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On the Decomposability of Selje Topological Spaces

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Abstract. The development of generalized closed set theory has significantly expanded the structural foundations of modern topology. In this paper, we investigate decomposition properties within Selje Topological Spaces, a relationally structured extension of classical topology. By introducing and analyzing SJ-locally closed and SJ-$R^{\#}\alpha$-locally closed sets, we establish a systematic decomposition framework that extends classical locally closed set representations into the Selje setting. Structural characterizations are obtained using Selje closure and interior operators, and inclusion relationships among Selje closed, $R^{\#}\alpha$-closed, and locally closed classes are rigorously examined. The study further introduces SJ-$R^{\#}\alpha$-LC continuous mappings and investigates their functional behavior under inverse images and composition. These results provide a refined hierarchical structure within Selje topology and strengthen its decomposition theory. The proposed framework enhances the theoretical depth of Selje Topological Spaces and offers a foundation for further research in relational topology, generalized continuity, and interdisciplinary applications such as network analysis and data-driven topological modeling.

Keywords: Selje Topological Space; $R^{\#}\alpha$-closed sets; SJ-locally closed sets; decomposition theory; generalized continuity; relational topology.

AMS Mathematics Subject Classification (2010): 54A05, 54D10, 54C08

1. Introduction

1.1. Statement of the problem

The theory of generalized closed sets has played a fundamental role in the advancement of modern topology. Beginning with the introduction of generalized closed sets by Levine (1), subsequent contributions by Maki et al. (2), Dontchev and Ganster (3), and

Veerakumar (4) expanded the structural understanding of closure operators and associated topological constructions. These developments provided refined characterizations of classical topological notions and significantly influenced the study of generalized continuity, separation axioms and decomposition principles.

As generalized structures evolved, new classes such as semi-open and pre-open sets (6,7) enriched the flexibility of boundary analysis and local structural behavior. Parallel investigations into generalized continuity and irresolute mappings (8,9) further decomposed classical continuity into weaker yet structurally meaningful components. Decomposition approaches were also examined in normality axioms and regular closed mappings (10,11), strengthening the analytical foundation of generalized topology.

Within this broader theoretical evolution, Selje Topological Space was introduced as a relationally structured extension of classical topology (12). Unlike conventional topological models based solely on open-set axioms, Selje topology incorporates relational mechanisms that refine the interpretations of closure, interior, and continuity. This relational extension creates new possibilities for decomposition analysis and structural stability.

1.2. Research gap

Although generalized closed set theory has been extensively studied in classical and extended topological frameworks, decomposition structures within Selje Topological Spaces remain insufficiently explored. A comparative structural study was presented in (20), and novel generalized closed sets in Selje spaces were introduced in (21). However, the systematic development of locally closed-type decompositions within the Selje relational structure has not been fully investigated.

In particular, while locally closed sets play an important role in classical decomposition theory, their relational counterparts in Selje topology have not been formally defined or structurally characterized. Moreover, the interaction between local decomposition behavior and relational closure mechanisms intrinsic to Selje spaces requires further analytical clarification. This indicates a need for introducing new locally structured set classes that align with the Selje relational framework and for establishing their structural and functional properties rigorously.

1.3. Proposed work and scope

Motivated by these theoretical gaps, the present paper investigates decomposition theory in Selje Topological Spaces. Specifically, we introduce two new structural classes: SJ-locally closed sets and SJ-R#α-locally closed sets. These concepts are formulated to unify local decomposition behavior with relational closure structures inherent in Selje topology. The study establishes:

- Structural characterizations of the newly defined classes
- Inclusion relations among existing generalized closed sets

- Preservation properties under suitable mappings
- Hierarchical positioning within the broader framework of generalized topology

Through this systematic development, the work strengthens the theoretical foundation of Selje topology and extends decomposition principles into a relational topological setting.

1.4. Related work

Selje topology has gradually expanded beyond its foundational definition. A structural comparison with other topological spaces was presented in (20), highlighting its relational stability and decomposition behavior. Subsequent studies introduced new generalized closed sets within Selje spaces and examined their implications (21).

Beyond theoretical exploration, Selje topology has demonstrated interdisciplinary applicability. Applications to electricity network optimization using topological data analysis and R programming were investigated in (23), establishing connections with network theory and data-driven modeling (15,17,18). These works demonstrated how relational topological structures can model transformer criticality, network stability and load distribution patterns. Furthermore, integration with blockchain-based systems was explored in (22), supported by foundational blockchain studies (13,14), thereby extending Selje topology into decentralized computational frameworks.

Collectively, these studies establish Selje topology as both a theoretically rich and practically adaptable framework. However, decomposition theory within Selje spaces remains underdeveloped, particularly in relation to locally closed-type structures. The present work addresses this gap by providing a systematic decomposition framework within the Selje relational context.

2. Literature survey

The development of generalized closed sets has been central to the expansion of classical topology. The foundational work of Levine (1) introduced generalized closed sets, which weakened the traditional notion of closedness while preserving essential structural properties. This concept stimulated extensive research into alternative closure operators and their role in decomposing topological structures. Maki et al. (2) further investigated generalized closed sets within modified topological frameworks, while Dontchev and Ganster (3) explored their interactions with separation axioms. Veerakumar (4) contributed additional structural properties and relationships between generalized closed sets and other weakened forms of closedness.

The evolution of semi-open and pre-open sets significantly enriched generalized topology. The systematic studies in (6,7) established fundamental properties of these sets and examined their role in refining boundary and interior operations. These classes provided a bridge between open and closed structures and enabled the formulation of intermediate decomposition results. In parallel, generalized continuity and irresolute mappings were introduced and studied in (8,9), where function behavior was analyzed under weaker structural constraints. These investigations decomposed classical continuity into combinations of generalized properties, thereby expanding the analytical scope of topological mappings.

Further developments examined decomposition principles in relation to normality axioms and regular closed mappings (10,11). These studies highlighted the structural interplay between closure operators and separation properties, strengthening the theoretical foundation of generalized decomposition theory. Collectively, these works established a robust framework for analyzing weaker forms of closedness and continuity within classical and extended topological spaces.

Within this broader research landscape, Selje Topological Space emerged as a relational extension of classical topology (12). Unlike traditional spaces defined purely by open set axioms, Selje topology incorporates relational structures that refine closure and interior mechanisms. A comparative structural analysis with existing topological spaces was conducted in (20), demonstrating distinctive decomposition behavior and relational stability properties. Subsequent research introduced novel generalized closed sets within Selje spaces and analyzed their inclusion relationships and structural consequences (21).
The applicability of Selje topology has also been explored beyond theoretical abstraction. Studies in (23) applied Selje structures to electricity network optimization using topological data analysis and R programming, linking relational topology with network modeling (15,17,18). These applications demonstrated how Selje-based frameworks can model transformer criticality, voltage instability regions, and load distribution dynamics. Additionally, the integration of Selje topology with blockchain-based computational systems was examined in (22), supported by foundational blockchain studies (13,14), thereby extending its relevance to secure and decentralized architectures.

Despite these advances, the literature reveals that decomposition theory within Selje Topological Spaces remains in an early stage of development. While generalized closed sets and their relational counterparts have been introduced, locally structured decomposition classes analogous to classical locally closed sets have not been systematically formulated within the Selje framework. In particular, the interaction between relational closure mechanisms and local structural behavior requires further analytical investigation.

Therefore, the present study builds upon the established body of work in generalized topology and Selje spaces by introducing SJ-locally closed and SJ-R#α-locally closed sets. This contribution aims to extend decomposition theory within relational topological systems and to position Selje topology within the broader hierarchy of generalized topological structures.
Let X be a non-empty set.

Definition 2.1. A pair (X, τ) is called a topological space if:

1. $\emptyset, X \in \tau$,
2. Arbitrary unions of members of τ belong to τ,
3. Finite intersections of members of τ belong to τ.

Definition 2.2. A subset $A \subseteq X$ is generalized closed (g-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.

Definition 2.3. A subset $A \subseteq X$ is α-closed if $cl(int(cl(A))) \subseteq A$.

Definition 2.4. A subset $A \subseteq X$ is locally closed if $A = U \cap F$, where U is open and F is closed.

Definition 2.5. A pair (X, τ_R) is called a Selje Topological Space if:

1. $\emptyset, X \in \tau_R$,
2. Arbitrary unions of Selje open sets belong to τ_R,
3. Finite intersections of Selje open sets belong to τ_R.

Let SJ- cl(A) and SJ- int(A) denote the Selje closure and interior respectively.

Definition 2.6. A subset $A \subseteq X$ is $R^{\#}\alpha$-closed if $SJ\text{-}cl(SJ\text{-}int(SJ\text{-}cl(A))) \subseteq A$.

Definition 2.7. A subset $A \subseteq X$ is SJ-$R^{\#}\alpha$-locally closed if $A = U \cap F$, where U is Selje open and F is $R^{\#}\alpha$-closed.

4. Decomposition of Selje topological space

Definition 3.1. *A subset* A *of a Selje topological space* $(U, \mu_R(X), SJ_R(X))$ *is called a Selje-locally closed (briefly SJ-lc) set if* $A = C \cap D$ *where* C *is SJ-open and* D *is SJ-closed. The family of all Selje-locally closed sets in a Selje topological space is denoted by SJ-LC(U, X).*

Remark 3.2. *Every SJ-closed (resp.SJ-open) set is SJ-locally closed set but the converse need not always be true and is shown in the following example.*

Example 3.3. *Let* $U = \{i,j,k,l,m\}$, $U/R = \{\{i,j\},\{k\},\{l,m\}\}$ $X=\{k,l\}\subseteq U$.
$SJ_R(X)=\{\Phi,U,\{j\},\{k\},\{m\},\{i,m\},\{j,m\},\{k,m\},\{j,k\},\{j,l\},\{k,l\},\{l,m\},\{i,j,m\},\{i,k,m\},\{i,l,m\},\{j,k,m\},\{j,l,m\},\{j,k,l\},\{k,l,m\},\{i,j,k,m\},\{i,j,l,m\},\{i,k,l,m\},\{j,k,l,m\}\}$.
$SJ\text{-}LC(U,X) = \{\Phi,U,\{j\},\{k\},\{l\},\{i,m\},\{j,k\},\{j,l\},\{k,l\},\{i,j,m\},\{i,k,m\},\{i,l,m\},\{j,k,l\},\{i,j,k,m\},\{i,j,l,m\},\{i,k,l,m\}\}$
Here $\{i\}$ *is in SJ-LC(U, X) but not in SJ-CL(U, X).*

Definition 3.4. *A subset* A *of a Selje topological space* $(U, \mu_R(X), SJ_R(X))$ *is called a SJ-*$R^{\#}\alpha$*-locally closed (briefly SJ-*$R^{\#}\alpha$*-lc) set if* D = E ∩ F *where* E *is SJ-*$R^{\#}\alpha$*-open and* F *is SJ-*$R^{\#}\alpha$ *closed.*
The family of all SJ-$R^{\#}\alpha$*-locally closed sets in a Selje topological space is denoted by SJ-*$R^{\#}\alpha$*-LC(U, X).*

Remark 3.5. *Every SJ-closed (resp.SJ-open) set is SJ-locally closed set but the converse need not always be true and is shown in the following example.*

Example 3.6. *Let* $U = \{i,j,k,l,m\}$, $U/R = \{\{i,j\},\{k\},\{l,m\}\}$ $X=\{k,l\}\subseteq U$.
$SJ_R(X)=\{\Phi,U,\{j\},\{k\},\{m\},\{i,m\},\{j,m\},\{k,m\},\{j,k\},\{j,l\},\{k,l\},\{l,m\},\{i,j,m\},\{i,k,m\},$
$\{i,l,m\},\{j,k,m\},\{j,l,m\},\{j,k,l\},\{k,l,m\},\{i,j,k,m\},\{i,j,l,m\},\{i,k,l,m\},\{j,k,l,m\}\}$.
SJ-$R^{\#}\alpha$-LC$(U,X) = \{\Phi,U,$
$\{j\},\{k\},\{l\},\{i,m\},\{j,k\},\{j,l\},\{k,m\},\{i,j,m\},\{i,k,m\},\{i,l,m\},$
$\{j,k,l\},\{i,j,k,m\},\{i,j,l,m\},\{i,k,l,m\}\}$
Here $\{j,l,m\}$ *is in SJ-*$R^{\#}\alpha$*-LC (U, X) but not in SJ-*$R^{\#}\alpha$*-CL (U, X).*

Definition 3.7. *A subset* A *of a Selje topological space* $(U, \mu_R(X), SJ_R(X))$ *is called a SJ-r*α*-locally closed (briefly SJ-r*α*-lc) set if* A = B ∩ C *where* B *is SJ-r*α*-open and* C *is SJ-r*α*-closed.*
The family of all SJ-rα*-locally closed sets in a Selje topological space is denoted by SJ-r*α*-LC(U, X).*
Definition 3.8. *A space (X ,T) is called SJ-*$R^{\#}\alpha$*-* $T_R\#_\alpha$*-space if every SJ-*$R^{\#}\alpha$*-closed is SJ-regular closed.*

5. Characteristics of locally closed sets

Theorem 4.1. *Every SJ-*$R^{\#}\alpha$*-lc set is SJ-Locally closed when* $SJ_R(X)$ *is SJ-*$R^{\#}\alpha$*-*$T_R\#_\alpha$*-space.*
Proof: Let A be a SJ-$T_R\#_\alpha$-lc set then A = B ∩ C where B is SJ-$T_R\#_\alpha$-open and C is SJ-$T_R\#_\alpha$-closed. Since $SJ_R(X)$ is SJ-$R^{\#}\alpha$-$T_R\#_\alpha$- space, SJ-$R^{\#}\alpha$-closed set is SJ-rα-closed and SJ-$R^{\#}\alpha$-open set is SJ-rα-open. Hence, A is SJ-Locally closed. □

Remark 4.2. *The converse of the above theorem need not to be true and is shown in the following example.*

Example 4.3. *Let* $U = \{1,2,3,4,5\}$, $U/R = \{\{1,2\},\{3\},\{4,5\}\}$ $X=\{3,4\}\subseteq U$.
$SJ_R(X)=\{\Phi,U,\{2\},\{3\},\{5\},\{1,5\},\{2,5\},\{3,5\},\{2,3\},\{2,4\},\{3,4\},\{4,5\},\{1,2,5\},\{1,3,5\},\{1,4,5\},\{2,3,5\},\{2,4,5\},\{2,3,4\},\{3,4,5\},\{1,2,3,5\},\{1,2,4,5\},\{1,3,4,5\},\{2,3,4,5\}\}$
SJ-$R^{\#}\alpha$-LC$(U,X) =$
$\{\Phi,U,\{2\},\{3\},\{5\},\{1,5\},\{2,5\},\{3,5\},\{2,3\},\{2,4\},\{3,4\},$
$\{4,5\},\{1,2,5\},\{1,3,5\},\{1,4,5\},\{2,3,5\},\{2,4,5\},\{2,3,4\},$
$\{3,4,5\},\{1,2,3,5\},\{1,2,4,5\},\{1,3,4,5\}\}$
Here $\{2,3,4,5\}$ *is inSJ-*$R^{\#}\alpha$*-CL (U, X) but not in SJ-*$R^{\#}\alpha$*-LC (U, X).*

Definition 4.3. *A subset* A *of a Selje topological space* $(U, \mu_{\mathrm{R}}(\mathrm{X}), SJ_{\mathrm{R}}(\mathrm{X}))$ *is called*

1. *SJ-rα-lc* set if* $\mathrm{A} = \mathrm{B} \cap \mathrm{E}$ *where* B *is SJ-rα-open and* C *is SJ-closed.*
2. *SJ-rα-lc** set if* $\mathrm{A} = \mathrm{B} \cap \mathrm{E}$ *where* B *is open and* C *is SJ-rα-closed.*
3. *SJ-$R^\#\alpha$-lc* set if* $\mathrm{A} = \mathrm{B} \cap \mathrm{E}$ *where* B *is* $R^\#\alpha$*-open and* C *is closed.*
4. *SJ-$R^\#\alpha$-lc** set if* $\mathrm{A} = \mathrm{B} \cap \mathrm{E}$ *where* B *is open and* C *is* $R^\#\alpha$*-closed.*

The family of all SJ-rα-lc*(resp. SJ-rα-lc**, SJ-$R^\#\alpha$-lc*, SJ-$R^\#\alpha$-lc**) sets in a Selje topological space is denoted by SJ-rα-LC *(U, X)(resp. SJ-rα-LC **(U, X), SJ-$R^\#\alpha$-LC *(U, X), SJ-$R^\#\alpha$-LC **(U, X)).

Remark 4.4.

1. *Every SJ-rα-lc set need not to be SJ-$R^\#\alpha$-lc * set.*
2. *Every SJ-rα-lc set need not to be SJ-$R^\#\alpha$-lc ** set.*
3. *Every SJ-rα-lc* set need not to be SJ-$R^\#\alpha$-lc set.*
4. *Every SJ-rα-lc** set need not to be SJ-$R^\#\alpha$-lc set.*

These implications are shown in the following example.

Example 4.5. *Let* $U = \{1,2,3,4,5\}$, $U/R = \{\{1,2\},\{3\},\{4,5\}\}$ $\mathrm{X}=\{3,4\}\subseteq U$.
$SJ_{\mathrm{R}}(\mathrm{X})=\{\Phi,U,\{2\},\{3\},\{5\},\{1,5\},\{2,5\},\{3,5\},\{2,3\},\{2,4\},\{3,4\},\{4,5\},\{1,2,5\},\{1,3,5\},$
$\{1,4,5\},\{2,3,5\},\{2,4,5\},\{2,3,4\},$
$\{3,4,5\},\{1,2,3,5\},\{1,2,4,5\},\{1,3,4,5\},\{2,3,4,5\}\}$

1. *Here* $\{2,3,4,5\} \in$ *SJ-$R^\#\alpha$-LC * but* $\{2,3,4,5\} \notin$ *SJ-LC.*
2. *Here* $\{5\} \in$ *SJ-$R^\#\alpha$-LC ** but* $\{5\} \notin$ *SJ-LC set.*
3. *Here* $\{5\} \in$ *SJ-$R^\#\alpha$-LC set but* $\{5\} \notin$ *SJ-$R^\#\alpha$-LC * set.*
4. *Here* $\{2,3,4,5\} \in$ *SJ-$R^\#\alpha$-LC set but* $\{2,3,4,5\} \notin$ *SJ-$R^\#\alpha$-LC ** set.*

Theorem 4.6. *For a subset* A *of* $(U, \mu_{\mathrm{R}}(\mathrm{X}), SJ_{\mathrm{R}}(\mathrm{X}))$, *the following statements are equivalent:*

- $\mathrm{A} \in SJ\text{-}R^\#\alpha\text{-}LC(U, \mathrm{X})$.
- $\mathrm{A} = V \cap SJ\text{-}R^\#\alpha\text{-}cl(\mathrm{A})$ *for some SJ-$R^\#\alpha$-open set* V.
- $SJ\text{-}R^\#\alpha\text{-}cl(\mathrm{A}) - \mathrm{A}$ *is SJ-$R^\#\alpha$-closed.*
- $\mathrm{A} \cup [SJ\text{-}R^\#\alpha\text{-}cl(\mathrm{A})]^c$ *is SJ-$R^\#\alpha$-open.*
- $\mathrm{A} \subseteq SJ\text{-}R^\#\alpha\text{-}int[\mathrm{A} \cup (SJ\text{-}R^\#\alpha\text{-}cl(\mathrm{A}))^c]^c$.

Proof:

(1) ⇒ (2) Let $\mathrm{A} \in \mathrm{SJ}\text{-}R^\#\alpha\text{-LC}(U, \mathrm{X})$. Then $\mathrm{A} = V \cap W$, where V is SJ-$R^\#\alpha$-open and W is SJ-$R^\#\alpha$-closed. Since $\mathrm{A} \subseteq W$, we have SJ-$R^\#\alpha$-cl(A) $\subseteq W$, so $V \cap$ SJ-$R^\#\alpha$-cl(A) $\subseteq V \cap W = \mathrm{A}$. Also, since $\mathrm{A} \subseteq V$ and $\mathrm{A} \subseteq$ SJ-$R^\#\alpha$-cl(A), we get $\mathrm{A} \subseteq V \cap$ SJ-$R^\#\alpha$-cl(A). Hence, $\mathrm{A} = V \cap$ SJ-$R^\#\alpha$-cl(A).

(2) ⇒ (3) If $\mathrm{A} = V \cap$ SJ-$R^\#\alpha$-cl(A), thenSJ-$R^\#\alpha$-cl(A) $-$ A $=$ SJ-$R^\#\alpha$-cl(A) $\cap$

V^c,
which is SJ-R$^{\#}\alpha$-closed. Since V^c is SJ-$R^{\#}\alpha$-closed and SJ-$R^{\#}\alpha$-cl(A) is SJ-$R^{\#}\alpha$-closed. Hence, SJ-$R^{\#}\alpha$-cl(A) $-$ A is also SJ-$R^{\#}\alpha$-cl(A).

(3)$\Rightarrow$(4) A$\cup$[SJ-$R^{\#}\alpha$-cl(A)]^c = (SJ-$R^{\#}\alpha$-cl(A) $-$ A)^c and by assumption (SJ-$R^{\#}\alpha$-cl(A) $-$ A)^c

is SJ-$R^{\#}\alpha$-open and so is A $\cup$ [SJ-$R^{\#}\alpha$-cl(A)]^C

$\Rightarrow$(5) By assumption, A$\cup$ [SJ-$R^{\#}\alpha$-cl(A)]^C = SJ-$R^{\#}\alpha$-int (A $\cup$ [SJ-$R^{\#}\alpha$-cl(A)])^C

(3) hence A $\subseteq$ SJ-$R^{\#}\alpha$-int (A $\cup$ [SJ-$R^{\#}\alpha$-cl(A)]^C .

(4) $\Rightarrow$ (1) By assumption and since A $\subseteq$ SJ-$R^{\#}\alpha$-cl(A), we have A = SJ-$R^{\#}\alpha$-int (A $\cup$ [SJ-$R^{\#}\alpha$-cl(A)], and hence A $\in$ SJ-$R^{\#}\alpha$-LC(U, X).

Theorem 4.7. *For a subset* (U, μ_{R}(X), SJ_{R}(X)), *the following statements are equivalent:*

1. A $\in$ *SJ-R*$^{\#}\alpha$*-LC**(U, X).
2. A = V $\cap$ *SJ-cl*(A) *for some SJ-R*$^{\#}\alpha$*-open set* V.
3. *SJ-cl*(A) $-$ A *is SJ-R*$^{\#}\alpha$*-closed.*
4. A $\cup$ [*SJ-cl*(A)]^c *is SJ-R*$^{\#}\alpha$*-open.*

Proof:

(1) $\Rightarrow$(2) Let A $\in$ SJ-$R^{\#}\alpha$-LC*(U, X). Then A = V $\cap$ W, where V is SJ-$R^{\#}\alpha$-open and W is SJ-closed. Since A $\subseteq$ V, A $\subset$ SJ-cl (A),A $\subset$ V $\cap$ SJ-cl (A). Also, since SJ-$R^{\#}\alpha$-cl (A) $\subset$ W, V $\cap$ SJ-cl (A) $\subset$ V $\cap$ W =A. ThereforeA= V $\cap$ SJ-cl (A).

(2) $\Rightarrow$ (1) Since V is SJ-$R^{\#}\alpha$-open and SJ-cl(A) is SJ-closed, A = V $\cap$ SJ-cl(A) $\in$SJ-$R^{\#}\alpha$-LC*(U, X).

(2) $\Rightarrow$(3) Since SJ-cl (A) - A = SJ-cl (A) $\cap$ V^c, SJ-cl (A) - A is SJ-$R^{\#}\alpha$-closed, since V^c is SJ-$R^{\#}\alpha$-closed.

(3) $\Rightarrow$ (2) Let V = [SJ-cl(A) $-$ A]^c. Then V is SJ-$R^{\#}\alpha$-open, and clearly A = V $\cap$SJ-cl(A).

(3) $\Rightarrow$ (4) Let W = SJ-cl(A) $-$ A.
Then: W^c = A $\cup$ [SJ-cl(A)]^c , which is SJ-$R^{\#}\alpha$-open.

(4) $\Rightarrow$ (3) Let V = A $\cup$ [SJ-cl(A)]^c.
Then V^c is SJ-$R^{\#}\alpha$-closed and V^c = SJ-cl(A) $-$ A,
which is SJ-$R^{\#}\alpha$-closed.

□

Theorem 4.8. *Let* A *be a subset of* (U, μ_{R}(X), SJ_{R}(X)). *Then* A $\in$ *SJ-R*$^{\#}\alpha$*-LC***(U, X) *if and only if* A = V $\cap$ *SJ-R*$^{\#}\alpha$*-cl*(A) *for some SJ-open set* V .

Proof: Suppose A $\in$ SJ-$R^{\#}\alpha$-LC**(U, X). Then A = V $\cap$ W, where V is SJ-open and $W$is SJ-$R^{\#}\alpha$-closed. Since A $\subseteq$ W, we have SJ-$R^{\#}\alpha$-cl(A) $\subseteq$ W, which gives

A = A$\cap$SJ-$R^{\#}\alpha$-cl(A) = $V \cap W \cap$SJ-$R^{\#}\alpha-cl-$(A) = $V \cap$SJ-$R^{\#}\alpha-cl-$(A).
Conversely, suppose V is SJ-open and SJ-$R^{\#}\alpha$-cl(A) is SJ-$R^{\#}\alpha$-closed.
Then A = $V \cap$ SJ-$R^{\#}\alpha$-cl(A) $\in$ SJ-$R^{\#}\alpha$-LC**(U, X). $\square$

Corollary 4.9. *Let* A *be a subset of* (U, μ_R(X), SJ_R(X)). *If* A $\in$ *SJ-$R^{\#}\alpha$-LC***(U, X), *then SJ-$R^{\#}\alpha$-cl*(A) − A *is SJ-$R^{\#}\alpha$-closed, and* A $\cup$ [*SJ-$R^{\#}\alpha$-cl*(A)]^{C} *is SJ-$R^{\#}\alpha$-open.*

Proof: A $\in$ SJ-$R^{\#}\alpha$-LC**(U, X). By above theorem, there exists a SJ-open set V such that A = $V \cap$ SJ-$R^{\#}\alpha$-cl(A). Then,SJ-$R^{\#}\alpha$-cl(A) − A = SJ-$R^{\#}\alpha$-cl(A) $\cap$ V^{c}, which is SJ-$R^{\#}\alpha$-closed.

Let W = SJ-$R^{\#}\alpha$-cl(A) − A. Then W^{c} = A $\cup$ [SJ-$R^{\#}\alpha$-cl(A)]^{C} , and W^{c} is SJ-$R^{\#}\alpha$ open and so is A $\cup$ [SJ-$R^{\#}\alpha$-cl(A)]^{C} $\square$

Theorem 4.10. *Suppose that* (U, μ_R(X), SJ_R(X)) *is under finite union of SJ-$R^{\#}\alpha$-lc-closed sets.*
Let A *and* B *be SJ-$R^{\#}\alpha$-lc-closed. If* A *and* B *are separated, then* A $\cup$ B *is SJ-$R^{\#}\alpha$-lc-closed.*

Proof: Since A and B are SJ-$R^{\#}\alpha$-lc-closed, then A = Q $\cap$ SJ-$R^{\#}\alpha$-*cl*(A) and B = W $\cap$ SJ-$R^{\#}\alpha$-*cl*(B), where Q,W are open set of X .
Now, put U = Q $\cap$ (X - cl(B)) and V = W $\cap$ (X - SJ-cl(A)).
So, U $\cap$ SJ-$R^{\#}\alpha$-cl(A) = (Q $\cap$ (X-SJ-cl(B))) $\cap$ SJ-$R^{\#}\alpha$-cl(A) = A $\cap$ (X-SJ-cl(B)) = A,
since A $\subseteq$ X-cl(B). Similarly, V $\cap$ SJ-$R^{\#}\alpha$-cl(B) = B and
U $\cap$ SJ-$R^{\#}\alpha$-cl(B) $\subseteq$ U $\cap$ SJ-cl(B) = Φ and V $\cap$ SJ-$R^{\#}\alpha$-cl(A) $\subseteq$ V $\cap$ SJ-cl(A) = Φ.
Since U and V are open, it has
(U $\cap$ V) $\cap$ SJ-$R^{\#}\alpha$-cl(A $\cup$ B) = (U $\cup$ V) $\cap$ (SJ-$R^{\#}\alpha$-cl(A) $\cup$ SJ-$R^{\#}\alpha$-cl(B))
= (U $\cap$ SJ-$R^{\#}\alpha$-cl(A)) $\cup$ (U $\cap$ SJ-$R^{\#}\alpha$-cl(B)) $\cup$ (V SJ-$R^{\#}\alpha$-cl(A)) $\cup$ (V $\cap$ SJ-$R^{\#}\alpha$-cl(B))= A $\cup$ B. Therefore, A $\cup$ B is SJ-$R^{\#}\alpha$-lc-closed. $\square$

Definition 4.11. *A function* $f : (U, SJ_R(X)) \rightarrow (V, SJ_R(Y))$ *is said to be SJ-$R^{\#}\alpha$ LC-continuous (respectively, SJ-$R^{\#}\alpha$ LC*-continuous, SJ-$R^{\#}\alpha$ LC**-continuous) if* $f^{-1}(G) \in SJ-R^{\#}\alpha LC(U, SJ_R(X))$ *(respectively,* $f^{-1}(G) \in SJ-R^{\#}\alpha LC^{*}(U, SJ_R(X))$, $f^{-1}(G) \in SJ-R^{\#}\alpha LC^{**}(U, SJ_R(X))$*) for each Selje closed set G in* (V, SJ_R(Y)).

Theorem 4.12. *Let* $f : (U, SJ_R(X)) \rightarrow (V, SJ_R(Y))$ *be a function. Then we have the following:*

1) *If f is SJ-LC-continuous, then f is SJ-$R^{\#}\alpha$ LC-continuous, SJ-$R^{\#}\alpha$ LC*-continuous, and SJ-$R^{\#}\alpha$ LC**-continuous.*
2) *If f is SJ-$R^{\#}\alpha$ LC*-continuous, then f is SJ-$R^{\#}\alpha$ LC-continuous.*
3) *If f is SJ-$R^{\#}\alpha$ LC**-continuous, then f is SJ-$R^{\#}\alpha$ LC-continuous.*

Proof: (1) Suppose $f : (U, SJ_R(X)) \longrightarrow (V, SJ_R(Y))$ is SJ-LC-continuous. Let G be a Selje closed set in U. Then $f^{-1}(G)$ is a Selje locally closed set in U. By above theorem, it follows that f is SJ-$R^{\#}\alpha$ LC-continuous (respectively, SJ-$R^{\#}\alpha$ LC^{*}-continuous and SJ-$R^{\#}\alpha$ LC^{**}-continuous).

(2) Let $f : (U, SJ_R(X)) \longrightarrow (V, SJ_R(Y))$ be a SJ-$R^{\#}\alpha$ LC^{*}-continuous function. Let G be a Selje closed set of U. Then $f^{-1}(G)$ is a SJ-$R^{\#}\alpha$ LC^{*} set in U. By Theorem, it follows that f is SJ-$R^{\#}\alpha$ LC^{*}-continuous implies SJ-$R^{\#}\alpha$ LC-continuous.

(3) Let $f : (U, SJ_R(X)) \longrightarrow (V, SJ_R(Y))$ be a SJ-$R^{\#}\alpha$ LC^{**}-continuous function. Let G be a Selje closed set of U. Then $f^{-1}(G)$ is a SJ-$R^{\#}\alpha$ LC^{**} set in U. By Theorem, it follows that SJ-$R^{\#}\alpha$ LC^{**}-continuity implies SJ-$R^{\#}\alpha$ LC-continuity.

Remark 4.13. *The converse of the above theorem need not be true as seen from the following examples.*

Example 4.14.

1.Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$, $X = \{a, d\}$ and $V = \{a, b, c, d\}$ with $V/R = \{\{c\}, \{d\}, \{a, b\}\}$, $Y = \{b, c\}$. Define a function $f : (U, SJ_R(X)) \longrightarrow (V, SJ_R(Y))$ by f(a) = b, f(b) = a,f(c) = d, f(d) = c. Then f is SJ-$R^{\#}\alpha$ LC-continuous, SJ-$R^{\#}\alpha$ LC^{*}-continuous, and SJ-$R^{\#}\alpha$ LC^{**}-continuous, but not SJ-LC-continuous.

2. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b, c, d\}\}$, $X = \{a, d\}$ and $V = \{a, b, c, d\}$ with $V/R = \{\{b\}, \{c\}, \{a, d\}\}$, $Y = \{b, c\}$. Define a function $f : (U, SJ_R(X)) \longrightarrow (V, SJ'_R(Y))$ by f(a) = a, f(b) = b, f(c) = c, f(d) = d. Then f is SJ-$R^{\#}\alpha$ LC-continuous but not SJ-$R^{\#}\alpha$ LC^{*}-continuous.

3. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b, c, d\}\}$, $X = \{b, c\}$ and $V = \{a, b, c, d\}$ with $V/R = \{\{c\}, \{d\}, \{a, b\}\}$, $Y = \{b, c\}$. Define a function $f : (U, SJ_R(X)) \longrightarrow (V, \tau'_R(Y))$ by f(a) = a, f(b) = b, f(c) = d,f(d) = c. Then f is SJ-$R^{\#}\alpha$ LC-continuous but not SJ-$R^{\#}\alpha$ LC^{**}-continuous.

Theorem 4.15. *Let $f : (U, SJ_R(X)) \longrightarrow (V, SJ_R(Y))$ be two functions. Then:*

i. *$g \circ f$ is SJ-$R^{\#}\alpha$ LC-continuous if f is SJ-$R^{\#}\alpha$ LC-continuous and g is Selje continuous.*

ii. *$g \circ f$ is SJ-$R^{\#}\alpha$ LC^{*}-continuous if f is SJ-$R^{\#}\alpha$ LC^{*}-continuous and g is Selje continuous.*

iii. *$g \circ f$ is SJ-$R^{\#}\alpha$ LC^{**}-continuous if f is SJ-$R^{\#}\alpha$ LC^{**}-continuous and g is Selje continuous.*

Proof: (i) Let G be a Selje closed set in $(W, SJ''_R(Y))$. Since g is Selje continuous, $g^{-1}(G)$ is a Selje closed set in $(V, SJ'_R(Y))$. Since f is SJ-$R^{\#}\alpha$ LC-continuous, $f^{-1}(g^{-1}(G))$ is in SJ-$R^{\#}\alpha$ LC in $(U, SJ_R(X))$. Thus $g \circ f$ is SJ-$R^{\#}\alpha$ LC-continuous.

(ii) Let G be a Selje closed set in $(W, SJ''_R(Y))$. Since g is Selje continuous,

$g^{-1}(G)$ is Selje closed in $(V, SJ'_R(\mathrm{Y}))$. Since f is SJ-$R^{\#}\alpha$ LC^{*}-continuous, $f^{-1}(g^{-1}(G))$ is in SJ-$R^{\#}\alpha$ LC^{*} in $(U, SJ_R(\mathrm{X}))$. Thus $g \circ f$ is SJ-$R^{\#}\alpha$ LC^{*}-continuous.

(iii) Let G be a Selje closed set in $(W, SJ''_R(\mathrm{Z}))$. Since g is Selje continuous, $g^{-1}(G)$ is Selje closed in $(V, SJ'_R(\mathrm{Y}))$.Since f is SJ-$R^{\#}\alpha$ LC^{**}-continuous,$f^{-1}(g^{-1}(G))$ is $SJ-R^{\#}\alpha$ LC^{**} in $(U, SJ_R(\mathrm{X}))$. Thus $g \circ f$ is $SJ-R^{\#}\alpha$ LC^{**}-continuous function.

Definition 4.16. *A function $f : (U, SJ_R(\mathrm{X})) \longrightarrow SJ'_R(\mathrm{Y}))$ is said to be $SJ - R^{\#}\alpha$ LC-irresolute (resp. SJ-$R^{\#}\alpha$ LC^{*}-irresolute, $SJ - R^{\#}\alpha$ LC^{**}-irresolute) if the inverse image of SJ-$R^{\#}\alpha$ LC set (resp. $SJ - R^{\#}\alpha$ LC^{*} set, $SJ - R^{\#}\alpha$ LC^{**} set) in $(V, SJ'_R(\mathrm{Y}))$ is a $SJ - R^{\#}\alpha$ LC set (resp. $SJ - R^{\#}\alpha$ LC^{*}, $SJ - R^{\#}\alpha$ LC^{**} set) in $(U, SJ_R(\mathrm{X}))$.*

Theorem 4.17. *If a function $f : (U, SJ_R(\mathrm{X})) \longrightarrow (V, SJ'_R(\mathrm{Y}))$ is SJ-LC-irresolute, then f is $SJ - R^{\#}\alpha LC$ -irresolute (resp. $SJ - R^{\#}\alpha$ LC^{*}-irresolute and $SJ - R^{\#}\alpha$ LC^{**}-irresolute).*

Proof: *Suppose* that f is SJ-LC-irresolute. Let G be a Selje locally closed set of $(U, SJ_R(\mathrm{X}))$. Then $f^{-1}(G)$ is a Selje locally closed set in $(U, SJ_R(\mathrm{X}))$. By Theorem, it follows that f is $SJ - R^{\#}\alpha$LC -irresolute (resp. $SJ - R^{\#}\alpha$ LC^{*}-irresolute and $SJ - R^{\#}\alpha$ LC^{**}-irresolute).

Remark 4.18. *The converse of the above theorem need not be true as seen from the following examples.*

Example 4.19. *Let $\{a, b, c, d\}$ with $U/R = \{\{a\}, \{b, c, d\}\}$ and $X = \{a, c\}$. Let $V = \{a, b, c, d\}$ with $V/R' = \{\{a\}, \{b\}, \{c, d\}\}$ and $Y = \{a, d\}$. Define a function $f : (U, SJ_R(\mathrm{X})) \longrightarrow (V, SJ_R(\mathrm{X}))$ as $f(a) = c$, $f(b) = d$, $f(c) = a$, and $f(d) = b$. Then f is $SJ - R^{\#}\alpha LC$ -irresolute, $SJ - R^{\#}\alpha$ LC^{*}-irresolute, $SJ - R^{\#}\alpha$ LC^{**}-irresolute but not SJ-LC-irresolute.*

Theorem 4.20. *Let $f : (U, SJ_R(\mathrm{X})) \longrightarrow (V, SJ_R(\mathrm{Y}))'$ and $g : (V, SJ_R(\mathrm{Y}))' \longrightarrow (W, SJ_R(\mathrm{Z}))''$ be any two functions. Then*

1. *$g \circ f : (U, SJ_R(\mathrm{X})) \longrightarrow (W, SJ_R(\mathrm{Z}))$ is $SJ - R^{\#}\alpha LC$ -irresolute if g is $SJ - R^{\#}\alpha LC$-irresolute and f is $SJ - R^{\#}\alpha LC$ -irresolute.*
2. *$g \circ f : (U, SJ_R(\mathrm{X})) \longrightarrow (W, SJ_R(\mathrm{Z}))$ is $SJ - R^{\#}\alpha LC$ -continuous if g is $SJ - R^{\#}\alpha LC$-continuous and f is $SJ - R^{\#}\alpha LC$ -irresolute.*

Proof:

1. Let G $\in SJ - R^{\#}\alpha$ LC $(W, SJ''_R(\mathrm{Z})))$. Since g is $SJ - R^{\#}\alpha$LC -irresolute,

$g^{-1}(G)$ is $SJ - R^{\#}\alpha$LC in $(V, SJ'_R(\mathrm{Y}))$. As f is $SJ - R^{\#}\alpha$LC -irresolute, $f^{-1}(g^{-1}(G))$ is $SJ - R^{\#}\alpha$LC in $(U, SJ_R(\mathrm{X}))$. That is $(g \circ f)^{-1}(G) \in SJ - R^{\#}\alpha$LC $(U, SJ_R(\mathrm{X}))$.
Thus $g \circ f$ is $SJ - R^{\#}\alpha$LC -irresolute.

2. Let G be a Selje closed set in $(W, SJ''_R(\mathrm{Z}))$. Since g is $SJ - R^{\#}\alpha$LC -continuous, $g^{-1}(G)$ is $SJ - R^{\#}\alpha$LC in $(V, SJ'_R(\mathrm{Y}))$. Again, since f is $SJ - R^{\#}\alpha$LC -irresolute, $f^{-1}(g^{-1}(G))$ is $SJ - R^{\#}\alpha$LC in $(U, SJ_R(\mathrm{X}))$. Thus $g\circ f$ is $SJ - R^{\#}\alpha$ LC -continuous.

Theorem 4.21. *$f : (U, SJ_R(\mathrm{X})) \to (V, SJ_R(\mathrm{Y}))$ and $g : (V, SJ_R(\mathrm{Y})) \to (W, SJ_R(\mathrm{Z}))$ be any two functions. Then:*

- i. *$g \circ f$ is $SJ - R^{\#}\alpha$ LC^{*}-irresolute if f and g are $SJ - R^{\#}\alpha$ LC^{*}-irresolute.*
- ii. *$g \circ f$ is $SJ - R^{\#}\alpha$ LC^{**}-irresolute if f and g are $SJ - R^{\#}\alpha$ LC^{**}-irresolute.*
- iii. *$g \circ f$ is $SJ - R^{\#}\alpha$ LC^{*}-continuous if f is $SJ - R^{\#}\alpha$ LC^{*}-irresolute and g is $SJ - R^{\#}\alpha$ LC^{*}-continuous.*
- iv. *$g \circ f$ is $SJ - R^{\#}\alpha$ LC^{**}-continuous if f is $SJ - R^{\#}\alpha$ LC^{**}-irresolute and g is $SJ - R^{\#}\alpha$ LC^{**}-continuous.*

Proof.

(i) Let G $\in SJ - R^{\#}\alpha$ LC^{*} $(W, SJ''_R(\mathrm{Z})))$. Since g is $SJ - R^{\#}\alpha$ LC^{*}-irresolute, $g^{-1}(G)$ is $SJ - R^{\#}\alpha$ LC^{*} in $(V, SJ'_R(\mathrm{Y}))$. As f is $SJ - R^{\#}\alpha$ LC^{*}-irresolute, $f^{-1}(g^{-1}(G))$ is $SJ - R^{\#}\alpha$ LC^{*} in $(U, SJ_R(\mathrm{X}))$. That is, $(g \circ f)^{-1}(G) \in SJ - R^{\#}\alpha$ $LC^{*}(U, SJ_R(\mathrm{X})$.
Thus $g \circ f$ is $SJ - R^{\#}\alpha$ LC^{*}-irresolute.

(ii) Let $G \in SJ - R^{\#}\alpha$ $LC^{**}(W, SJ''_R(\mathrm{Z})))$. Since g is $SJ - R^{\#}\alpha$ LC^{**}-irresolute, $g^{-1}(G)$ is $SJ - R^{\#}\alpha$ LC^{**} in $(V, SJ'_R(\mathrm{Y}))$. As f is $SJ - R^{\#}\alpha$ LC^{**}-irresolute, $f^{-1}(g^{-1}(G))$ is $SJ - R^{\#}\alpha$ LC^{**} in $(U, SJ_R(\mathrm{X}))$. Hence $g \circ f$ is $SJ - R^{\#}\alpha$ LC^{**}-irresolute.

(iii) Let G be a Selje closed set in $(W, SJ''_R(\mathrm{Z})))$. Since g is $SJ - R^{\#}\alpha$ LC^{*}-continuous, $g^{-1}(G)$ is $SJ - R^{\#}\alpha$ LC^{*} in $(V, SJ'_R(\mathrm{Y}))$. Again, since f is $SJ - R^{\#}\alpha$ LC^{*}-irresolute,
$f^{-1}(g^{-1}(G))$ is $SJ - R^{\#}\alpha$ LC^{*} in $(U, SJ_R(\mathrm{X}))$. Hence $g\circ f$ is $SJ - R^{\#}\alpha$ LC^{*}-continuous.

(iv) Let G be a Selje closed set in $(W, SJ''_R(\mathrm{Z})))$. Since g is $SJ - R^{\#}\alpha$ LC^{**}-continuous, $g^{-1}(G)$ is $SJ - R^{\#}\alpha$ LC^{**} in $(V, SJ'_R(\mathrm{Y}))$. Since f is $SJ - R^{\#}\alpha$ LC^{**}-irresolute,
$f^{-1}(g^{-1}(G))$ is $SJ - R^{\#}\alpha$ LC^{**} in $(U, SJ_R(\mathrm{X}))$. Hence $g\circ f$ is $SJ - R^{\#}\alpha$ LC^{**}-continuous.

6. Conclusion and future work

In this paper, a comprehensive decomposition framework has been developed within the setting of Selje Topological Spaces, contributing significantly to the advancement of relational topology. By introducing the new classes of SJ-locally closed sets and SJ-R#α-locally closed sets, the study successfully extends the classical notion of locally closed sets into a more generalized and relationally structured environment. This extension not only fills a notable gap in the existing literature but also provides a unified perspective that connects local decomposition behavior with the intrinsic relational mechanisms that define Selje topology.

The structural properties of these newly defined classes have been rigorously investigated using Selje closure and interior operators, leading to precise characterizations that strengthen their theoretical validity. In addition, inclusion relationships among various classes of generalized closed sets, particularly Selje closed sets, R#α-closed sets and the introduced locally closed variants, have been systematically established. These relationships reveal a refined hierarchical organization that enhances the understanding of structural dependencies within Selje spaces.

A significant contribution of this work lies in the introduction of SJ-R#α-LC continuous mappings, which provide a functional framework for analyzing the behavior of these decomposition structures under mappings. The preservation properties under inverse images and compositions demonstrate that the proposed concepts are not only structurally consistent but also functionally robust. This dual structural-functional perspective adds depth to the decomposition theory and aligns it with broader developments in generalized continuity and mapping theory.

Moreover, the results presented in this study highlight the flexibility and adaptability of Selje Topological Spaces as a powerful extension of classical topology. By integrating relational characteristics with decomposition principles, the framework offers improved analytical capabilities for handling complex systems where classical topological tools may be insufficient. This positions Selje topology as a promising area for further theoretical exploration and interdisciplinary applications.

Despite these advancements, the present work opens several promising directions for future research. One immediate extension would be to study separation axioms in the context of SJ-locally closed and SJ-R#α-locally closed structures, which may lead to refined classifications of Selje spaces. Similarly, compactness and connectedness properties can be re-examined through the lens of these decomposition classes to obtain deeper insights into global structural behavior.

Another important direction is the exploration of stronger and weaker forms of continuity associated with SJ-based decompositions. The development of new classes of mappings, such as SJ-irresolute, SJ-precontinuous or hybrid continuity types, may further enrich the functional framework of Selje topology. In addition, investigating the algebraic and categorical aspects of SJ-R#α-LC mappings could provide a more abstract and unified understanding of these structures, potentially linking Selje topology with category theory and algebraic topology.

From an applied perspective, integrating the proposed decomposition framework with computational and numerical methods represents a highly promising avenue. The incorporation of Selje topology into topological data analysis, machine learning models and network optimization problems could lead to innovative methodologies for analyzing complex datasets. In particular, applications in electricity network optimization, medical data analysis, and dynamic system modeling may benefit from the relational and decomposition-based insights provided by this framework.

Furthermore, the development of algorithmic approaches and visualization techniques for SJ-based structures can enhance their practical usability. Implementing these concepts using computational tools such as R or Python would facilitate simulation, modeling and real-time analysis, thereby bridging the gap between theoretical topology and real-world applications.

In conclusion, this work not only strengthens the theoretical foundation of Selje Topological Spaces but also establishes a robust decomposition framework that integrates structural and functional aspects coherently. The introduction of SJ-locally closed and SJ-R#α-locally closed sets, along with their associated mappings, provides a meaningful extension of classical topological concepts into a relational setting. The study thus lays a strong groundwork for future research, encouraging further exploration in both pure and applied domains of topology.

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