzzy steepest descent method

Algorithm 3.4.

Step 1: Consider the unconstrained optimization problem with intuitionistic fuzzy triangular coefficient $\tilde{g}^N\left(\tilde{x}^{(k)^N}\right)$.

Step 2: Input $\tilde{x}^{(0)^N}, \in$ let $k \leftarrow 0$

Step 3: Calculate $\tilde{S}_i^N$ at $\tilde{X}_i^N$ by $\tilde{S}_i^N = \nabla\tilde{g}_i^N$ where $\tilde{g}^N = \tilde{Q}^N\tilde{x}^N$

Step 4: (i) Calculate $\tilde{\lambda}_i^N$ by using $\tilde{\lambda}_i^N = \frac{\langle \tilde{g}^{k+1}, \tilde{g}^{k+1} \rangle}{\langle \tilde{g}^k, \tilde{g}^k \rangle}$(or)

(ii) Calculate Hessian matrix A from the given $\tilde{g}_i^N$, then $\tilde{\lambda}_i^N = \frac{\nabla\tilde{g}_i^{N^T}\nabla\tilde{g}_i^N}{\tilde{s}_i^N A\tilde{s}_i^{N^T}}$

Step 5: Find the $\tilde{d}^{(k+1)^N} = \tilde{g}^{(k+1)^N} + \tilde{\lambda}_i^N\tilde{d}^k$ then $k \leftarrow k+1$.

Step 6: Repeat the process until $\left\|(\tilde{x}^{(k)})^N - (\tilde{x}^{(k+1)})^N\right\| < \in$. Then stop the process or go to step 3.

Step 7: Check the optimum $\tilde{x}^{(*)^N} \leftarrow \tilde{x}^{(k)^N}$

4. Numerical illustrations

Some numerical examples are provided here to check the robustness of the proposed algorithms.

Example 1:

Case (i)

Let us consider the unconstrained optimization problem with Neutrosophic triangular fuzzy coefficients.,

$\tilde{\mathcal{T}}(x,y) = (0.5,1,1.5); (0.68,0.51,0.55)x - (0.5,1,1.5); (0.67,0.54,0.54)y + (1.5,2,2.5); (0.69,0.51,0.52)xy + (1.5,2,2.5); (0.7,0.65,0.72)x^2 + (0.5,1,1.5); (0.67,0.5,0.51)y^2.$

Solving this problem by the algorithm proposed in section 3.4,3.8, the MATLAB outputs are tabulated here.

Iteration	(x_i, y_i)	(x_{i+1}, y_{i+1}) **(Algorithm 3.3)**	(x_{i+1}, y_{i+1}) **(Algorithm 3.3)**
1	(0,0,0); (0.65,0.51,0.55) (0,0,0); (0.65,0.51,0.55)	$\begin{pmatrix}(-1.5,-1,-0.5);\\(0.67,0.54,0.55)\\(0.5,1,1.5);\\(0.67,0.54,0.55)\end{pmatrix}$	$\begin{pmatrix}(-1.5,-1,-0.5);\\(0.67,0.54,0.55)\\(0.5,1,1.5);\\(0.67,0.54,0.55)\end{pmatrix}$
2.	$\begin{pmatrix}(-1.5,-1,-0.5);\\(0.67,0.54,0.55)\\(0.5,1,1.5);\\(0.67,0.54,0.55)\end{pmatrix}$	$\begin{pmatrix}(-01.2,-0.8,-0.4);\\(0.67,0.54,0.55)\\(0.6,1.2,1.8);\\(0.67,0.54,0.55)\end{pmatrix}$	$\begin{pmatrix}(-01.2,-0.8,-0.4);\\(0.67,0.54,0.55)\\(0.6,1.2,1.8);\\(0.67,0.54,0.55)\end{pmatrix}$
3	$\begin{pmatrix}(-01.2,-0.8,-0.4);\\(0.67,0.54,0.55)\\(0.6,1.2,1.8);\\(0.67,0.54,0.55)\end{pmatrix}$	$\begin{pmatrix}(-1.5,-1,-0.5);\\(0.67,0.54,0.55)\\(0.7,1.4,2.1);\\(0.67,0.54,0.55)\end{pmatrix}$	$\begin{pmatrix}(-1.5,-1,-0.5);\\(0.67,0.54,0.55)\\(0.7,1.4,2.1);\\(0.67,0.54,0.55)\end{pmatrix}$
4	$\begin{pmatrix}(-1.5,-1,-1.4);\\(0.67,0.54,0.55)\\(0.7,1.4,2.1);\\(0.67,0.54,0.55)\end{pmatrix}$	$\begin{pmatrix}(-1.44,-0.96,-0.48);\\(0.67,0.54,0.55)\\(0.72,1.44,2.16);\\(0.67,0.54,0.55)\end{pmatrix}$	$\begin{pmatrix}(-1.44,-0.96,-0.48);\\(0.67,0.54,0.55)\\(0.72,1.44,2.16);\\(0.67,0.54,0.55)\end{pmatrix}$
5	$\begin{pmatrix}(-1.44,-0.96,-0.48);\\(0.67,0.54,0.55)\\(0.72,1.44,2.16);\\(0.67,0.54,0.55)\end{pmatrix}$	$\begin{pmatrix}(-1.5,-1,-0.5);\\(0.67,0.54,0.55)\\(0.74,1.48,2.22);\\(0.67,0.54,0.55)\end{pmatrix}$	$\begin{pmatrix}(-1.488,-0.992,-0.696)\\(0.67,0.54,0.55)\\(1.144,1.488,2.232);\\(0.67,0.54,0.55)\end{pmatrix}$
6	$\begin{pmatrix}(-1.488,-0.992,-0.696); (0.67,0.54,0.55)\\(1.144,1.488,2.232);\\(0.67,0.54,0.55)\end{pmatrix}$	$\begin{pmatrix}(-1.488,-0.992,-0.696); (0.67,0.54,0.55)\\(1.144,1.488,2.232);\\(0.67,0.54,0.55)\end{pmatrix}$	$\begin{pmatrix}(-1.461,-1.0001,-0.697);\\(0.67,0.54,0.55)\\(0.748,1.496,2.244);\\(0.67,0.54,0.55)\end{pmatrix}$

Iteration	(x_i, y_i)	(x_{i+1}, y_{i+1}) **(Algorithm 3.4)**		(x_{i+1}, y_{i+1}) **(Algorithm 3.4)**
1	(0,0,0); (0.65,0.51,0.55) (0,0,0);	$\begin{pmatrix}(-1.5,-1,-0.5);\\(0.67,0.54,0.55)\\(0.5,1,1.5);\\(0.67,0.54,0.55)\end{pmatrix}$		$\begin{pmatrix}(-1.5,-1,-0.5);\\(0.67,0.54,0.55)\\(0.5,1,1.5);\\(0.67,0.54,0.55)\end{pmatrix}$

	(0.65,0.51,0.55)			
2.	$\begin{pmatrix}(-1.5,-1,-0.5);\\(0.67,0.54,0.55)\\(0.5,1,1.5);\\(0.67,0.54,0.55)\end{pmatrix}$	$\begin{pmatrix}(-1.5,-1,-0.5);\\(0.67,0.54,0.55)\\(0.75,1.5,2.25);\\(0.67,0.54,0.55))\end{pmatrix}$		$\begin{pmatrix}(-1.5,-1,-0.5);\\(0.67,0.54,0.55)\\(0.75,1.5,2.25);\\(0.67,0.54,0.55)\end{pmatrix}$

Example 2:

Case (i)

Let us consider the unconstrained optimization problem with Neutrosophic triangular fuzzy coefficients.,

$\tilde{\mathcal{T}}(x,y) = (0.5,1.,1.5); (0.8,0.75,0.6)x^2 + (0.5,1.0,1.5); (0.7,0.65,0.75)y^2 + (1,2,3); (0.74,0.75,0.68)xy$

Solving this problem by the algorithm proposed in section 3.4 and 3.8, the MATLAB outputs are tabulated here.

Iteration	(x_i, y_i)	(x_{i+1}, y_{i+1}) **(Algorithm 3.3)**	(x_{i+1}, y_{i+1}) **(Algorithm 3.3)**
1	(-10.5,-10,-9.5); (0.7,0.75,0.8) (9.5,10,10.5);(0.7,0.75,0.8)	$\begin{pmatrix}(6.53,6.87,7.21);\\(0.7,0.75,0.8)\\(-0.65,-0.62,-0.59);\\(0.7,0.75,0.8)\end{pmatrix}$	$\begin{pmatrix}(6.53,6.87,7.21);\\(0.7,0.75,0.8)\\(-0.65,-0.62,-0.59);\\(0.7,0.75,0.8)\end{pmatrix}$
2.	$\begin{pmatrix}(6.53,6.87,7.21)\\(-0.65,-0.62,-0.59);\\(0.7,0.75,0.8)\end{pmatrix}$	$\begin{pmatrix}(2.07,2.18,2.29)\\(-2.29,-2.1,-2.07);\\(0.7,0.75,0.8)\end{pmatrix}$	$\begin{pmatrix}(2.07,2.18,\\2.29);\\(0.7,0.75,0.8)\\(-2.29,-2.18,\\-2.078);\\(0.7,0.75,0.8)\end{pmatrix}$
3	$\begin{pmatrix}(2.07,2.18,2.29);\\(0.7,0.75,0.8)\\(-2.29,-2.18,-2.07);\\(0.7,0.75,0.8)\end{pmatrix}$	$\begin{pmatrix}(1.42,1.50,1.57);\\(0.7,0.75,0.8)\\(-0.14,-0.13,\\-0.12);\\(0.7,0.75,0.8)\end{pmatrix}$	$\begin{pmatrix}(1.42,1.50,1.57);\\(0.7,0.75,0.8)\\(-0.14,-0.13,\\-0.12);\\(0.7,0.75,0.8)\end{pmatrix}$
4	$\begin{pmatrix}(1.42,1.50,1.57);\\(0.7,0.75,0.8)\\(-0.14,-0.13,\\-0.12);\\(0.7,0.75,0.8)\end{pmatrix}$	…	…
…	…	…	…

Iteration	(x_i, y_i)	(x_{i+1}, y_{i+1}) (Algorithm 3.4)	(x_{i+1}, y_{i+1}) (Algorithm 3.4)
1	(-10.5,-10,-9.5);(0.7,0.75,0.8) (9.5,10,10.5);(0.7,0.75,0.8)	$\begin{pmatrix} (6.53,6.87,7.21); \\ (0.7,0.75,0.8) \\ (-0.65,-0.62,-0.59); \\ (0.7,0.75,0.8) \end{pmatrix}$	$\begin{pmatrix} (6.53,6.87,7.21); \\ (0.7,0.75,0.8) \\ (-0.65,-0.62,-0.59); \\ (0.7,0.75,0.8) \end{pmatrix}$
2.	$\begin{pmatrix} (6.53,6.87,7.21); \\ (0.7,0.75,0.8) \\ (-0.65,-0.62,-0.59); \\ (0.7,0.75,0.8) \end{pmatrix}$	$\begin{pmatrix} (0.00,0.00075,0.0001); \\ (0.7,0.75,0.8) \\ (-0.000,-0.000,0.00); \\ (0.7,0.75,0.8) \end{pmatrix}$	$\begin{pmatrix} (0.00,0.00075,0.0001); \\ (0.7,0.75,0.8) \\ (-0.000,-0.000,0.00); \\ (0.7,0.75,0.8) \end{pmatrix}$

4. Conclusion

In this paper, a new strategy for solving fuzzy unconstrained optimization issues was proposed. In addition, triangular fuzzy number coefficients, as well as triangular intuitionistic fuzzy number coefficients, are used. For tackling fuzzy unconstrained optimization problems, Steepest Descent method and Conjugate Gradient method approach is employed, and the validity of the proposed methods is tested using numerical examples and MATLAB programme outputs. In addition, we conducted a comparison study of fuzzy, and intuitionistic fuzzy Steepest Descent method and fuzzy Fletcher Reeves Method (Conjugate Gradient method) with unconstrained optimization problems and found that our suggested method converges quickly.

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