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A Fuzzy Inventory System of Inflation Effect and Time-Related Demand

Mohammad Anwar Hossen

Department of Mathematics, Comilla University, Comilla, Bangladesh
Email: mdahossen@yahoo.com

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Abstract. In this research work, a fuzzy inventory model was developed for deteriorating items with time-dependent demand considering the inflation effect. Shortages if any are allowed and partially backlogged with a variable rate dependent on the duration of waiting time up to the arrival of the next lot. The corresponding problem has been formulated as a nonlinearly constrained optimization problem, all the cost parameters are fuzzy valued and solved. Numerical examples have been considered to illustrate the model and the significant features of the results are discussed. Finally, based on these examples, sensitivity analyses have been studied by taking one parameter at a time keeping the other parameters as same.

Keywords: Fuzzy inventory, fuzzy optimization, inflation, time-dependent demand

AMS Mathematics Subject Classification (2010): 90B05

1. Introduction

Inventory optimization deals with the decision to minimize the total average cost or maximize the total average profit. According to the existing literature of inventory control system, most of the inventory models have been developed under the assumption. In real life situation, this assumption is not always true due to the effect of deterioration in the preservation of commonly used physical goods like wheat, paddy or any other type of food grains, vegetables, fruits, drugs, pharmaceuticals, etc. As a result, the loss due to this natural phenomenon (i.e., the deterioration, I inflation, time effect) can't be ignored in the analysis of the inventory system. Ghare and Schrader [3] first developed an inventory model for exponentially decaying inventory. Then Emmons [4] proposed this type of model with variable deterioration which follows two-parameter Weibull distribution. But in reality, the nature of these parameters is uncertain, so it is important to consider them as fuzzy numbers. Considering the fuzzy set theory in theinventory model brings authenticity to the model since fuzziness is the closest possible approach to reality. The concept of fuzzy set theory modeling was developed by Zadeh [17]. Jain [8] deliberated a fuzzy inventory model on decision making in the presence of fuzzy variables. In a fuzzy environment, data of stock are not clear. In 2002 Kao and Hsu [9] proposed a single period inventory model with fuzzy demand. An inventory model with total demand and storing cost as triangular fuzzy numbers was developed by Yao and Chiang. They applied the defuzzification by centroid method and signed distance method both. Mahapatra and Maiti [14] presented a multi-item, multi-objective inventory model for deteriorating items with stock- and time-dependent demand rate over a finite time horizon in fuzzy stochastic environment. Halim et. al. [5] deliberated by an EOQ model for perishable items with stochastic demand and partial backlogging. A fuzzy inventory model without shortages by using triangular fuzzy number was discussed by De and Rawat [1]. Some recent work in this direction is done by Jaggi et al. [7], Hossen et al. [13], Saha and Chakrabarti [16], Dutta and Kumar [2] and Kumar and Rajput [10], Fathalizadeh et al. [15], Jayaswal et al. [11] etc. Stock maintain has directly related to deterioration specially for perishable items and short expiry period goods. In 2016, Sangal et al. [6] developed a fuzzy environment inventory model with partial backlogging under the learning effect. Mondal and et al. [12] proposed a production-repairing inventory model with fuzzy rough coefficients under inflation and the time value of money. In the fuzzy approach, the parameters, constraints and goals are considered as fuzzy sets with known membership functions. On the other hand, in the fuzzy-stochastic approach, some parameters are viewed as fuzzy sets and others, as random variables. However, it is not always easy for a decision maker to specify the appropriate membership function or probability distribution. To overcome these difficulties, we have applied GMIV (Graded Mean Integration value) technique to de-fuzzyfi the fuzzy numbers.

In this work, a fuzzy inventory model developed for deteriorating items with time dependent demand considering the inflation effect. and the corresponding problem has been formulated as a nonlinearly constrained optimization problem, all the cost parameters are fuzzy valued and solved. Also sensitivity analyses have been studied.

Assumptions:

The following assumptions and notations are used to develop the proposed model:

a) The entire lot is delivered in one batch.
b) Inflation effect of the system.
c) The demand rate $D(t)$ is dependent on time. It is denoted by

$$D(t) = a + bt,\ a, b > 0\ .$$

d) The deteriorated units ware neither repaired nor refunded.
e) The inventory system involves only one item and one stocking point and the inventory planning horizon is infinite.
f) Replenishments are instantaneous and lead time is constant.
g) The replenishment cost (ordering cost) is constant and transportation cost does not include for replenishing the item.
h) The inventory costs parameters are fuzzy valued.

Notations:

$I(t)$	Inventory level at time t	S	Highest stock level at the beginning of stock-in period
R	Highest shortage level	θ	Deterioration rate ($0 < \theta << 1$)
$\tilde{C}_o$	Fuzzy replenishment cost per order	δ	Backlogging parameter

$\tilde{C}_p$	Fuzzy purchasing cost per unit	p	Selling price per unit of item
$D(t)$	Time dependent demand	$\tilde{C}_h$	Fuzzy opportunity cost due to lost sale
$\tilde{C}_b$	Fuzzy shortage cost per unit per unit time	$\tilde{C}_{ls}$	Fuzzy opportunity cost due to lost sale
t_2	Time at which the stock level reaches to zero	T	Time at which the highest shortage level reaches to the lowest point
r	Inflation rate	$\tilde{Z}$	The total average cost

2. Inventory model with shortages

In this model, it is assumed that after fulfilling the backorder quantity, the on-hand inventory level is S at $t=0$ and it declines continuously up to the time $t = t_2$ when it reaches the zero level. The decline in inventory during the closed time interval $0 \le t \le t_2$ occurs due to the customer's demand and deterioration of the item. After the time $t = t_2$, shortage occurs and it accumulates at the rate $\left[1+\delta(T-t)\right]^{-1}$, ($\delta > 0$) up to the time $t = T$ when the next lot arrives. At time $t = T$, the maximum shortage level is R. This entire cycle then repeats itself after the cycle length T.

Let $I(t)$ be the instantaneous inventory level at any time $t \ge 0$. Then the inventory level $I(t)$ at any time t satisfies the differential equations as follows:

$$\frac{dI(t)}{dt} + \theta I(t) = -D(t), \qquad 0 \le t \le t_1 \tag{1}$$

$$\frac{dI(t)}{dt} = \frac{-D(t)}{[1+\delta(T-t)]}, \quad t_1 < t \le T \tag{2}$$

with the boundary conditions

$$I(t) = S \text{ at } t = 0 \text{ , } I(t) = 0 \text{ at } t = t_1 . \tag{3}$$

$$\text{and } I(t) = -R \text{ at } t = T . \tag{4}$$

Also, $I(t)$ is continuous at $t = t_1$.

The solution of the differential equations (1) given by

$$\frac{dI(t)}{dt} + \theta I(t) = -D(t), \qquad 0 \le t \le t_1$$

$$\Rightarrow e^{\theta t}\left\{\frac{dI(t)}{dt} + \theta I(t)\right\} = -(a+bt)e^{\theta t}$$

$$\Rightarrow I(t) = -\frac{a}{\theta} - b\left(\frac{t}{\theta} - \frac{1}{\theta^2}\right) + C_1 e^{-\theta t}$$

Using condition (3), $I(t)=0$ at $t = t_1$,Then

$$\Rightarrow C_1 = e^{\theta t_1}\left\{\frac{a}{\theta} + b\left(\frac{t_1}{\theta} - \frac{1}{\theta^2}\right)\right\}$$

$$\Rightarrow I(t) = \frac{b}{\theta^2} - \frac{a}{\theta} - \frac{bt}{\theta} + \left(\frac{a}{\theta} + \frac{bt_1}{\theta} - \frac{b}{\theta^2}\right)e^{\theta(t_1 - t)} \qquad 0 < t \le t_1$$

Again using condition (3), *I(t)=S* at $t = 0$,Then

$$S = \left(\frac{a}{\theta} - \frac{b}{\theta^2}\right)\left(e^{\theta t_1} - 1\right) + \frac{e^{\theta t_1} bt_1}{\theta} \tag{5}$$

The solution of the differential equations (2) given by

$$\frac{dI(t)}{dt} = \frac{-D(t)}{1 + \delta(T - t)}, \qquad t_1 < t \le T$$

$$\Rightarrow \frac{dI(t)}{dt} = -\frac{a}{1 + \delta(T - t)} - \frac{bt}{1 + \delta(T - t)}$$

$$\Rightarrow I(t) = \frac{a}{\delta}\log\left|1 + \delta(T - t)\right| + \frac{bt}{\delta} + \frac{b(1 + \delta T)}{\delta^2}.\log\left|1 + \delta(T - t)\right| + C_2$$

Using condition (4), *I(t)=-R*at $t = T$,Then

$$\Rightarrow -R = \frac{bT}{\delta} + C_2 \quad \Rightarrow C_2 = -R - \frac{bT}{\delta}$$

Then, $I(t) = \dfrac{\{a\delta + b(1 + \delta T)\}\log\left|1 + \delta(T - t)\right|}{\delta^2} + \dfrac{bt}{\delta} - \left(R + \dfrac{bT}{\delta}\right)$

Again using condition (3), *I(t)=0* at $t = t_1$,Then

$$\Rightarrow R = \frac{\{a\delta + b(1 + \delta T)\}\log\left|1 + \delta(T - t_1)\right|}{\delta^2} + \frac{bt_1}{\delta} - \frac{bT}{\delta} \tag{6}$$

Now the total inventory holding cost for the entire cycle is given by

$$\tilde{C}_{hol} = \tilde{C}_h \int_0^{t_1} e^{-rt} I(t)dt \tag{7}$$

$$= \tilde{C}_h \int_0^{t_1} e^{-rt}.\left\{\frac{b}{\theta^2} - \frac{a}{\theta} - \frac{bt}{\theta} + \left(\frac{a}{\theta} + \frac{bt_1}{\theta} - \frac{b}{\theta^2}\right)e^{\theta(t_1 - t)}\right\}dt$$

$$= \tilde{C}_h.\left\{\left(\frac{b}{\theta^2} - \frac{a}{\theta}\right)\left(1 - e^{-rt_1}\right) + \left(\frac{a}{\theta} + \frac{bt_1}{\theta} - \frac{b}{\theta^2}\right)\frac{e^{\theta t_1}}{r + \theta}\left(1 - e^{-(r+\theta)t_1}\right)\right\} +$$

$$\tilde{C}_h\left\{\frac{b}{\theta}\left(\frac{1}{r^2} - \frac{e^{-rt_1}}{r^2} - \frac{t_1 e^{-rt_1}}{r}\right)\right\}$$

Again, the total shortage cost C_{Sho} over the entire cycle is given by

$$\tilde{C}_{sho} = \tilde{C}_b \int_{t_1}^{T} \left\{-e^{-rt} I(t)\right\}dt \tag{8}$$

$$= \tilde{C}_b \left[\begin{array}{l} \frac{\{a\delta + b(1+\delta T)\}}{6\delta^2} \left\{ \text{explog} rT^2 (-3 + 2\delta T - 3\delta T) - \text{explog} rt_1^{\,2} (-3 + 2\delta t_1 - 3\delta\, t_1) \right\} + \\ \frac{b}{\delta} \left\{ \frac{Te^{-rT}}{r} + \frac{e^{-rT}}{r^2} - \frac{t_1 e^{-rt_1}}{r} - \frac{e^{-rt_1}}{r^2} \right\} + \\ \left(R + \frac{bT}{\delta} \right) \left(\frac{e^{-rt_1} - e^{-rT}}{r} \right) \end{array} \right]$$

Cost of lost sale OCLS over the entire cycle is given by

$$\tilde{O}CLS = \tilde{C}_{ls} \int_{t_2}^{T} e^{-rt} \left\{ 1 - \frac{1}{1+\delta(T-t)} \right\} D(t)dt \qquad (9)$$

$$= \tilde{C}_{ls} e^{-rT} \left[\begin{array}{l} a(T - t_2) + \frac{b}{2}\left(T^2 - t_2^{\,2}\right) - \frac{a}{\delta}\{\log|1 + \delta(T - t_2|\} + \\ \frac{b}{\delta}\left\{ (T - t_2) - \frac{1+\delta T}{\delta}.\log\left|1 + \delta(T - t_{2)}\right| \right\} \end{array} \right]$$

Total cost during the entire cycle is given by

<ordering cost> + *<purchasing cost>* + *<inventory holding cost>* + *< cost of lost sale>*+*< inventory shortage cost>*

i.e., $\tilde{X} = \tilde{C}_4 + \tilde{C}_p(S+R) + \tilde{C}_{hol} + \tilde{O}CLS + \tilde{C}_{sho}$

Average cost during the entire cycle is given by $\tilde{Z} = \dfrac{\tilde{X}}{T}$

Hence the corresponding constrained optimization problem is given by

3. Fuzzy numerical examples

For numerical illustration of the proposed inventory model, we have considered the following examples.

Example 1:

$\tilde{C}_o = (495, 500, 505)$, $\tilde{C}_h = (1, 1.5, 2)$, $\tilde{C}_p = (25, 30, 35)$, $\tilde{C}_b = (10, 15, 20)$, $a = 45$, $b = 5$,

$\tilde{C}_{ls} = (10, 15, 20)$, $\theta = 0.5$, $r = 0.06$, $\delta = 1.5$.

Example 2:

$\tilde{C}_o = (490, 495, 500)$, $\tilde{C}_h = (2, 2.5, 3)$, $\tilde{C}_p = (25, 30, 35)$, $\tilde{C}_b = (10, 12, 14)$, $a = 50$, $b = 5$,

$\tilde{C}_{ls} = (10, 15, 20)$, $\theta = 0.5$, $r = 0.06$, $\delta = 1.5$.

Example 3:

$\tilde{C}_o = (540, 550, 560)$, $\tilde{C}_h = (1, 1.5, 2)$, $\tilde{C}_p = (30, 35, 40)$, $\tilde{C}_b = (10, 15, 20)$, $a = 45$, $b = 5$,

$\tilde{C}_{ls} = (10, 15, 20)$, $\theta = 0.5$, $r = 0.06$, $\delta = 0.6$.

According to the solution procedure, the optimal solution has been obtained with the help of LINGO software for different examples. The optimum values of t_1, T, S and R along with minimum average cost are displayed in **Table 1**.

Table 1: Optimal solution for different examples

Examples	S	R	t_1	T	Z
1	53.6081	24.8696	0.9515	1.6561	2121.981
2	91.8766	4.0366	1.7579	1.8300	2298.648
3	39.2629	36.5827	0.7014	1.5997	2139.171

4. Sensitivity analysis

For the given example mentioned earlier, sensitivity analysis has been performed to study the effect of changes (under or over estimation) of different parameters like demand, deterioration, inventory cost parameters and mark-up rate on maximum initial stock level, shortage level, cycle length, frequency of advertisement along with the maximum profit of the system. This analysis has been carried out by changing (increasing and decreasing) the parameters from – 20% to + 20%, taken one or more parameters at a time making the other parameters at their more parameters at a time and making the other parameters at their original values. The results of this analysis are shown in Figures.

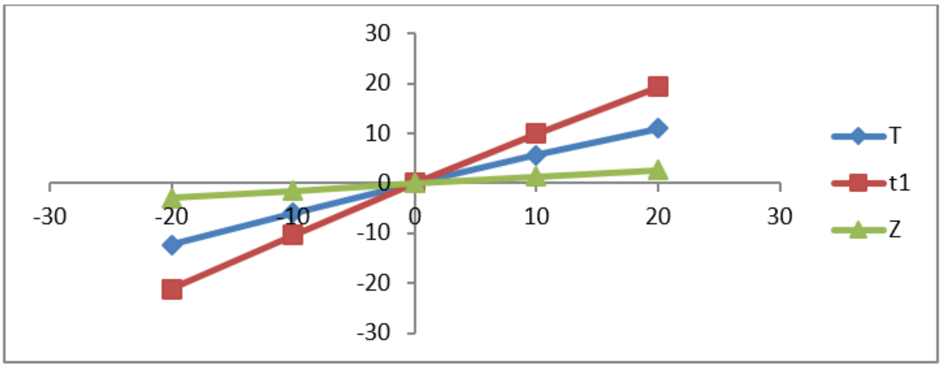

Figure 1: Effect of the variable and cost function if % change of parameter C_o

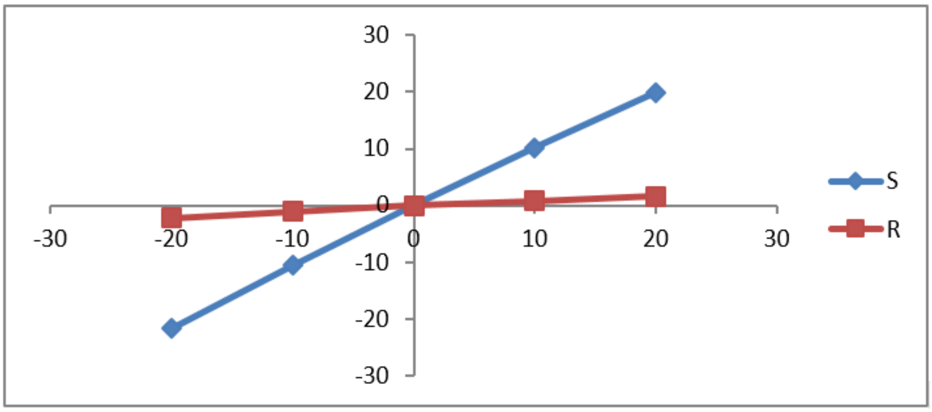

Figure 2: Effect of the variable *S* and *R* if % change of parameter C_o

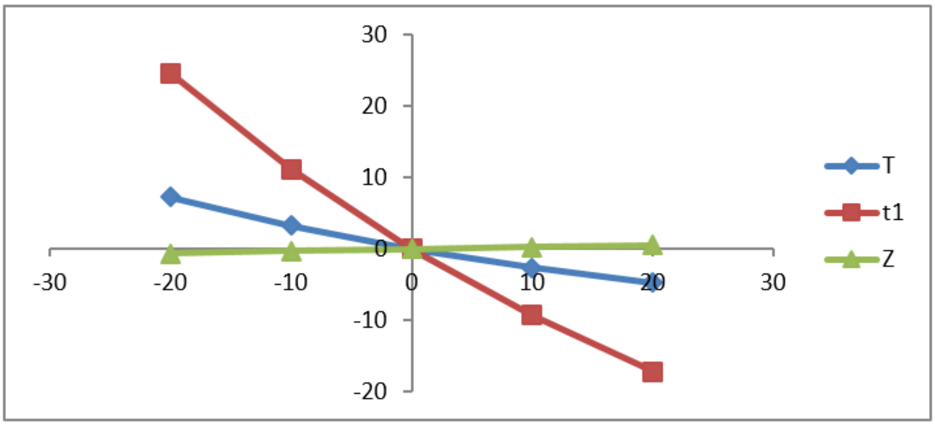

Figure 3: Effect of the variable and cost function if % change of parameter C_h

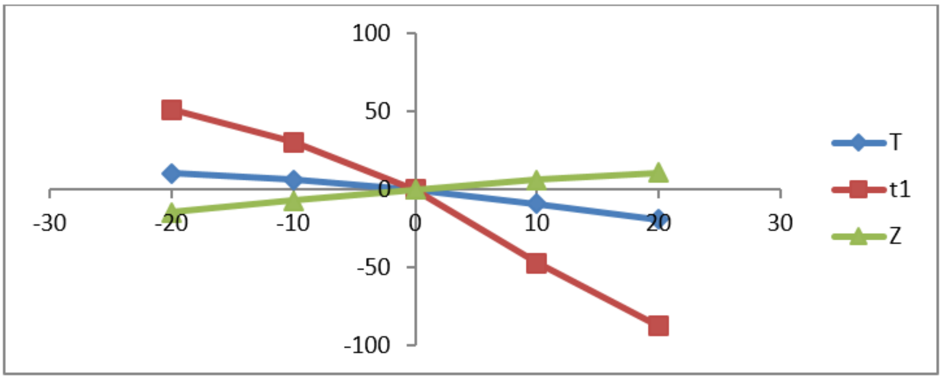

Figure 4: Effect of the variable *S* and *R* if % change of parameter C_h

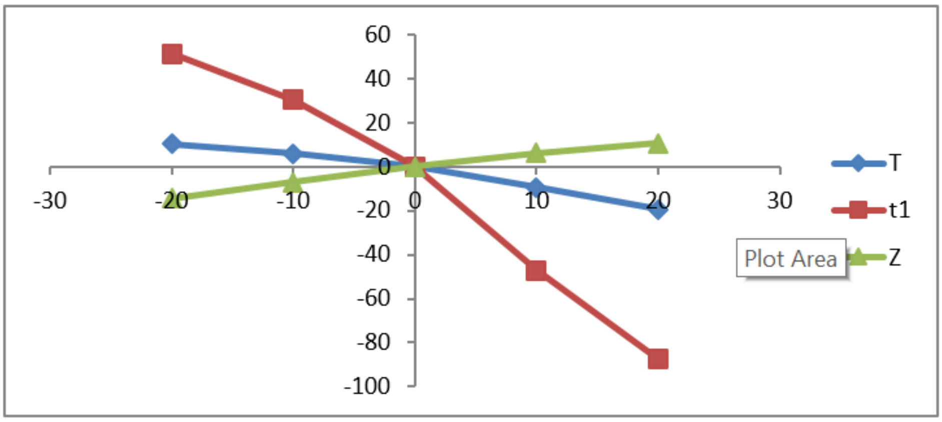

Figure 5: Effect of the variable and cost function if % change of parameter C_p

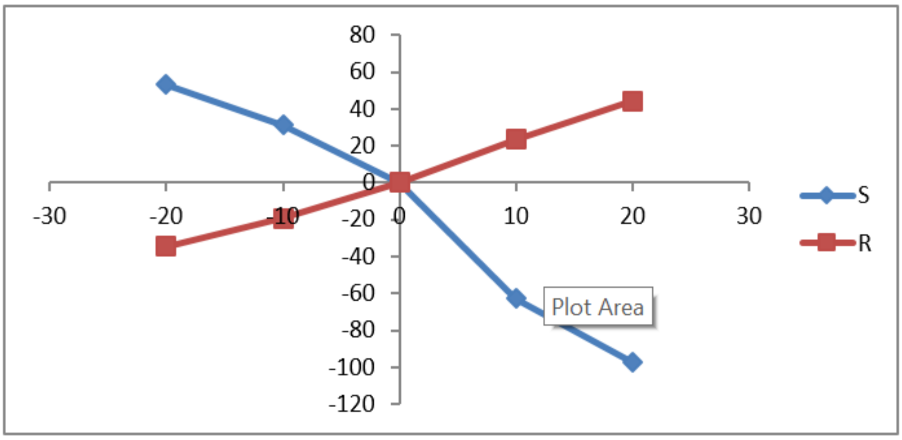

Figure 6: Effect of the variable and cost function if % change of parameter C_p

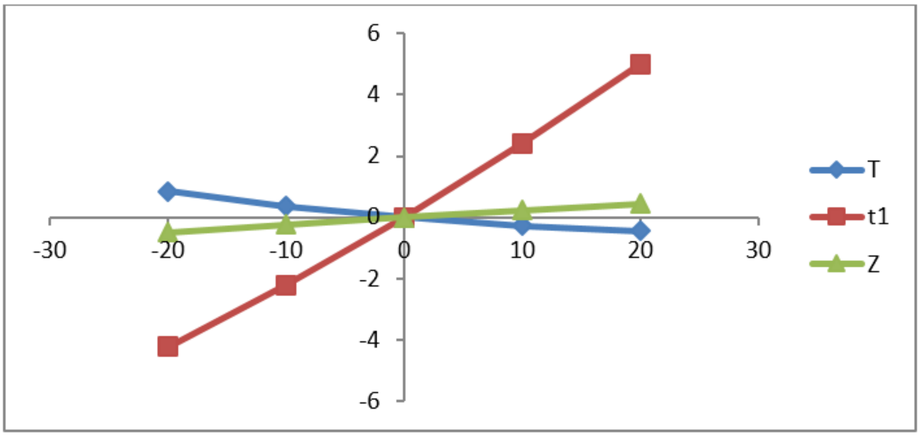

Figure 7: Effect of the variable and cost function if % change of parameter *r*

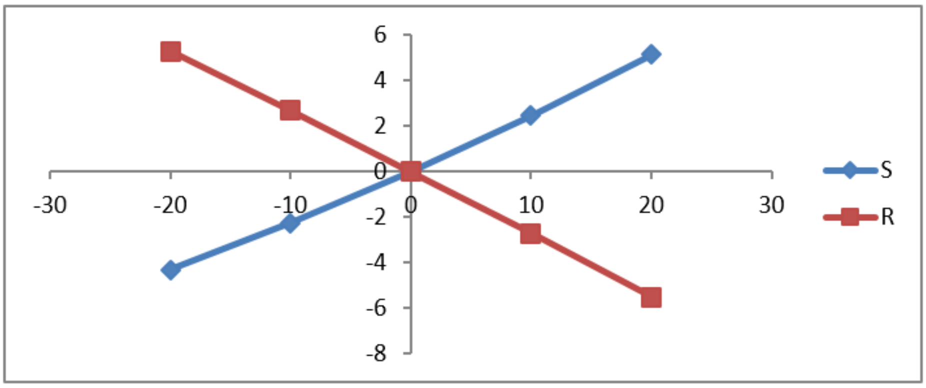

Figure 8: Effect of the variable and cost function if % change of parameter *r*

5. Concluding remarks

This work deals with a deterministic inventory model for deteriorating items with variable demand inflation effect of the system and fuzzy valued inventory costs. In most of the research works, the inventory costs like the carrying cost, ordering cost, shortage cost and purchase cost have been assumed to be known at a fixed level. However, in reality, these costs should be imprecise number instead of fixed real numbers because inventory costs are fluctuating due to different factors. Ordering cost is dependent on the transportation facilities, may also vary from season to season. Changes in the price of fuels, mailing charges and telephonic charges may also make the ordering cost fluctuating. Unit purchase cost is highly dependent on the costs of raw materials and labor charges, which may fluctuate over time. To solve the problem with such imprecise parameters, stochastic, fuzzy and fuzzy-stochastic approaches may be used. In stochastic approach, the parameters are assumed to be random variables with known probability distribution. The present model is also applicable to the problems where the selling prices of the items as well as the advertisement of items affect the demand. It is applicable for fashionable goods, two level and single level credit policy approach also.

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