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On Fuzzy Strongly g^*-Closed Sets in Fuzzy Topological Spaces

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Abstract. In this paper, a new class of fuzzy sets called fuzzy strongly g^*-closed sets is introduced and its properties are investigated.

Keywords: fuzzy generalized closed sets , fuzzy g^*-closed sets and fuzzy strongly g^*-closed sets

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1. Introduction

In 1965, Zadeh [16] introduced the concept of fuzzy sets . Subsequently many researchers have been worked in this area and related areas which have applications in different field based on this concept. As a generalization of topological spaces Chang [6] introduced the concept of fuzzy topological space in 1968. g^*-closed sets were introduced and studied by Veerakumar [14] for general topology. Recently Parimelazhagan and Subramonia pillai introduced strongly g^*-closed sets in topological space [9].

In the present paper, we introduce fuzzy strongly g^*-closed sets in fuzzy topological space and investigate certain basic properties of these fuzzy sets.

2. Basic Concepts

A family τ of fuzzy sets of X is called a fuzzy topology on X if 0 and 1 belong to τ and τ is closed with respect to arbitrary union and finite intersection. [6]. The members of τ are called fuzzy open sets and their complements are fuzzy closed sets.

Throughout the present paper, (X,τ) or simply X mean fuzzy topological space (abbreviated as fts) on which no separation axioms are assumed unless otherwise mentioned. We denote and define the closure and interior for a fuzzy set A by $Cl(A) = \wedge\{\mu : \mu \geq A, 1-\mu \in \tau\}$ and $Int(A) = \vee\{\mu : \mu \leq A, \mu \in \tau\}$ respectively.

Definition 2.1. A fuzzy set A of (X,τ) is called fuzzy semiopen (in short , fs-open) if $A \le Cl\left(Int\left(A\right)\right)$ and a fuzzy semi closed (in short ,fs-closed)if $Int\left(Cl\left(A\right)\right) \le A$ [1].

Definition 2.2. A fuzzy set A of (X,τ) is called fuzzy preopen (in short , fp-open) if $A \le Int\left(Cl\left(A\right)\right)$ and a fuzzy pre-closed (in short , fp-closed) if $Cl\left(Int\left(A\right)\right) \le A$[3].

Definition 2.3. A fuzzy set A of (X,τ) is called fuzzy α-open (in short , fα-open) if $A \le Int\left(Cl\left(Int\left(A\right)\right)\right)$and a fuzzy α-closed (in short , fα-closed) if

$$Cl\left(Int\left(Cl\left(A\right)\right)\right) \le A \text{ [3].}$$

Definition 2.4. A fuzzy set A of (X,τ) is called fuzzy semi-preopen (in short , fsp-open) if $A \le Cl\,Int\left(Cl\left(A\right)\right)$ and a fuzzy semi-preclosed(in short , fsp-closed) if

$$Int\left(Cl\left(Int\left(A\right)\right)\right) \le A \text{ [13].}$$

Definition 2.5. A fuzzy set A of (X,τ) is called fuzzy θ -open (in short , f θ -open) if $A = Int_\theta(A)$ and a fuzzy θ -closed (in short , f θ -closed) if $A = Cl_\theta(A)$ where $Cl_\theta(A) = \wedge\left\{cl(\mu) : A \le \mu, \mu \in \tau\right\}$ [7].

The semi closure [15] (respectively pre closure[3] ,α-closure [13] and semi preclosure[14]) of a fuzzy set A of (X,τ) is the intersection of all fs-Closed (respectively fp-closed , fα-closed and fsp-closed) sets that contain A and is denotedby $sCl\left(A\right)$ (respectively $pCl(A)$, $\alpha Cl\left(A\right)$ and $sp\,Cl\left(A\right)$).

Definition 2.6. A fuzzy set A of (X,τ) is called fuzzy generalized closed (in short, fg-closed) if $Cl\,A \le H$, whenever $A \le H$ and H is fuzzy open set in X [4]

Definition 2.7. A fuzzy set A of (X,τ) is called fuzzy generalized fuzzy semi closed (in short , gfs-closed) if $sCl\left(A\right) \le H$, whenever $A \le H$ and H is fs-open set in X . This set is also called generalized fuzzy weakly semi closed set. [2]

Definition 2.8. A fuzzy set A of (X,τ) is called fuzzy generalized semi closed (in short , fgs-closed) if $sCl\left(A\right) \le H$ whenever $A \le H$ and H is fuzzy open set in X [13].

Definition 2.9. A fuzzy set A of (X,τ) is called fuzzy generalized pre closed (in short , fgp-closed) if $pCl(A)\leq H$ whenever $A\leq H$ and H is fuzzy open set in X [8].

Definition 2.10. A fuzzy set A of (X,τ) is called fuzzy α-generalized closed (in short , fαg – closed) if $\alpha Cl(A)\leq H$ whenever $A\leq H$ and H is fuzzy open set in X [11] .

Definition 2.11. A fuzzy set A of (X,τ) is called fuzzy Fgα-closed if $\alpha Cl(A)\leq H$ whenever H is fuzzy open set in X [12].

Definition 2.12. A fuzzy set A of (X,τ) is called fuzzy generalized semi pre closed (in short,fsp-closed) if $spCl(A)\leq H$ whenever $A\leq H$ and H is fuzzy open set in X [8]

Definition 2.13. A fuzzy set A of (X,τ) is called fuzzy semi-pre-generalized closed (in short, fspg-closed) if $spCl(A)\leq H$, whenever $A\leq H$ and H is fs-open in X [11]

Definition 2.14. A fuzzy set A of (X,τ) is called fuzzy θ-generalized closed (in short, fθg-closed) if $Cl_{\theta}(A)\leq H$, whenever $A\leq H$ and H is fuzzy open in X [7] .

Definition 2.15. A fuzzy set A of (X,τ) is called fuzzy g^{*}- closed (in short, fg^{*}-closed) if $Cl(A)\leq H$, whenever $A\leq H$ and H is fg- open in X [5] .

Definition 2.16. A fuzzy point $x_p\in A$ is said to be quasi-coincident with the fuzzy set A denoted by $x_p qA$ iff $P+A(x)>1$. A fuzzy set A is quasi-coincident with a fuzzy set B denoted by AqB iff there exists $x\in X$ such that $A(x)+B(x)>1$. If A and B are not quasi-coincident. Then we write $A\bar{q}B$. Note that $A\leq B\Leftrightarrow A\bar{q}(1-B)$. [10]

3. Fuzzy strongly g*-closed sets in fuzzy topological spaces

Definition 3.1. Let (X,τ) be a fuzzy topological space. A fuzzy set A of (X,τ) is called fuzzy strongly g*-closed if $Cl(Int(A))\leq H$ whenever $A\leq H$ and H is fg –open in X.

Theorem 3.1. Every fuzzy closed set is fuzzy strongly g*- closed set in a fuzzy topological space (X,τ).

Proof: Let A be fuzzy closed set in X. Let H be a fg- open set in X such that $A\leq H$ .Since A is fuzzy closed, $Cl(A)=A$.Therefore $Cl(A)\leq H$ .Now $Cl(Int(A))\leq Cl(A)\leq H$. Hence A is fuzzy strongly g*-closed set in X .

The converse of the above theorem need not be true in general which can be seen from the following example.

Example 3.1. Let $X = \{a,b,c\}$. Fuzzy sets A and B are defined by
A (a) = 0.7 , A(b)=0.3, A(c)=0.5; B(a)=0.2 , B(b) = 0.1 , B(c) = 0.3.
Let τ ={0, A,1}.
Then B is a fuzzy strongly g* - closed set but it is not a fuzzy closed set in (X,τ).

Theorem 3.2. Every fuzzy g - closed set is fuzzy strongly g *- closed sets in X.
Proof: Obvious

Converse of the above theorem need not be true it can be seen by the following example

Example 3.2. Let $X = \{a,b\}$ and the fuzzy sets A and B be defined as follows A (a) = 0.3 , A(b)=0.3; B(a)=0.5 , B(b) = 0.4. Let τ ={0, B,1}. Then A is fuzzy strongly g^*-closed but it is not fg-closed, since $Cl(B) \not\leq B$.

Theorem 3.3. Every fuzzy g* - closed set is a fuzzy strongly g *- closed sets in X.
Proof: Suppose that A is fg* - closed in X. Let H be a fg-open set in X such that $A \leq H$ .Then $Cl(A) \leq H$, since A is fg * - closed. Now $Cl(Int(A)) \leq Cl(A) \leq H$. Hence A is fuzzy strongly g* - closed set in X.

However the converse of the above theorem need not be true as seen from the following example:

Example 3.3. Let X={a,b}, τ = {0, A, B, D,1} and fuzzy sets A, B, D and H are defined as follows.
A(a) = 0.2 , A(b)=0.4; B(a) = 0.6 , B(b)=0.7; D(a)=0.4 , D(b)=0.6 H(a)=0.4, H(b)=0.5.
Then H is fuzzy strongly g* -closed set but it not fg*-closed in (X,τ), because $Cl(H) \not\leq D$ where as $H \leq D$ and D is fg-open.

Theorem 3.4. A fuzzy set A of fuzzy topological space (X,τ) is fuzzy strongly g* -closed iff A $\overline{q}$ B $\Rightarrow$ $Cl(Int(A))\overline{q}B$ for every fg -closed set B of X
Proof: Suppose that A is a fuzzy strongly g* - closed set of X such that A $\overline{q}$ B
Then $A \leq 1-B$ and $1-B$ is a fuzzy g- open set X. which implies that $Cl(Int(A)) \leq 1-B$, since A is fuzzy strongly g* -closed. Hence $Cl(Int(A))\overline{q}B$.

Conversely, let E be a fuzzy g- open let in X such that $A \leq E$. Then $A\overline{q}1-E$ and $1-E$ is fg- closed set in X. By hypothesis, $Cl(Int(A))\overline{q}(1-E)$ which implies $Cl(Int(A)) \leq E$. Hence A is fuzzy strongly g*- closed in X.

Theorem 3.5. Let A be fuzzy strongly g* - closed set in (X,τ) and x_p be a fuzzy point of (X,τ) such that $x_p q Cl(Int(A))$ then $Cl(Int(x_p))qA$

Proof: Let A be a fuzzy strongly g* - Closed set (X,τ) and x_p be a fuzzy point of (X,τ) such that $x_p q Cl(Int(A))$. Suppose $Cl(Int(x_p))\overline{q}A$, then $Cl(Int(x_p))q1-A$ and hence $A\leq 1-Cl(Int(x_p))$

Now, $1-Cl(Int(x_p))$ is fuzzy open. Moreover, since A is fuzzy strongly g*-closed, $Cl(Int(A))\leq 1-Cl(Int(x_p))\leq 1-x_p$ .Hence $x_p\overline{q}Cl(Int(A))$, which is a contradiction.

Theorem 3.6. If A is a fuzzy strongly g* - closed set in (X,τ) and $A\leq B\leq Cl(Int(A))$, then B is fuzzy strongly g *- closed in (X,τ).

Proof: Let A be a fuzzy strongly g* - closed set in (X,τ). Let $B\leq H$ where H is a fuzzy g-open se in X. Then $A\leq H$. Since A is fuzzy strongly g* - closed set, it follows that $Cl(Int(A))\leq H$.

Now $B\leq Cl(Int(A))$ implies $Cl(Int(B))\leq Cl(Int(Cl(Int(A))))=Cl(Int(A))$
We get, $Cl(Int(B))\leq H$. Hence, B is fuzzy strongly g* - closed set in (X,τ).

Definition 3.2. A fuzzy set A of (X,τ) is called fuzzy strongly g* - open set in X iff $1-A$ is fuzzy strongly g* - closed in X. In other words , A is fuzzy strongly g*-open iff $H\leq Cl(Int(A))$ whenever $H\leq A$ of H is fg closed in X.

Theorem 3.7. Let A be a fuzzy strongly g* - open in X and $Int(Cl(A))\leq B\leq A$ then B is fuzzy strongly g *-open in X.

Proof: Suppose that A is fuzzy strongly g* - open in X and $Int(Cl(A))\leq B\leq A$. Then 1-A is fuzzy strongly g* - closed in X and $1-A\leq 1-B\leq Cl(Int(1-A))$. Then by theorem 3.6, $1-B$ is fuzzy strongly g* - closed, Hence B is fuzzy strongly g*-open in X.

Theorem 3.8. Let (Y,τ_Y) be a subspace of a fuzzy topological space (X,τ) and A be a fuzzy set of Y. If A is fuzzy strongly g* - closed in X , then A is a fuzzy strongly g* - closed in Y.

Proof: Let Y be a subspace of X. Let H be a fg – open get in Y such that $A\leq H$
We have to prove that $Cl_y(Int_y(A))\leq H$ .Since H is fg-open in Y. We have H=G $\cap Y$ where G is fg- open in X. Hence $A\leq H=G\cap Y$ Implies $A\leq G$ and A is fuzzy strongly g*-open in X. We get $Cl(Int(A))\leq G$. Therefore $Cl(Int(A))\cap Y\leq G\cap Y=H$ .Thus $Cl(Int(A))\leq H$ whenever $A\leq H$ and H is fuzzy g – open in Y. Hence A is fuzzy strongly g^* – open in Y.

Theorem 3.9. If a fuzzy set A of a fuzzy topological space X is both fuzzy open and fuzzy strongly g * - closed then it is fuzzy closed.

Proof: Suppose that a fuzzy set A of X is both fuzzy open and fuzzy strongly g*-closed. Now $A \geq Cl(Int(A)) \geq Cl(A)$ .That is $A \geq Cl(A)$. Since $A \leq Cl(A)$. We get $A = Cl(A)$. Hence A is fuzzy closed in X.

Theorem 3.10. If a fuzzy set A of a fuzzy topological space X is both fuzzy strongly g * - closed and fuzzy semi-open then it is fg*-closed.
Proof: Suppose a fuzzy set A of X is both fuzzy strongly g* - closed and fuzzy semi-open in X. Let H be a fg-open set such that $A \leq H$. Since A is fuzzy strongly g*-closed, therefore $Cl(Int(A)) \leq H$. Also since A is fs-open, $A \leq Cl(Int(A))$.
We have $Cl(A) \leq Cl(Int(A)) \leq H$. Hence A is fg*-closed in X.

Theorem 3.11. Every f θ – closed set is a fuzzy strongly g * - closed set.
Proof: Obvious

The following example shows that the converse of the above theorem is not true in general.

Example 3.4. Let X={a,b}, τ ={0, A,1} and fuzzy sets A & B are defined as follows.
$A(a) = 0.3$, A(b)=0.7; B(a)=0.6, B(b)=0.5;

Then B is strongly g* - closed but it is not f θ- closed , because $Cl_{\theta}(B) = 1 \neq B$.

Observation: Every fp-closed, fsp-closed, gfs-closed, fg α-closed and fspg- closed sets are fuzzy strongly g* - closed. But the converse may not be true in general.

Example 3.5. Let X={a,b} and τ ={0, A,1} and fuzzy sets A and B are in X defined by A(a)=0.8,A(b)=0.2; B(a)=0.9,B(b)=0.6.

Then B is a fuzzy strongly g*-closed in (X,τ) but it is not fp-closed set in (X,τ), because $Cl(Int(B)) \not\leq B$.

Example 3.6. Let X={a,b}. Define fuzzy sets A and B are in X defined by A(a)=0.2,A(b)=0.6; B(a)=0.5,B(b)=0.7.

Let τ ={0, A,1}. Then B is a fuzzy strongly g*-closed in (X,τ) but it is not fsp-closed set in (X,τ), since $Int(Cl(Int(B))) \not\leq B$.

Example 3.7. Let . X={a,b,c}. Define fuzzy sets A,B & C in X as follows
A(a)=0.1,A(b)=0.2,A(c)=0.7; B(a)=0.9,B(b)=0.1,B(c)=0.6; D(a)=0.9,D(b)=0.2,D(c)=0.8

Let τ ={0, A,1} then B is strongly g*- closed set in (X,τ) but it not gfs- closed set in (X,τ).For, $sCl(B) \not\leq D$ where $B < D$ and D is fs-open in (X,τ).

Example 3.8. Let X={a,b,c}, τ ={0, A,1} and fuzzy sets A and B are defined by
A(a)=0.8,A(b)=0.3,A(c)=0.1; B(a)=0.8,B(b)=0.1,B(c)=0.1

Then B is strongly g*- closed set in (X,τ) but it is neither fgs-closed, because $sCl(B)=1\not\leq A$ where as $B\leq A$ and A is fuzzy open and nor fgα-closed, since $\alpha Cl(B)=1\not\leq A$ where as $B\leq A$ and A is fα-open in X.

Example 3.9. Let X={a,b} and τ ={0, A,1} . Define fuzzy set A, B and D in X by

A(a)=0.3,A(b)=0.6 ; B(a)=0.6,B(b)=0.6 ; D(a)=0.9,D(b)=0.8

Then B is fuzzy strongly g* - closed but it is not Fspg- closed because, $spCl(B)\not\leq D$ where as $B\leq D$ and D is fs-open in X.

From the above discussions and known results we have the following implications.

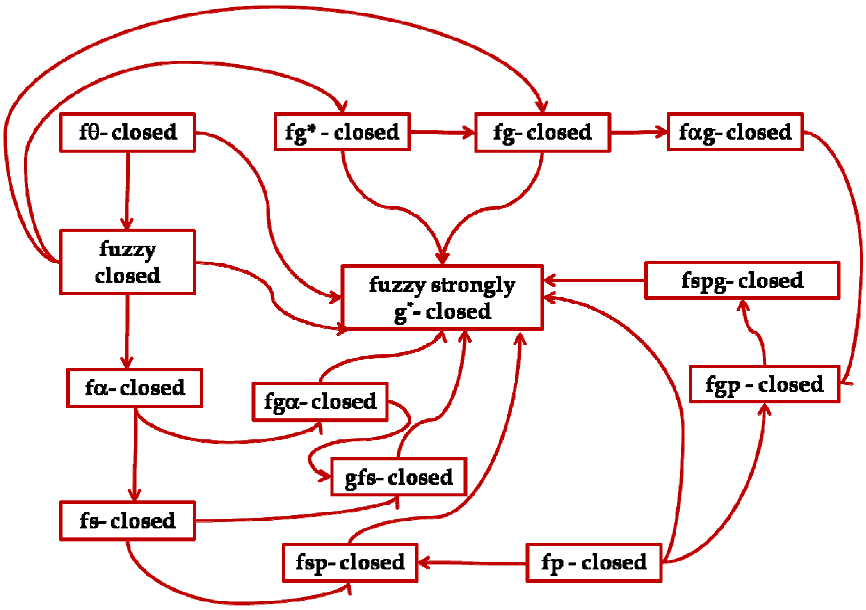

Figure 3.1

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