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A Study of an EOQ Model under Cloudy Fuzzy Environment

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Abstract. This article deals with an economic order quantity (EOQ) model under non-random uncertain demand. Firstly, we have solved the crisp model and then the model is converted into a fuzzy environment. For a comparative study, we have considered the demand rate as a cloudy fuzzy number. The numerical result is obtained by LINGO 16.0 software. Finally, sensitivity analysis and graphical illustration have been given for better justification of the model.

Keywords: Cloudy fuzzy number; Backorder Inventory; Defuzzification; Optimization.

AMS Mathematics Subject Classification (2010): 90B05

1. Introduction

In 1965, Zadeh [8] introduced us to a precarious world. He claimed that except for some global truth, there is uncertainly behind every fact. He organised this uncertainly by using the membership function of each element in a set. This type of set is called fuzzy set. After that, Bellmann and Zadeh [14] used the fuzzy set perception in operations research decision-making. After that, Atanassov [7] define intuitionistic fuzzy set, which contains not only membership value but also non-membership value of each element in a set. Baez-Sanchez et al. [3] developed a mathematical formalization to define polygonal fuzzy numbers with an extension of fuzzy sets. De and Beg [28] introduced the concept of the dense fuzzy set by incorporating learning experiences in the fuzzy set. They considered each member of a fuzzy set that converge to a crisp number n tends to infinity. They also define a new defuzzification formulae of triangular dense fuzzy set. Also De and Beg [27] described the dense fuzzy Neutrosophic set. De and Mahanta [26] introduced the concept of cloudy fuzzy number, which converge to a crisp number after infinite time's. They used this concept to optimize a backorder economic order quantity (EOQ) model with uncertain demand rate. We know every decision–maker maker used some planning to optimize the inventory profit keeping this in mind, De [33] introduced the dense fuzzy lock set concept, which is a lot more helpful to manage the inventory control problems for the decision-maker. Faritha and Priya [2] defined parabolic fuzzy number for optimizing a backorder EOQ model using nearest interval approximation. Then Garg and Ansha [5] developed some arithmetic operation over generalized parabolic fuzzy numbers with its applications.

Maity et al. [18] introduced the concept of non-linear heptagonal dense fuzzy set in inventory problems with an application. Maity et al. [22] also defined cloud type non-linear intuitionistic dense fuzzy set with symmetry and asymmetry cases. De [32] gave a new concept over the degree of fuzziness and its application in decision-making problems. From the above literature review of fuzzy numbers, we observed that every researcher considered one type of fuzzy number with an application. None of them didn't give a comparative study to examine which fuzzy number is more profitable for inventory management. This model provided a comparative study among dense fuzzy, cloudy fuzzy, parabolic fuzzy and degree of fuzziness over the inventory profit function.

Harris [4]first developed the classical inventory model. After that, several researchers produced many research articles on inventory problems. All of them considered the demand rate as a deterministic one. First time Karlin[15] introduced fuzziness in one stage inventory problems. Nowadays modern researchers are showing their interest in solving inventory problems in fuzzy environments. De [31] developed an EOQ model under daytime non-random uncertain demand. De et al.[12] solved an EOQ model using a step order fuzzy approach. He considered daytime uncertain demand where demand in the backorder period depends on time. De et al. [30] studied an EOQ model where demand depends on selling price and informational efforts. The model has been optimized using the intuitionistic fuzzy technique. Kazemi er al. [11] solved an EOQ model fordefective quality items by considering the learning effects on some fuzzy parameters. In 2016, De and Sana [29] developed an economic production quantity (EPQ) model with variable lead time and stochastic demand. They considered all parameters the intuitionistic fuzzy number and used the intuitionistic fuzzy aggregation Bonferroni mean for defuzzified the model. Karmakar et al. [16, 17]considered an EPQ model and used the dense fuzzy lock set concept to reduce pollution by reusing waste items of sponge iron industry. Maity et al. [21] developed an EOQ model with dense fuzzy demand rate where two decision –makers make a single decision to optimize the model. Maity et al. [23] studied an EOQ model under daytime uncertain demand rate. They made a computer- based algorithm and flowchart to optimize the model by updating key vectors automatically. De and Mahata [24] considered an EOQ model to imperfect-quality items with considerable discounts. They optimized the model using cloudy fuzzy numbers. Laing and Wang [13] presented an integrated decision support model to buy their desired products online for the customers. Traditionally, De and Mahata [25] developed an EOQ model of defective quality items where the retailer first screened the items. Then the defective items are sold separately at a discount. Several researchers (Khan et al. [9], Lin and Chain [34], Naimi and Tahavori [10], Sofiana and Rosvidi [1], Wangsa and Wee[6] and Zhou et al. [35]) studied over imperfect quality items. Maity et al. [19] developed an EOQ model of growing items under parabolic dense fuzzy lock demand rate. Maity et al. [20] also presented an EOQ model of imperfect items under cloudy fuzzy environment. In this article, we have considered an EOQ model with cloudy fuzzy demand rate. Here, firstly we have solved the crisp model and then solve the model in cloudy fuzzy environment. Sensitivity analysis and graphical illustration have been made for better justification of the model. The numerical result has been taken by Lingo 16.0 software.

2. Preliminaries

Definition 1. Fuzzy set. If X= $\{x\}$ is a collection of objects denoted generically by x, then a fuzzy set $\tilde{A}$ in X is a set of order pairs

$$\tilde{A} = \left\{\left(x, \mu_{\tilde{A}}(x)\right) : x \in X\right\}$$

where $\mu_{\tilde{A}}(x)$ is called the "membership value" of x in $\tilde{A}$ in each pair $\left(x, \mu_{\tilde{A}}(x)\right)$ is called a singleton.

Definition 2. α- cut set of fuzzy set A. A fuzzy set A is defined on X. The α-cut set $^{\alpha}A$ is a set whose membership grade is greater equals to α. Here it is defined by
$^{\alpha}A$= $\{x \in X, A(X) \geq \alpha\}$,
where α *lies between* $[0,1]$.

The α-cut set of a fuzzy set A is the crisp set. Theα- cut set is also called the α- level set.

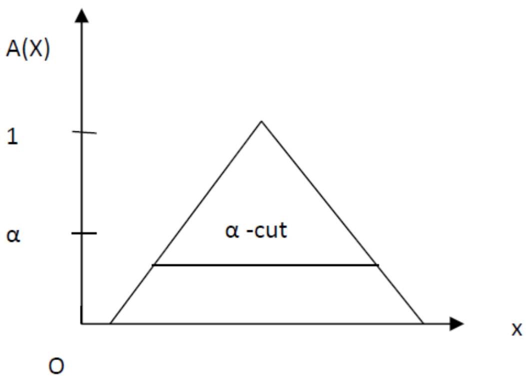

Figure 1. An α- cut of a triangular fuzzy number

Definition 3. Fuzzy number. A fuzzy number A is a fuzzy set of the real line with a normal, (fuzzy) convex and continuous membership function of bounded support. The family of fuzzy numbers will be denoted by F.

Definition 4. Triangular fuzzy number. A fuzzy set A is called triangular fuzzy number with peak (or centre) a, left width $\alpha > 0$ and right width $\beta > 0$ if its membership function has the following form

$$A(X) = \begin{cases} 1 - \dfrac{(a - x)}{\alpha} & \text{if } a - \alpha \leq x \leq a \\ 1 - \dfrac{x - a}{\beta} & \text{if } a \leq x \leq a + \beta \\ 0 & \text{otherwise} \end{cases}$$

and we use the notation A= (a, α, β). It can easily be verified that

$$[a - (1 - \gamma)\alpha \; + (1 - \gamma)\beta], \forall \gamma \in [0,1]$$

The support of A is (a-α, b+β).

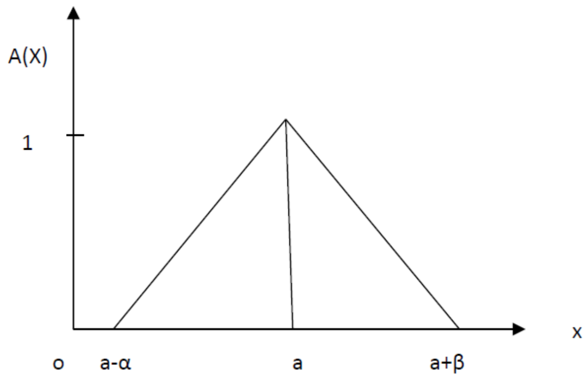

Figure 2: Triangular fuzzy number

A triangular fuzzy number with centre ‘a’ may be seen as a fuzzy quantity “x is approximately equal to a”.

Definition 5. Cloudy fuzzy number. let $\tilde{X} = \langle a_n, a_{n-1} \ldots\ldots a_1, a, a_1' \ldots a_{n-1}', a_n' \rangle$ be fuzzy number containing 2n+1 component.If the set itself converges to a crisp singleton set after an infinite time then the fuzzy number is called 2n+1 tuple cloudy fuzzy number**.**

Definition 6. Triangular cloudy fuzzy number. Let $\tilde{B}$ =$\langle b\left(1-\frac{\gamma}{1+t}\right), b, b\left(1+\frac{\delta}{1+t}\right)\rangle$ be a triangular fuzzy number. When$t \to \infty$ the fuzzy number converges to a crisp singletone set. So $\tilde{B}$ is called cloudy triangular fuzzy number. Then its membership function is given by

$$\mu(y,t) = \begin{cases} \dfrac{y-b\left(1-\frac{\gamma}{1+t}\right)}{\frac{b\gamma}{1+t}} & for b\left(1-\frac{\gamma}{1+t}\right) \leq y \leq b \\ 1 & for\ y = b \\ \dfrac{b\left(1+\frac{\delta}{1+t}\right)-y}{\frac{b\delta}{1+t}} & for b \leq y \leq b\left(1+\frac{\delta}{1+t}\right) \\ 0 & for\ elsewhere \end{cases} \tag{1}$$

Defuzzification formula of cloudy fuzzy number. Letμ_L^α and μ_R^α be left and right α-cuts of μ(y, t) respectively obtained from (1). Then the defuzzification formulae is given by

$$I(\tilde{B}) = \frac{1}{2T}\int_{\alpha=0}^{1}\int_{t=0}^{T}\left({\mu_L}^\alpha + {\mu_R}^\alpha\right) d\alpha dT \tag{2}$$

3. Crisp mathematical model

3.1. Assumptions and notation

Notations

H_1 **:** Holding cost per quantity per unit time ($)
H_2: Shortage cost per unit quantity per unit time($)

H_3 : Set up cost per unit time period percycle ($)
D_2: Demand in shortage period ($D_2 = D_1 e^{-t_2}$)
S_1 : Inventory level within time (days)
S_2 : Shortage quantity during the time t_2(days)
t_1: Inventory run time(days)
t_2: Shortage time (days)
T : Cycle time ($= t_1 + t_2$)(days)
TAC: Total average cost ($)

Assumptions

The following assumptions are made to develop our proposed model.
1. Demand rate is uniform and known
2. Rate of replenishment is finite
3. Lead time is zero/negligible
4. Shortage are allowed and fully backlogged

3.2. Formulation of crisp mathematical model

Let the inventory starts at time $t = 0$ with order quantity S_1 and demand rate D. After time $t = t_1$ the inventory reaches zero level and the shortage starts and it continues up to time $T = t_1 + t_2$. Let S_2 be the shortage quantity during that time period t_2. Also, we assume that the shortage time demand rate is depending on the duration of shortage time t_2.Therefore, the mathematical problem associated to the proposed model is shown in the figure and the necessary calculations are given below:

$$\text{Inventory holding cost} = \frac{1}{2} H_1 S_1 t_1 \tag{3}$$

$$\text{Shortage cost} = 1/2 H_2 D_1 (1 - e^{-t_2}) t_2 \tag{4}$$

$$\text{Set up cost} = H_3 \tag{5}$$

$$\text{Order quantity } S_1 = D_1 t_1 \tag{6}$$

$$\text{and backorder quantity } S_2 = \int_0^{t_2} D_1 e^{-t_2} dt_2 \quad = D_1 (1 - e^{-t_2}) \tag{7}$$

$$T = t_1 + t_2 \tag{8}$$

Therefore, the average inventory cost is given by

$$Z = \frac{1}{2T} H_1 S_1 t_1 + \frac{1}{2T} H_2 S_2 t_2 + \frac{1}{T} H_3$$

$$= \frac{1}{2} H_1 D_1 \frac{{t_1}^2}{t_1 + t_2} + \frac{1}{2} H_2 D_1 \frac{(1 - e^{-t_2})}{t_1 + t_2} t_2 + \frac{H_3}{t_1 + t_2}$$

$$= D_1 \left\{ \frac{1}{2} H_1 \frac{{t_1}^2}{t_1 + t_2} + \frac{1}{2} H_2 \frac{(1 - e^{-t_2})}{t_1 + t_2} \right\} + \frac{H_3}{t_1 + t_2} \tag{9}$$

Therefore, our problem is redefined as: -

$$\text{Minimize } Z = D_1 f_1 + f_2 \tag{10}$$

$$\text{where } \begin{cases} f_1 = \frac{1}{2} H_1 \frac{{t_1}^2}{t_1 + t_2} + \frac{1}{2} H_2 \frac{(1 - e^{-t_2})}{t_1 + t_2} t_2 \\ f_2 = \frac{H_3}{t_1 + t_2} \end{cases} \tag{11}$$

Subject to the condition (6-8).

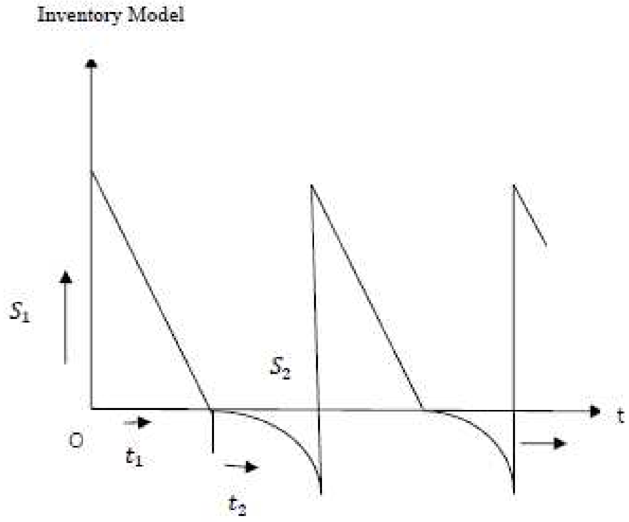

Figure 3: Logistic diagram of crisp mathematical model

4. Cloudy fuzzy model

Since, demand rate follows an important role in defining the objective function in an inventory process. So, we consider the demand rate assumes flexible values in the propose model and it can be reduce by means of dense fuzzy set.So, the objective function of the crisp model (10) can be written as

$$Z = D_1 f_1 + f_2 \tag{12}$$

where f_1 and f_2 are given by (11)

Now, (10) can be written as $d_1 = \frac{z-f_2}{f_1}$ and its fuzzy equivalent is given by$d_1 = \frac{z-f_2}{f_1}$ (13)

$$\mu(\tilde{d}) = \begin{cases} \frac{d-d_1\left(1-\frac{\gamma}{1+t}\right)}{\frac{d_1\gamma}{1+t}} for d_1\left(1-\frac{\gamma}{1+t}\right) \le d \le d_1 \\ \frac{d_1\left(1+\frac{\delta}{1+t}\right)-d}{\frac{d_1\delta}{1+t}} for d_1 \le d \le d_1\left(1+\frac{\delta}{1+t}\right) \\ 0 \quad elsewhere \end{cases} \tag{14}$$

Then the left and right α- cuts are $\left(\mu_L{}^{\alpha}, \mu_R{}^{\alpha}\right) = \left[d_1 - \frac{d_1\gamma}{1+t}(1-\alpha), d_1 + \frac{d_1\delta}{1+t}(1-\alpha)\right]$

Now, $I(\tilde{d}) = \frac{1}{2T}\int_{\alpha=0}^{1}\int_{t=0}^{T}\left(\mu_L{}^{\alpha} + \mu_R{}^{\alpha}\right) d\alpha dT$

$= \frac{1}{2T}\int_{\alpha=0}^{1}\int_{t=0}^{T}\left[2d_1 - \frac{d_1\gamma}{1+t}(1-\alpha) + \frac{d_1\delta}{1+t}(1-\alpha)\right] d\alpha dT$

$= \frac{1}{2T}\left[\int_{\alpha=0}^{1}\int_{t=0}^{T} 2d_1\, \mathrm{d\alpha dT} - \int_{\alpha=0}^{1}\int_{t=0}^{T} d_1 \frac{(\gamma-\delta)}{1+t} d\alpha dT + \int_{\alpha=0}^{1}\int_{t=0}^{T} 2d_1\, \alpha \frac{(\gamma-\delta)}{1+t} d\alpha dT\right]$

$= \left[d_1 - \frac{1}{4T} d_1(\gamma-\delta) log(1+T)\right]$ (15)

From (13) and (14) we obtain the membership function of the fuzzy objective function as follows:

$$\mu(\check{Z}) = \begin{cases} \dfrac{\frac{z-f_2}{f_1}-d_1\left(1-\frac{\gamma}{1+t}\right)}{\frac{d_1\gamma}{1+t}} for\ d_1\left(1-\frac{\gamma}{1+t}\right) \le \frac{z-f_2}{f_1} \le d_1 \\ \dfrac{d_1\left(1+\frac{\delta}{1+t}\right)-\frac{z-f_2}{f_1}}{\frac{d_1\delta}{1+t}} for\ d_1 \le \frac{z-f_2}{f_1} \le d_1\left(1+\frac{\delta}{1+t}\right) \\ \quad 0 \quad , \quad elsewhere \end{cases}$$

Then, the left α- cuts $\mu_L{}^{\alpha} = \frac{d_1\gamma f_1}{1+t}(\alpha-1)+d_1f_1+f_2$

and the right α-cuts $\mu_R{}^{\alpha} = d_1f_1+\frac{d_1\delta}{1+t}(1-\alpha)f_1+f_2$

Now the index value is:

$$I(\tilde{Z}) = \frac{1}{2T}\int_{\alpha=0}^{1}\int_{t=0}^{T}\left(\mu_L{}^{\alpha}+\mu_R{}^{\alpha}\right)d\alpha dT$$

$$= \frac{1}{2T}\int_{\alpha=0}^{1}\int_{t=0}^{T}\left[2d_1f_1+2f_2+\frac{d_1\gamma f_1}{1+t}(\alpha-1)+\frac{d_1\delta}{1+t}(1-\alpha)f_1\right]$$

$$= \frac{1}{2T}\left[\int_{\alpha=0}^{1}\int_{t=0}^{T}2d_1f_1d\alpha dT+\int_{\alpha=0}^{1}\int_{t=0}^{T}2f_2d\alpha dT+\int_{\alpha=0}^{1}\int_{t=0}^{T}\frac{d_1f_{1(\gamma-\delta)}\alpha}{1+t}d\alpha dT+\frac{d_1f_1}{1+t}(\delta-\gamma)d\alpha dT\right]$$

$$= d_1f_1+f_2-\frac{1}{4T}d_1f_1(\gamma-\delta)\log(1+T)$$

So, the fuzzy model becomes:

$$\begin{cases} Minimize I[\tilde{Z}] = d_1f_1+f_2-\frac{1}{4T}d_1f_1(\gamma-\delta)\log(1+T) \\ \qquad I[\tilde{d}] = d_1-\frac{d_1}{4T}(\gamma-\delta)\log(1+T) \\ \qquad\qquad Sub.tocondition\ (6-8) \end{cases} \tag{16}$$

4.1. Numerical result

4.1.1. Case study

We visited a Toy producing company situated at Kolkata, the capital of West Bengal, India last year. After long discussion with the manager (DM) it is found that the company has normal run time and a shortage run time in their total cycle time. During shortage period, the demand depletes exponentially with the duration of shortage period. We have collected some valuable data regarding our proposed model. The holding cost, shortage cost, set up cost, and usual demand of the items are $H_1 = \$2.4$,$H_2 = \$1.7$, $H_3 = \$1300$,$D_1 = 150$ units, respectively.

Table 1: Optimal solution of backorder EOQ model

Approach	Time(T_1^*)	Time(T_2^*)	S_1^*	S_2^*	Minimum cost(Z^*)
Crisp	3.656	1.656	548.400	121.3654	729.814
Cloudy Fuzzy	4.145	1.855	549.158	111.759	701.288

From Table 1, it is clear that in crisp case the average inventory cost is \$729.81for order quantity 548.40 unit and backorder quantity 121.36 unit where as for cloudy fuzzy model the average inventory cost is \$701.28 when order quantity is 549.16 units and backorder quantity is 111.75 units. So, from table 1 we can claim that the cloudy fuzzy model gives a finer optimum compared with the crisp model.

4.2. Sensitivity analysis

We have observed that cloudy fuzzy gives the finer optimum value. So, for sensitivity analysis we have changes the values of all parameters from -40% to +40% one-by-one and get the following results.

Table 2: Sensitivity analysis with changing parameters from (-40% to +40)

ra-meter	% Changes	Time (T_1^*)	Time (T_2^*)	S_1^*	S_2^*	Z^*	$\frac{Z^*-Z}{Z} \times 100$
H_1	+40	4.145000	1.855000	549.1578	111.7589	883.3885	25.97
	+20	4.145000	1.855000	549.1578	111.7589	792.3382	12.98331
	-20	4.145000	1.855000	549.1578	111.7589	610.2374	-12.983
	-40	4.145000	1.855000	549.1578	111.7589	519.1871	-25.97
H_2	+40	4.145000	1.855000	549.1578	111.7589	713.0355	1.6751
	+20	4.145000	1.855000	549.1578	111.7589	707.1617	0.8376
	-20	4.145000	1.855000	549.1578	111.7589	695.4139	-0.8376
	-40	4.145000	1.855000	549.1578	111.7589	689.5401	-1.6752
H_3	+40	4.145000	1.855000	549.1578	111.7589	787.9545	12.3582
	+20	4.145000	1.855000	549.1578	111.7589	744.6211	6.17910
	-20	4.145000	1.855000	549.1578	111.7589	657.9545	-6.17810
	-40	3.887866	1.597866	523.3681	107.3793	613.8803	-12.463
D_1	+40	4.145000	1.855000	768.8210	156.4624	895.1362	27.6418
	+20	4.145000	1.855000	658.9894	134.1106	798.2120	13.8289
	-20	4.145000	1.855000	439.3263	89.40709	604.3696	-13.8200
	-40	4.145000	1.855000	329.4947	67.05531	507.4393	-27.641
$\boldsymbol{\gamma}$	+40	4.145000	1.855000	302.35444	61.52997	483.4793	-31.058
	+20	4.145000	1.855000	425.7511	86.64442	592.3838	-15.529
	-20	3.559984	1.269984	565.8276	114.3047	795.1595	13.3856
	-40	3.211399	0.9213993	559.8604	104.9565	856.4954	22.1317
$\boldsymbol{\delta}$	+40	3.272486	0.9824865	560.1817	107.0933	843.5446	20.2850
	+20	3.625286	1.335286	567.8706	115.4319	786.4912	12.1496
	-20	4.145000	1.855000	440.2696	89.59906	605.1960	-13.7021
	-40	4.145000	1.855000	331.3813	67.43926	509.1042	-27.4044

5. Graphical illustrations

From Table 2 we have drawn the following graphs.

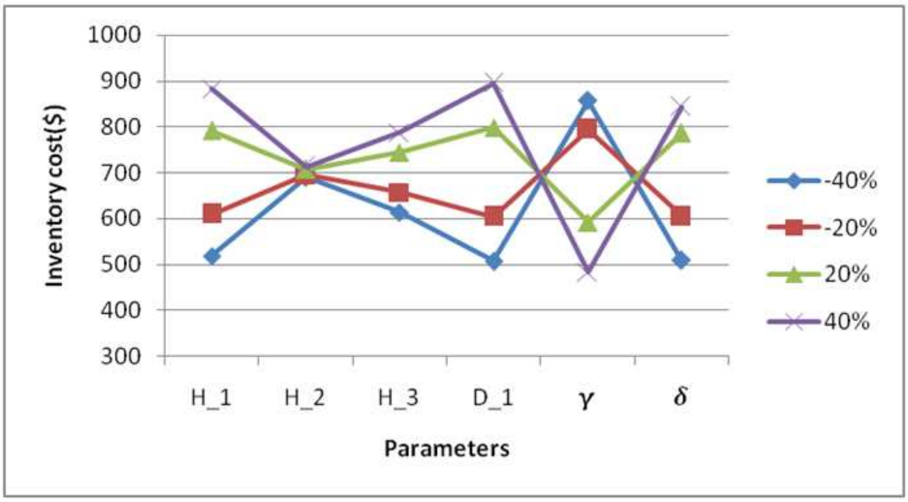

Figure 4: Inventory cost versus parametric change

From Figure 4, it is clear that the shortage cost parameter $H_2$is almost insensitive and setup cost parameter H_3 is low sensitive within the variation. But the holding cost parameter H_1, demand rate D_1, and fuzzy deviation parameters γ and δ are highly sensitive when the values changes from -40% to +40%.

Figure 5 indicates the surface like curve of the inventory cost function with respect to the variation of order quantity and backorder quantity. It is seen that, when order quantity assumes values about 302 units and backorder quantity assumes values nearly 61 units the average inventory cost gets minimum value. However, it is observe that when order quantity assumes values about 768 units and backorder quantity assumes values nearly 156 units the average inventory cost gets maximum value.

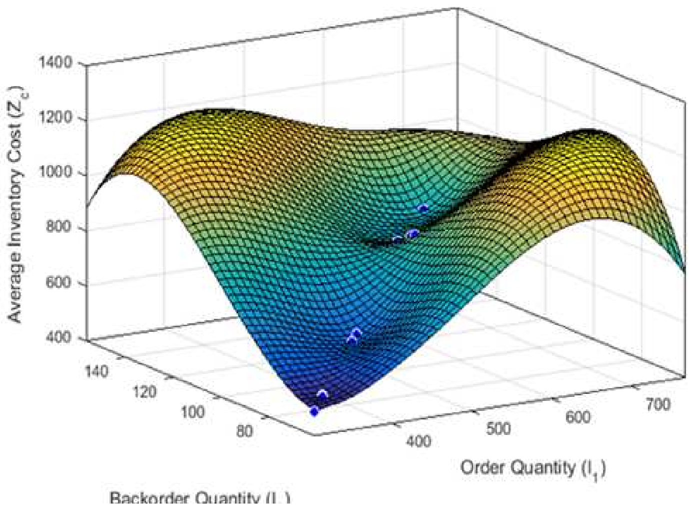

Figure 5: Average Inventory cost vs Order and Backorder quantity

6. Conclusion

Here, we have studied over a simple backorder EOQ model under cloudy fuzzy environment. For a comparative study we also consider demand rate as cloudy fuzzy number. The numerical result and graphical illustration claims that the model gives finer optimum for cloudy fuzzy demand rate. So, we may conclude that cloudy fuzzy is much more suitable for decision-makers to make decision compared with crisp environment.

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