Intern. J. Fuzzy Mathematical Archive
Vol. 23, No. 1, 2025, 67-73
ISSN: 2320 –3242 (P), 2320 –3250 (online)
Published on 21 July 2025
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/ijfma.v23n1a06251

International Journal of Fuzzy Mathematical Archive

Difference-Mean Edge Properties in Some Products of Two Fuzzy Graphs

***K. Radha*[1*] and *S. Sri Harini*[2]**

[1,2]PG and Research Department of Mathematics,
Thanthai Periyar Government Arts and Science College,
(Affiliated to Bharathidasan University), Trichy-23, Tamil Nadu, India.
[1]Email: radhagac@yahoo.com; [2]Email: sriharini.suresh@gmail.com
*Corresponding author

Received 2 May 2025; accepted 20 July 2025

Abstract. In this paper, we investigate the difference-mean edge properties in the context of specific fuzzy graph products, like the Strong product, the Residue product, and the Maximal product. The requirements for different mean edge properties in these products of fuzzy graphs are also explored.

Keywords: Strong Product, Maximal Product, Residue Product and Complement.

AMS Mathematics Subject Classification (2010): 05C72, 05C78

1. Introduction

Like set theory, Rosenfeld proposed fuzzy graph theory in 1975. Zadeh's fuzzy set theory from 1965 serves as the foundation for fuzzy graph theory [8]. A fuzzy labeling and its attributes were studied by Nagoorgani and Rajalaxmi [1]. Effective fuzzy semi-graphs were the subject of research by Radha and Renganathan [3]. The residue product of two fuzzy graphs was analyzed by Radha and Arumugam [2], while the maximal product and strong product of two fuzzy graphs were examined by Radha and Uma [7]. Radha and Sri Harini constructed a "Difference mean fuzzy graph" by placing the constraint on the membership value of the edges and some other qualities of the difference mean edge involved in path and star graphs [4]. In addition to examining the characterisations of difference mean edges in some binary operations of fuzzy graphs, and the requirements of difference mean edges in the direct sum of two fuzzy graphs, suggestions were made [[5], [6]]. This study gives detailed definitions for each fuzzy graph product operation, including the Strong, Residue, and Maximal products, and examines them in connection to these different mean edge properties. Edges in the resulting graph fulfil the difference-mean criterion under certain required and sufficient conditions, which are determined for each fuzzy product.

2. Preliminaries

Definition 2.1. Let V be a non-empty set fuzzy subset and E ⊆ V×V. A fuzzy graph $G:(\sigma,\mu)$ is a pair of functions, $\sigma: V \to [0,1]$ and $\mu: E \to [0,1]$ such that $\mu(xy) \le \sigma(x) \wedge \sigma(y)$ for all $x, y \in V$. Underlying crisp graph is denoted by $G^*:(V,E)$[3].

Definition 2.2. [7] The strong product $G_1 \circ G_2 : (\sigma, \mu)$ of G_1 and G_2 on (V, E), where V = $V_1 \times V_2$ and E = $\{((u,v)(x,y)) \,/\, u = x,\ vy \in E_2 \ or\ v = y,\ ux \in E_1 \ or\ ux \in E_1,\ vy \in E_2\}$, is given by $\sigma((u,v)) = \sigma_1(u) \wedge \sigma_2(u)$ and

$$\mu((u,v)(x,y)) = \begin{cases} \sigma_1(u) \wedge \mu_2(vy), & if u = x,\ vy \in E_2 \\ \sigma_2(v) \wedge \mu_1(ux), & if v = y,\ ux \in E_1 \\ \mu_1(ux) \wedge \mu_2(vy), & if ux \in E_1,\ vy \in E_2 \end{cases}.$$

Definition 2.3. [7] The maximal product $G_1 \circ G_2 : (\sigma, \mu)$ of G_1 and G_2 on (V, E), where V = $V_1 \times V_2$ and E = $\{((u,v)(x,y)) u = x,\ vy \in E_2 \ or\ v = y,\ ux \in E_1\}$, is given by $\sigma((u,v)) = \sigma_1(u) \vee \sigma_2(u)$ and

$$\mu((u,v)(x,y)) = \begin{cases} \sigma_1(u) \vee \mu_2(vy), & if u = x, & vy \in E_2 \\ \sigma_2(v) \vee \mu_1(ux), & if v = y, & ux \in E_1 \end{cases}$$

Definition 2.4. Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs with underlying crisp graphs $G_1{}^* : (V_1, E_1)$ and $G_2{}^* : (V_2, E_2)$ respectively. Define $G : (\sigma, \mu)$ with underlying crisp graph where $G^* : (V, E)$ where V = $V_1 \times V_2$ and E = $\{((u,v)(x,y))\ u = x,\ vy \in E_2 \ or\ v = y,\ ux \in E_1 \ or\ ux \in E_1,\ vy \in E_2\}$, is given by,

$\sigma(u_1, v_1) = \sigma_1(u_1) \vee \sigma_2(v_1)$ for all $u_1, v_1 \in V$

If $u_1 u_2 \in E_1$ and $v_1 \neq v_2$ then

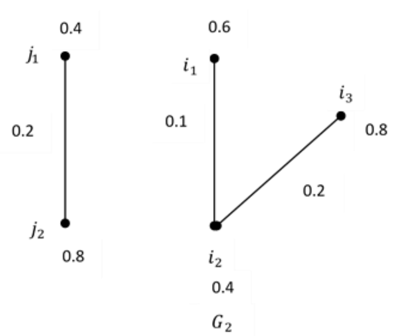
$\mu((u_1, v_1), (u_2, v_2) = \mu(u_1, u_2)$ for all $(u_1, v_1), (u_2, v_2) \in E$

$$\mu((u_1, v_1), (u_2, v_2) = \mu(u_1, u_2) \leq \sigma_1(u_1) \wedge \sigma_2(v_1)$$
$$\leq [\sigma_1(u_1) \vee \sigma_2(v_1)] \wedge [\sigma_1(u_2) \vee \sigma_2(v_2)] = \sigma(u_1, v_1) \wedge \sigma(u_2, v_2)$$

Hence $\mu\big((u_1, v_1), (u_2, v_2)\big) = \sigma(u_1, v_1) \wedge \sigma(u_2, v_2)$. Therefore, G:(σ, μ) is a fuzzy graph. This is called the residue product of the fuzzy graphs G_1 and G_2 and denoted by $G_1 \bullet G_2$ [7].

3. Difference mean edge properties in strong product of two fuzzy graphs

Remark 3.1. If ih is a difference mean edge of G_1 or G_2, then the edge $(i,j)(h,j)$ or$(j,i)(j,h)$ in the strong product $G_1 \circ G_2$ need not be a difference mean edge. In the following figure 3.1, all the edges of G_1 and G_2 are difference mean edges. But the edges $(j_1, i_1)(j_1, i_2), (j_1, i_2)(j_1, i_3), (j_1, i_1)(j_2, i_1), (j_1, i_1)(j_2, i_2), (j_1, i_2)(j_2, i_1), (j_1, i_3)(j_2, i_2)$ are not difference mean edges of the strong product.



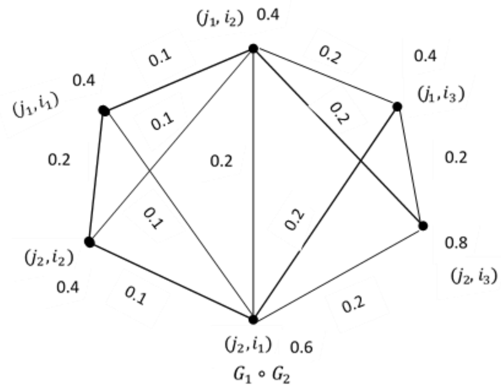


Figure 3.1:

Theorem 3.2. Consider a fuzzy graph $G_i: (\sigma_i, \mu_i, V_i, E_i)$, for $i = 1,2$. Let $g_j g_l$ be a difference mean edge in G_2. If h_i is a vertex of G_1 such that $\sigma_1(h_i) \geq \sigma_2(g_j)$ and $\sigma_1(h_i) \geq \sigma_2(g_l)$, then $(h_i, g_j)(h_i, g_l)$ is a difference mean edge in the strong product $G_1 \circ G_2$.

Proof: Since $g_j g_l$ is a difference mean edge, $2\mu_2(g_j g_l) = |\sigma_2(g_j) - \sigma_2(g_l)|$

Also, since $\sigma_1(h_i) \geq \sigma_2(g_j)$ and $\sigma_1(h_i) \geq \sigma_2(g_l)$,

$$|\sigma(h_i, g_j) - \sigma(h_i, g_l)| = |\sigma_1(h_i) \wedge \sigma_2(g_j) - \sigma_1(h_i) \wedge \sigma_2(g_l)|$$
$$= |\sigma_2(g_j) - \sigma_2(g_l)| = 2\mu_2(g_j g_l)$$

Now $\sigma_1(h_i) \geq \sigma_2(g_j) \wedge \sigma_2(g_l) \geq \mu_2(g_j g_l)$

Therefore $\mu((h_i, g_j)=(g_k, g_l)) = \sigma_1(h_i) \wedge \mu_2(g_j g_l) = \mu_2(g_j g_l)$

Hence $|\sigma(h_i, g_j) - \sigma(h_i, g_l)| = 2\mu((h_i, g_j)(h_i, g_l))$

Therefore $(h_i, g_j)(h_i, g_l)$ is a difference mean edge of the strong product $G_1 \circ G_2$.

Theorem 3.3. For any vertex h of G_1 and any edge jg of G_2 such that $\sigma_1(h) \leq \sigma_2(g)$ and $\sigma_1(h) \leq \sigma_2(j)$ the edge $(h, g)(h, j)$ of the strong product $G_1 \circ G_2$ cannot be a difference mean edge of $G_1 \circ G_2$.

Proof: Hence $\sigma_1(h) \leq \sigma_2(g)$ and $\sigma_1(h) \leq \sigma_2(j)$. Therefore,

$(\sigma_1 \circ \sigma_2)(h, g) = \sigma_1(h) \wedge \sigma_2(g) = \sigma_1(h)$

$(\sigma_1 \circ \sigma_2)(h, j) = \sigma_1(h) \wedge \sigma_2(j) = \sigma_1(h)$

Therefore, $|(\sigma_1 \circ \sigma_2)(h, g) - (\sigma_1 \circ \sigma_2)(h, g)| = |\sigma_1(h) - \sigma_1(h)| = 0$

But from the elucidation of the strong product,

$(\mu_1 \circ \mu_2)((h, g)(h, j)) = \sigma_1(h) \wedge \mu_2(gj) > 0$

Therefore, $2(\mu_1 \circ \mu_2)((h, g)(g, j)) \neq |(\sigma_1 \circ \sigma_2)(h, g) - (\sigma_1 \circ \sigma_2)(h, j)|$

Hence $(h, g)(h, j)$ is not a difference mean edge of $G_1 \circ G_2$.

Theorem 3.4. If $h_i h_k$ is a difference mean edge of $G_1$and g_j is a vertex of G_2 such that $\sigma_1(h_i) \leq \sigma_2(g_j)$ and $\sigma_1(h_k) \leq \sigma_2(g_j)$, then $(h_i, g_j)(h_k, g_j)$ is a difference mean edge in the strong product $G_1 \circ G_2$.

Theorem 3.5. For any edge gj of G_1 and any vertex h of G_2 such that $\sigma_1(g) \geq \sigma_2(h)$ and $\sigma_1(j) \geq \sigma_2(g)$, the edge $(g,h)(j,h)$ of the strong product $G_1 \circ G_2$ cannot be a difference mean edge of $G_1 \circ G_2$.

4. Difference mean edge properties in maximal product of two fuzzy graphs

Remark 4.1. If gh is a difference mean edge of G_1 (or G_2), then the edge $(g,u)(h,u)$ [or $(u,g)(u,h)$] need not be a difference mean edge of the maximal product $G_1 \vee G_2$. For example, consider G_1 and G_2 in figure 4.1. All the edges of G_1 and G_2 are difference mean edges. But no edge of the maximal product $G_1 \vee G_2$ is a difference mean edge.

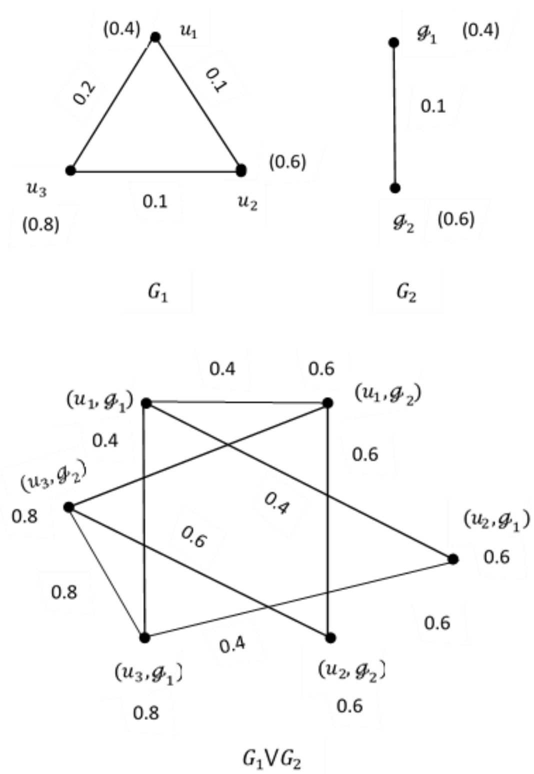


Figure 4.1:

Theorem 4.2. If h is a vertex of a fuzzy graph G_1 and gj is a difference mean edge of G_2, such that $\sigma_1(h) \leq \mu_2(gj)$ then $(h,g)(h,j)$ is a difference mean edge of $G_1 \vee G_2$ of G_1 and G_2.

Proof: Since $\sigma_1(h) \leq \mu_2(gj) \leq \sigma_2(g) \leq \sigma_2(j)$, it follows that $\sigma_1(h) \leq \sigma_2(g)$ and $\sigma_1(h) \leq \sigma_2(j)$,

$(\sigma_1 \vee \sigma_2)(h,g) = \sigma_1(h) \vee \sigma_2(g) = \sigma_2(g)$ and

$(\sigma_1 \vee \sigma_2)(h,j) = \sigma_1(h) \vee \sigma_2(j) = \sigma_2(j)$

Therefore, $|(\sigma_1 \vee \sigma_2)(h,g) - (\sigma_1 \vee \sigma_2)(h,j)| = |\sigma_2(g) - \sigma_2(j)| = 2\mu_2(gj)$,

since gj is the difference mean edge of G_2.

Now since $\sigma_1(h) \leq \mu_2(gj)$, $(\mu_1 \vee \mu_2)((h,g)(h,j)) = \sigma_1(z) \vee \mu_2(gj) = \mu_2(gj)$

Hence $|(\sigma_1 \vee \sigma_2)(h, g) - (\sigma_1 \vee \sigma_2)(h, j)| = 2(\mu_1 \vee \mu_2)((h, g)(h, j))$.
Therefore, $(h, g)(h, j)$ is a difference mean edge of the maximal product $G_1 \vee G_2$.

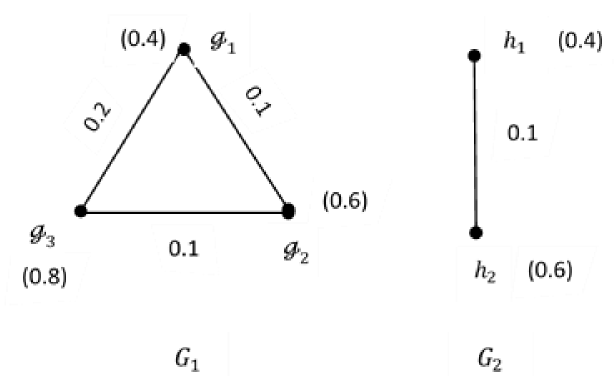
Theorem 4.3. If gj is a difference mean edge of G_1 and h is a vertex of G_2, such that $\sigma_2(h) \leq \mu_1(gj)$, then $(g, j)(g, h)$ is a difference mean edge of $G_1 \vee G_2$ of G_1 and G_2.

Theorem 4.4. For any vertex h of G_1 and any edge gj of G_2 such that $\sigma_1(h) \geq \sigma_2(g)$ and $\sigma_1(h) \geq \sigma_2(j)$, the edge $(h, g)(h, j)$ cannot be a difference mean edge of $G_1 \vee G_2$ of G_1 and G_2.
Proof: Since $\sigma_1(h) \geq \sigma_2(g)$ and $\sigma_1(h) \geq \sigma_2(j)$,
$(\sigma_1 \vee \sigma_2)(h, g) = \sigma_1(h) \vee \sigma_2(g) = \sigma_1(h)$ and
$(\sigma_1 \vee \sigma_2)(h, j) = \sigma_1(h) \vee \sigma_2(j) = \sigma_1(h)$.
Therefore, $|(\sigma_1 \vee \sigma_2)(h, g) - (\sigma_1 \vee \sigma_2)(h, j)| = |\sigma_1(h) - \sigma_1(h)| = 0$.
Also, from the elucidation of the maximal product of fuzzy graphs,
$(\mu_1 \vee \mu_2)((h, g)(h, j)) = \sigma_1(h) \vee \mu_2(gj) > 0$
Therefore $2(\mu_1 \vee \mu_2)((h, g)(h, j)) \neq |(\sigma_1 \vee \sigma_2)(h, g) - (\sigma_1 \vee \sigma_2)(h, j)|$.
Hence, $(h, g)(h, j)$ cannot be a difference mean edge of the maximal product $G_1 \vee G_2$.

Theorem 4.5. For any edge gj of the fuzzy graph G_1 and a vertex h of G_2 such that $\sigma_1(g) \leq \sigma_2(h)$ and $\sigma_1(j) \geq \sigma_2(h)$, the edge $(g, h)(j, h)$ cannot be a difference mean edge of $G_1 \vee G_2$ of G_1 and G_2.

5. Difference mean edge properties in residue product of two fuzzy graphs

Remark 5.1. If gh is a difference mean edge of G_1, then $(g, x)(h, y)$, where x and y are distinct vertices of G_2, need not be a difference mean edge of the residue product $G_1 \cdot G_2$. For example, consider G_1, G_2 and their residue product in figure 5.1, G_1 and G_2 are both difference-mean fuzzy graphs. But the edges $(g_1, h_2)(g_2, h_1)$ and $(g_1, h_2)(g_3, h_1)$ are not difference mean edges in the residue product $G_1 \cdot G_2$.



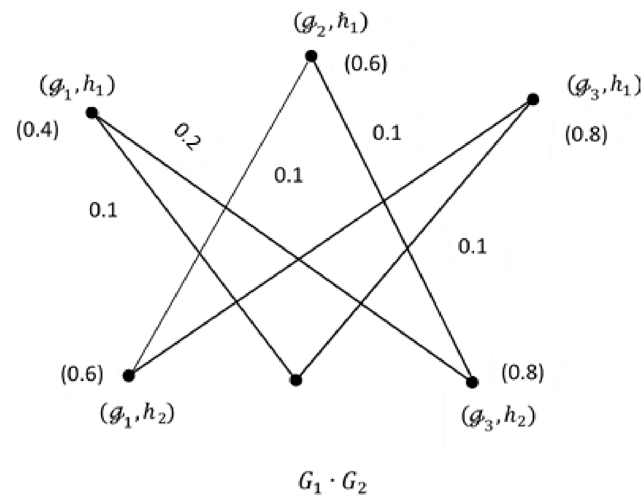


Figure 5.1:

Theorem 5.2. Consider two fuzzy graphs $G_1: (\sigma_1, \mu_1, V_1, E_1)$ and $G_2: (\sigma_2, \mu_2, V_2, E_2)$ such that $\sigma_1(h) \geq \sigma_2(j)$ for every h in V_1 and for every j in V_2. Then the residue product $G_1 \cdot G_2$ is a difference mean fuzzy graph if and only if G_1 is a difference mean fuzzy graph.

Proof: Assume that $G_1 \cdot G_2$ is a difference mean fuzzy graph.

Consider any edge hj of G_1. Let g_1 and g_2 be two separate vertices of G_2.

Then $(h, g_1)(j, g_2)$ is an edge of $G_1 \cdot G_2$.

Since $G_1 \cdot G_2$ is a difference mean fuzzy graph, it is a difference mean edge.

Therefore, $2(\mu_1 \cdot \mu_2)((h, g_1)(j, g_2)) = |(\sigma_1 \cdot \sigma_2)(h, g_1) - (\sigma_1 \cdot \sigma_2)(j, g_2)|$

which implies $2\mu_1(hj) = |\sigma_1(h) \wedge \sigma_2(g_1) - \sigma_1(j) \wedge \sigma_2(g_2)|$

$2\mu_1(hj) = |\sigma_1(h) - \sigma_1(j)|$, since $\sigma_1(h) \geq \sigma_2(g_1)$ and $\sigma_1(j) \geq \sigma_2(g_2)$ by hypothesis.

Hence hj is a difference mean edge of G_1. Since the edge hj is arbitrary, G_1 is a difference mean fuzzy graph.

Conversely, assume that G_1 is a difference mean fuzzy graph.

Consider any edge $(h_i, j_j)(h_k, j_l)$ of the residue product.

Then $h_i h_k \in X_1$ and $j_l \neq j_k$ and $(\mu_1 \cdot \mu_2)\left((h_i, j_j)(h_k, j_l)\right) = \mu_1(h_i h_k)$

By hypothesis, $\sigma_1(h_i) \geq \sigma_2(j_j)$ and $\sigma_1(h_k) \geq \sigma_2(j_l)$.

Therefore $(\sigma_1 \cdot \sigma_2)(h_k, j_l) = \sigma_1(h_k) \vee \sigma_2(j_l) = \sigma_1(h_k)$.

Since $h_i h_k$ is a difference mean fuzzy graph,

$2\mu_1(h_i h_k) = |\, \sigma_1(h_i) - \sigma_2(h_k)\, |$.

$$\begin{aligned}\text{Now } \left|(\sigma_1 \cdot \sigma_2)(h_i, j_j) - (\sigma_1 \cdot \sigma_2)(h_k, j_l)\right| &= |\, \sigma_1(h_i) - \sigma_1(h_k)| \\ &= 2\mu_1(h_i h_k) \\ &= 2(\mu_1 \cdot \mu_2)\left((h_i, j_j)(h_k, j_l)\right)\end{aligned}$$

Therefore $(h_i, j_j)(h_k, j_l)$ is a difference mean edge. Since $(h_i, j_j)(h_k, j_l)$ is an arbitrary edge, $G_1 \cdot G_2$ is a difference mean fuzzy graph.

6. Conclusion

This study contributes a comprehensive interpretation of difference mean edge properties within various fuzzy graph products, including the Strong, Residue, and Maximal products, as well as their complements. The research delineates the conditions under which difference mean edges exist in Strong, Residue, and Maximal products of fuzzy graphs. It highlights how these properties are influenced by the structural characteristics and membership functions of the original graphs. These findings enhance the understanding of how difference mean edge properties behave under various fuzzy graph operations and transformations. These insights are valuable for applications in network analysis, decision-making processes, and modeling complex systems where uncertainty and partial relationships are inherent.

REFERENCES

1. A.Nagoorgani and D. Rajalaxmi, A note on fuzzy labeling, *International Journal of Fuzzy Mathematical Archive*, 4(2) (2014) 2320 –3250.
2. K. Radha and S. Arumugam, On residue product of two fuzzy graphs, International *Journal of multidisciplinary Research and Development*, 2(11) (2015).
3. K. Radha and P. Renganathan, Effective fuzzy semigraphs, *Advances and Applications in Mathematical Sciences*, 20(5) (2021) 895-904.
4. K. Radha and S. Sri Harini, Effective and difference mean fuzzy graphs, *Journal of Computational Analysis and Applications*, 33(6) (2024) 754 – 759.
5. K. Radha and S. Sri Harini, Characterizations of the Direct Sum of Two Difference-mean fuzzy graphs, *Indian Journal of Science and Technology*, 17(20) (2024) 2043-2049.
6. K. Radha and S. Sri Harini, Characterizations of the Difference-mean edges in some binary operations of fuzzy graphs, *Indian Journal of Natural Science*, 17(20) (2024) 2043- 2049.
7. K. Radha and A. Uma, Simplicial vertices in the direct sum, strong product, lexicographic product and maximal product of two fuzzy graph, *Advances and Applications in Mathematical Sciences*, 21(6) (2022) 3357-3369.
8. L.A. Zadeh, Fuzzy sets, *Information and Control*, 8(3) (1965) 338-353.