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Equivalence of Interval Valued Fuzzy Sets

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Abstract. In this paper, we introduce and study the equivalence of i-v fuzzy subsets of a finite set X. In order to study on equivalence of i-v fuzzy subsets of X we introduce the notion of i- pinned flag on X. First we establish the condition for the one to one correspondence between equivalence classes of i-v fuzzy subsets of X and i-pinned flags on X.

Keywords: Equivalence, i-v fuzzy subset, i-keychain, i-pinned flag, comparability.

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1. Introduction

Zadeh [7] made an extension of the concept of a fuzzy set by an interval–valued fuzzy set with an interval–valued membership function. Interval–valued fuzzy sets have many applications in several areas. For example, Zadeh [7] constructed a method of approximate inference using his interval – valued fuzzy set. Gorzalczany [1] studied the interval–valued fuzzy sets for approximate reasoning, Roy and Biswas [4] studied interval–valued fuzzy relations and applied these in Sanxhez's approach for medical diagnosis. Murali and Makamba [2, 3] studied and computed the number of fuzzy subsets, up to a naturally defined equivalence on the set of all fuzzy subsets. Singaram and Kandasamy [5] studied the interval valued fuzzy ideals of regular and intraregular semigroups. In this paper, we introduced the notion of equivalent interval – valued fuzzy sets that is two interval – valued fuzzy sets are equivalent if they maintain the same relative degrees of membership between any pair of elements (refer definition 3.2). In general equivalence identifies objects similar to each other in some sense. Equality of interval – valued fuzzy set is too strong to be of any in the enumeration process. The equivalence that we consider here is weaker than the equality of interval – valued fuzzy set, but reduces to equality of crisp set when two truth values {0,1} are used.

The theme of this paper is equivalent interval–valued fuzzy sets and their associated i- pinned flags. We study here these two related ideas: One is the representation of interval – valued fuzzy set of a set X by an increasing chain of subsets of X, with degrees of membership in the descending order. We call such an object an i-pinned flag. The other is the Equivalence of interval – valued fuzzy subsets. In section 2, we gather all the preliminaries of interval – valued fuzzy sets and fix notations. In section 3, we introduce the notion of i-keychain and study the condition for the one to one correspondence between equivalence classes of i-v fuzzy subsets of X and i-pinned flags on X.

2. Preliminaries

Throughout the paper we use notations as in [2].

Let $I = [0,1]$ be the real unit interval as a chain with the usual ordering in which $\wedge$ stands for infimum and $\vee$ stands for supremum. Throughout this paper we take X to be a non - empty finite set with n elements labeled as $\{x_1, x_2, \dots x_n\}, n \geq 2$.

A fuzzy subset of a set X is a mapping $\mu: X \to I$. We denote the set of all fuzzy subset of X by I^X. Further we denote a fuzzy set by Greek letters μ, ν, η, etc., by an α-cut of μ for a real no α in I, we mean a crisp subset $\mu^\alpha = \{x \in X / \mu(x) \geq \alpha\}$ of X. We remark that for $0 \leq \alpha \leq \beta \leq 1$, we have $\mu^\beta \subseteq \mu^\alpha$. Generally the 1-cut of any fuzzy subset μ of any given set may or may not be empty and 0-cut will be the full set X.

Definition 2.1. [3] Let μ and γ be fuzzy sets of a set X. μ is said to be equivalent to γ denoted as $\mu \sim \gamma$ if and only if $\forall x, y \in X$

(i) $\mu(x) = 1$ if and only if $\gamma(x) = 1$

(ii) $\mu(x) \geq \mu(y)$ if and only if $\gamma(x) \geq \gamma(y)$

(iii) $\mu(x) = 0$ if and only if $\gamma(x) = 0$.

Definition 2.2. [3]

(i) A flag C on a set X is a maximal chain C of subsets of X of the form
$$X_0 \subset X_1 \subset X_2 \subset \cdots \subset X_n = X.$$

(ii) An n-chain is an $(n+1)$- tuple of distinct real numbers $\lambda_i \in I$ always including 1, and not necessarily including 0 of the form $1 = \lambda_0 > \lambda_1 > \lambda_2 > \cdots > \lambda_n$.

(iii) A keychain l of an n-chain is a set of real numbers $\lambda_i \in I$ of the form $1 = \lambda_0 \geq \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n \geq 0$.

The λ_i's are called pins. We write a keychain l as $l = 1, \lambda_1, \lambda_2, \dots, \lambda_n$.

In terms of labeled elements, X_i can be taken to be $\{x_1, x_2, \dots, x_i\}$. Each X_i is called the component of the flag C.

Definition 2.3. [3] By a pinned flag on X, we mean a pair (C, l) of a flag C on X and a keychain l from I written suggestively as follows:
$$(C, l): {X_0}^1 \subset {X_1}^{\lambda_1} \subset {X_2}^{\lambda_2} \subset \cdots \subset {X_n}^{\lambda_n}.$$

Theorem 2.4. [3] A pinned flag (C, l): ${X_0}^1 \subset {X_1}^{\lambda_1} \subset {X_2}^{\lambda_2} \subset \cdots \subset {X_n}^{\lambda_n}$ on X represents a fuzzy subset μ on X defined as $\mu(x) = \{\lambda_i : \ x \in X_i \backslash X_{i-1}\}$. Conversely Suppose μ is a fuzzy subset of X then we can decompose μ into a pinned flag (C, l).

Proposition 2.5. [2] Let $\mu, \gamma \in I^X$. If for each $t > 0$ there exists an $s > 0$ such that $\mu^t = \gamma^s$, then $\mu \sim \gamma$ on X.

Proposition 2.6. [3] Let the pinned flags corresponding to two fuzzy sets μ and γ be given by
(C_μ, l_μ): ${X_0}^1 \subset {X_1}^{\lambda_1} \subset {X_2}^{\lambda_2} \subset \cdots \subset {X_n}^{\lambda_n}$ and
(C_γ, l_γ): ${Y_0}^1 \subset {Y_1}^{\beta_1} \subset {Y_2}^{\beta_2} \subset \cdots \subset {Y_n}^{\beta_n}$
Then $\mu \sim \gamma$ on X if and only if

i) $n = m$
ii) $X_i = Y_i$ for $i = 0,1, \dots, n$; provided the $\lambda_k's$ are distinct and $\beta_k's$ are distinct
iii) $\lambda_i > \lambda_j$ if and only if $\beta_i > \beta_j$ for $1 \le i, j \le n$ and $\lambda_k = 0$ if and only if $\beta_k = 0$ for some k between 1 and n.

3. Equivalence on *i-v fuzzy sets*

In this section we shall extend the concept of equivalence relation on fuzzy sets to interval valued fuzzy sets. We shall follow the definition of $\bar{t}$ –cuts and comparability of interval numbers as in [6].

Let $D[0,1]$be the family of all closed subintervals of [0,1] with $\bar{0} = [0,0]$ and $\bar{1} = [1,1]$.
Any element $\bar{a} \in D[0,1]$ is called an interval number.

Definition 3.1. Any two elements $\bar{a} = [a^-, a^+]$ and $\bar{b} = [b^-, b^+]$, in $D[0,1]$ are said to be comparable if either $\bar{a} \le \bar{b}$ or $\bar{a} \ge \bar{b}$ where $\bar{a} \le \bar{b}$ if and only if $a^- \le b^-$ and $a^+ \le b^+$ and $\bar{a} = \bar{b}$ if and only if $a^- = b^-$ and $a^+ = b^+$.

An interval - valued fuzzy set (briefly, an i-v fuzzy set) of X is a mapping $\bar{A}: X \to D[0,1]$, where $\bar{A}(x) = [A^-(x), A^+(x)]\ \forall x \in X, A^-$ and A^+ are fuzzy sets in X such that $A^-(x) \le A^+(x)\ \forall x \in X$.

Let $\bar{A}$ be an i-v fuzzy set of X and $\bar{t} = [t_1, t_2] \in D[0,1]$. Then the set $\bar{A}_{\bar{t}} = \{x \in X: \bar{A}(x) \ge \bar{t}\ \}$ is called a level sub set of $\bar{A}$.

Note that if $\bar{t} = [t_1, t_2]$, then $\bar{A}_{\bar{t}} = \bar{A}_{[t_1,t_2]} = A^-_{t_1} \cap A^+_{t_2}$, where $A^-_{t_1} = \{x \in X: A^-(x) \ge t_1\}$ and $A^+_{t_2} = \{x \in X: A^+(x) \ge t_2\}$.

Definition 3.2. Let $\bar{A}$ and $\bar{B}$ be i-v fuzzy sets of X. Then $\bar{A}$ is said to be equivalent to $\bar{B}$ on X if and only if for all $x, y \in X$,

(i) $\bar{A}(x) = \bar{1}$ if and only if $\bar{B}(x) = \bar{1}$
(ii) $\bar{A}(x) = \bar{0}$ if and only if $\bar{B}(x) = \bar{0}$
(iii) $\bar{A}(x) \ge \bar{A}(y)$ if and only if $\bar{B}(x) \ge \bar{B}(y)$.

Note. The condition $\bar{A}(x) = \bar{0}$ if and only if $\bar{B}(x) = \bar{0}$ simply says that the support of $\bar{A}$ and $\bar{B}$ are equal whereby support of an i-v fuzzy set we mean a crisp set $supp\bar{A} = \{\ x \in X: \bar{A}(x) > \bar{0}\}$. The above condition can be made redundant since it is an essential part of the equivalence relation. The following example illustrates the point.

Example 3.3. Let X={a,b,c,d,e}. Define i-v fuzzy sets $\bar{A}$ and $\bar{B}$ in X as follows:

$$\bar{A}(x)=\begin{cases}\bar{1} & if\ x=a\\ [0.4,0.7] & if\ x=b,c\\ [0.1,0.5] & otherwise.\end{cases} \quad \text{and} \quad \bar{B}(x)=\begin{cases}\bar{1} & if\ x=a\\ [0.3,0.8] & if\ x=b,c\\ \bar{0} & otherwise.\end{cases}$$

It is clear that $\bar{A}(x)>\bar{A}(y)$ if and only if $\bar{B}(x)>\bar{B}(y)$ for all $x,y\in X$. But $supp\bar{A}\neq supp\bar{B}$. Here $\bar{A}\nsim\bar{B}$.

Proposition 3.4. Suppose $\bar{A}$ and $\bar{B}$ are two i-v fuzzy subsets of X which assumes only comparable interval numbers such that $\bar{A}\sim\bar{B}$ then $|Im(\bar{A})|=|Im(\bar{B})|$.

Proof. Assume that $\bar{A}$ and $\bar{B}$ are equivalent i-v fuzzy sets of a set X which assumes only comparable interval numbers. Define a function $f: Im(\bar{A})\longrightarrow Im(\bar{B})$ by $f\left(\bar{A}(x)\right)=\bar{B}(x)$. Then it is easy to check that f is well defined, one to one and onto. Hence $|Im(\bar{A})|=|Im(\bar{B})|$.

The converse is not true, Viz., if $|Im(\bar{A})|=|Im(\bar{B})|$ or even $Im(\bar{A})=Im(\bar{B})$ and $supp\bar{A}=supp\bar{B}$, it is not necessary to have $\bar{A}\sim\bar{B}$. This is illustrated in the following example.

Example 3.5. Let $X=\{a,b,c,d,e\}$.
Define i-v fuzzy sets $\bar{A}$ and $\bar{B}$ are as follows:

$$\bar{A}(x)=\begin{cases}\bar{1} & if\ x=a,\\ [0.4,0.8] & if\ x=b,c,\\ [0.2,0.6] & otherwise,\end{cases} \quad \text{and} \quad \bar{B}(x)=\begin{cases}\bar{1} & if\ x=a,\\ [0.4,0.8] & if\ x=d,e,\\ [0.2,0.6] & otherwise.\end{cases}$$

We can easily verify that $Im(\bar{A})=Im(\bar{B})$ and $supp\bar{A}=supp\bar{B}$. However $\bar{A}(b)>\bar{A}(d)$ but $\bar{B}(b)\ngtr\bar{B}(d)$.
Therefore $\bar{A}\nsim\bar{B}$.

Proposition 3.6. Let $\bar{A}=[A^-,A^+]$ and $\bar{B}=[B^-,B^+]$ be any i-v two fuzzy sets of a set X such that $A^-\sim B^-$ and $A^+\sim B^+$ then $\bar{A}\sim\bar{B}$.

Proof. Let us assume $A^-\sim B^-$ and $A^+\sim B^+$ then,

$$\begin{aligned}\bar{A}(x)=\bar{1} &\Leftrightarrow A^-(x)=1 \text{ and } A^+(x)=1,\\ &\Leftrightarrow B^-(x)=1 \text{ and } B^+(x)=1,\ \text{ since } A^-\sim B^- \text{ and } A^+\sim B^+\\ &\Leftrightarrow \bar{B}(x)=\bar{1},\\ \bar{A}(x)=\bar{0} &\Leftrightarrow A^-(x)=0 \text{ and } A^+(x)=0,\\ &\Leftrightarrow B^-(x)=0 \text{ and } B^+(x)=0,\ \text{ since } A^-\sim B^- \text{ and } A^+\sim B^+\\ &\Leftrightarrow \bar{B}(x)=\bar{0},\\ \text{and } \bar{A}(x)\geq\bar{A}(y) &\Leftrightarrow [A^-(x),A^+(x)]\geq[A^-(y),A^+(y)],\\ &\Leftrightarrow A^-(x)\geq A^-(y) \text{ and } A^+(x)\geq A^+(y),\\ &\Leftrightarrow B^-(x)\geq B^-(y) \text{ and } B^+(x)\geq B^+(y),\\ &\Leftrightarrow [B^-(x),B^+(x)]\geq[B^-(y),B^+(y)],\\ &\Leftrightarrow \bar{B}(x)\geq\bar{B}(y).\end{aligned}$$

Hence $\bar{A}\sim\bar{B}$.

Remark: We observe that the converse of Proposition 3.6 is not true. This is illustrated in the following:

Example 3.7. Let $X = \{a, b, c, d\}$. Define i-v fuzzy sets $\bar{A}$ and $\bar{B}$ in X by

$$\bar{A}(a) = \bar{0}, \bar{A}(b) = \bar{1}, \bar{A}(c) = [0.2,0.8], \bar{A}(d) = [0.4,0.6],$$
$$\bar{B}(a) = \bar{0}, \bar{B}(b) = \bar{1}, \bar{B}(c) = [0.5,0.7], \text{ and } \bar{B}(d) = [0.3,0.8].$$

Since $\bar{A}(b) = \bar{1} = \bar{B}(b)$ and $\bar{A}(a) = \bar{0} = \bar{B}(a)$ (i) and (ii) of Definition 3.2 hold.
Now $\bar{A}(c)$ and $\bar{A}(d)$ are not comparable and similarly $\bar{B}(c)$ and $\bar{B}(d)$ are also not comparable. Therefore (iii) of Definition 3.2 holds trivially. Hence $\bar{A} \sim \bar{B}$
Now $A^+(c) = 0.8 \geq 0.6 = A^+(d)$ and $B^+(c) = 0.7 \ngeq 0.8 = B^+(d)$, we have
$A^+(x) \geq A^+(y) \nLeftrightarrow B^+(x) \geq B^+(y)$.
Therefore $A^+ \nsim B^+$.
Similarly $A^-(d) = 0.4 \geq 0.2 = A^-(c)$ and $B^-(d) = 0.3 \ngeq 0.5 = B^-(c)$, we have
$A^-(x) \geq A^-(y) \nLeftrightarrow B^-(x) \geq B^-(y)$.
Therefore $A^- \nsim B^-$.
This shows that $\bar{A} \sim \bar{B} \nRightarrow A^- \sim B^-$ or $A^+ \sim B^+$.

Definition 3.8.

i) An i- n-chain is an n- tuple of comparable and distinct interval numbers $\overline{\lambda_k} \in D[0,1]$ always including $\bar{1}$ not necessarily including $\bar{0}$ of the form $\bar{1} = \overline{\lambda_0} > \overline{\lambda_1} > \overline{\lambda_2} > \cdots > \overline{\lambda_n}$ written in the decending order of magnitude.
ii) An i- keychain $\bar{l}$ of an i- n-chain is a set of interval numbers $\overline{\lambda_k} \in D[0,1]$ of the form $\bar{1} = \overline{\lambda_0} \geq \overline{\lambda_1} \geq \overline{\lambda_2} \geq \cdots \geq \overline{\lambda_n} \geq \bar{0}$. the $\overline{\lambda_k}'s$ are called i-pins.
iii) An i-pinned flag on X is a pair $(C, \bar{l}\,)$ of a flag C on X and an i- keychain $\bar{l}$ from $D[0,1]$ written as $(C, \bar{l}\,):\ {X_0}^{\bar{1}} \subset {X_1}^{\overline{\lambda_1}} \subset {X_2}^{\overline{\lambda_2}} \subset \cdots \subset {X_n}^{\overline{\lambda_n}}$.

Theorem 3.9. We can associate an i-v fuzzy set $\bar{A}$ with a given i-pinned flag $(C, \bar{l}\,)$.
Proof. Let $\bar{l} = \{\bar{1} = \overline{\lambda_0}, \overline{\lambda_1}, \overline{\lambda_2}, \dots, \overline{\lambda_n}\}$ be the comparable and distinct interval numbers then we got two key chains l^- and l^+ as follows:
$l^- = \{1 = {\lambda_0}^-, {\lambda_1}^-, {\lambda_2}^-, \dots, {\lambda_n}^-\}$ and $l^+ = \{1 = {\lambda_0}^+, {\lambda_1}^+, {\lambda_2}^+, \dots, {\lambda_n}^+\}$.
For the two key chains we have two pinned flags (C, l^-) and (C, l^+) such that
(C, l^-): ${X_0}^1 \subset {X_1}^{{\lambda_1}^-} \subset {X_2}^{{\lambda_2}^-} \subset \cdots \subset {X_n}^{{\lambda_n}^-}$ and
(C, l^+): ${X_0}^1 \subset {X_1}^{{\lambda_1}^+} \subset {X_2}^{{\lambda_2}^+} \subset \cdots \subset {X_n}^{{\lambda_n}^+}$.
We can associate two fuzzy sets with the above pinned flags respectively

$$\text{i.e., } A^-(x) = \begin{cases} 1, & x \in X_0 \\ {\lambda_1}^-, & x \in X_1 \backslash X_0 \\ {\lambda_2}^-, & x \in X_2 \backslash X_1 \\ \vdots & \\ {\lambda_n}^-, & x \in X_n \backslash X_{n-1} \end{cases} \text{ and } A^+(x) = \begin{cases} 1, & x \in X_0 \\ {\lambda_1}^+, & x \in X_1 \backslash X_0 \\ {\lambda_2}^+, & x \in X_2 \backslash X_1 \\ \vdots & \\ {\lambda_n}^+, & x \in X_n \backslash X_{n-1} \end{cases}$$

Now $\bar{A}(x) = [A^-(x), A^+(x)]$ is an i-v fuzzy set associated with the given i-pinned flag $(C, \bar{l}\,)$.

Theorem 3.10. Suppose $\bar{A}$ be an i-v fuzzy set of X which assumes only comparable interval numbers then we can decompose $\bar{A}$ into an i- pinned flag

$$(C, \bar{l}\,): {X_0}^{\bar{1}} \subset {X_1}^{\overline{\lambda_1}} \subset {X_2}^{\overline{\lambda_2}} \subset \cdots \subset {X_n}^{\overline{\lambda_n}}.$$

Proof. Since X is finite, $\bar{A}(X) \subset D[0,1]$.
Let $\bar{A}(X) = \{\overline{\lambda_0}, \overline{\lambda_1}, \overline{\lambda_2}, \dots, \overline{\lambda_k}\}$ where the sequence is decreasing (strictly).
Let $Y_m = \bar{A}^{\overline{\lambda_m}}$ be the $\bar{t}$ –cut corresponding to $\bar{t} = \overline{\lambda_m}$ for $m = 1,2, \dots, k$.
Then as the $\overline{\lambda_m}'s$ are comparable three facts are well known:

i) Every Y_m is a subset of X.
ii) $\overline{\lambda_m} > \overline{\lambda_n}$ implies that $Y_m \subset Y_n$ for $1 \le m, n \le k$.
iii) The chain $C_1: Y_1 \subset Y_2 \subset \cdots \subset Y_k$ can be refined to yield a flag $C: X_0 \subset X_1 \subset X_2 \subset \cdots \subset X_n = X$.

As we refine C_1 to C we may have to repeat some of the i-pins $\overline{\lambda_m}$ correspondingly. Once this process carried out, we arrive at a i-pinned flag $X_0{}^{\bar{1}} \subset X_1{}^{\overline{\lambda_1}} \subset X_2{}^{\overline{\lambda_2}} \subset \cdots \subset X_n{}^{\overline{\lambda_n}}$ which represents $\bar{A}$.

Note that if $\bar{A}$ is an i-v fuzzy set of X which assumes some non comparable interval numbers then we cannot decompose $\bar{A}$ into an i- pinned flag which represents $\bar{A}$. We will see this by an example.

Example 3.11. Let $X = \{a, b, c, d\}$. Define an i-v fuzzy set $\bar{A}$ on X by
$\bar{A}(a) = \bar{0}$, $\bar{A}(b) = \bar{1}$, $\bar{A}(c) = [0.2,0.8]$, and $\bar{A}(d) = [0.4,0.6]$
Since $\bar{A}(c)$ and $\bar{A}(d)$ are not comparable, the fact (ii) in the Theorem 3.10 fails. Therefore we cannot decompose $\bar{A}$ into an i- pinned flag which represents $\bar{A}$.

Proposition 3.12. Let $\bar{A}$ and $\bar{B}$ be two i-v fuzzy subsets of X. If for each $\bar{t} = [t^-, t^+] > \bar{0}$, there exists $\bar{s} = [s^-, s^+] > \bar{0}$ such that $\bar{A}^{\bar{t}} = \bar{B}^{\bar{s}}$ then $\bar{A} \sim \bar{B}$ *on* X.
Assume for each $\bar{t} = [t^-, t^+] > \bar{0}$, there exists $\bar{s} = [s^-, s^+] > \bar{0}$ such that $\bar{A}^{\bar{t}} = \bar{B}^{\bar{s}}$.
That is for each $t^- > 0$ there exists an $s^- > 0$ such that $A^{-t^-} = B^{-s^-}$ and for each $t^+ > 0$ there exists an $s^+ > 0$ such that $A^{+t^+} = B^{+s^+}$ .Therefore by Proposition 2.5 , $A^- \sim B^-$ and $A^+ \sim B^+$ and by Proposition 3.6, $\bar{A} \sim \bar{B}$.

Proposition 3.13. Let the i-pinned flags corresponding to two i-v fuzzy sets $\bar{A}$ and $\bar{B}$ be given by $(C_{\bar{A}}, \bar{l}_{\bar{A}})$: $X_0{}^{\bar{1}} \subset X_1{}^{\overline{\lambda_1}} \subset X_2{}^{\overline{\lambda_2}} \subset \cdots \subset X_n{}^{\overline{\lambda_n}}$ and
$(C_{\bar{B}}, \bar{l}_{\bar{B}})$: $Y_0{}^{\bar{1}} \subset Y_1{}^{\overline{\beta_1}} \subset Y_2{}^{\overline{\beta_2}} \subset \cdots \subset Y_n{}^{\overline{\beta_n}}$
Then $\bar{A} \sim \bar{B}$ on X if and only if

(i) $n = m$
(ii) $X_i = Y_i$ for $i = 0,1, \dots, n$; provided the $\overline{\lambda_k}'s$ are distinct and $\overline{\beta_k}'s$ are distinct
(iii) $\overline{\lambda_i} > \overline{\lambda_j}$ if and only if $\overline{\beta_i} > \overline{\beta_j}$ for $1 \le i, j \le n$ and $\overline{\lambda_k} = \bar{0}$ if and only if $\overline{\beta_k} = \bar{0}$ for some k between 1 and n.

Proof. $\bar{A}$ is an i-v fuzzy set on X and $(C_{\bar{A}}, \bar{l}_{\bar{A}}) = X_0{}^{\bar{1}} \subset X_1{}^{\overline{\lambda_1}} \subset X_2{}^{\overline{\lambda_2}} \subset \cdots \subset X_n{}^{\overline{\lambda_n}}$ is the corresponding i-pinned flag.
Therefore we have two fuzzy sets A^- and A^+ and two pinned flags
$(C_{A^-}, l_{A^-}) = X_0{}^1 \subset X_1{}^{\lambda_1{}^-} \subset X_2{}^{\lambda_2{}^-} \subset \cdots \subset X_n{}^{\lambda_n{}^-}$ and
$(C_{A^+}, l_{A^+}) = X_0{}^1 \subset X_1{}^{\lambda_1{}^+} \subset X_2{}^{\lambda_2{}^+} \subset \cdots \subset X_n{}^{\lambda_n{}^+}$.
Similarly for $\bar{B}$, we have
$(C_{B^-}, l_{B^-}) = Y_0{}^1 \subset Y_1{}^{\beta_1{}^-} \subset Y_2{}^{\beta_2{}^-} \subset \cdots \subset Y_n{}^{\beta_n{}^-}$ and

$(C_{B^+}, l_{B^+}) = {Y_0}^{1} \subset {Y_1}^{{\beta_1}^+} \subset {Y_2}^{{\beta_2}^+} \subset \cdots \subset {Y_n}^{{\beta_n}^+}$.
Then by our assumption and Proposition 2.6 $A^- \sim B^-$ and $A^+ \sim B^+$.
Therefore by Proposition 3.6 $\bar{A} \sim \bar{B}$.

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