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Interval-valued Fuzzy Ideals of Regular and Intraregular Semigroups

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Abstract. Let S be a semigroup. A mapping $\bar{A}: S \longrightarrow D[0,1]$ is called an interval-valued fuzzy subset of S where $D[0,1]$ denotes the family of all closed sub intervals of $[0,1]$. A semigroup S is called an intraregular semigroup if for each element $a \in S$ there exist $x, y \in S$ such that $a = xa^2y$. In this paper, intraregular semigroups are characterized by means of interval-valued fuzzy left ideals (resp. right ideals, bi-ideals, interior ideals).

Keywords: Semigroup, Interval-valued fuzzy subsemigroup, Interval-valued fuzzy ideal, Interval-valued fuzzy bi-ideal, Interval-valued fuzzy interior ideal

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1. Introduction

L.A. Zadeh [11] made an extension of the concept of a fuzzy set by an interval-valued fuzzy set with an interval-valued membership function. Interval-valued fuzzy sets have many applications in several areas. For example, Zadeh [11] constructed a method of approximate inference using his interval-valued fuzzy set. Gorzalczany [3] studied the interval-valued fuzzy sets for approximate reasoning, Roy and Biswas [1] studied interval -valued fuzzy relations and applied these in Sanxhez's approach for medical diagnosis. X.Y. Xie and J. Tang [10] studied regular and intraregular semigroups in terms of fuzzy sets. Y. Hang and X. Fang [4] characterized intraregular semigroups by intuitionistic fuzzy sets. In [8, 9] AL. Narayanan and T. Manikantan introduced the notions of interval-valued fuzzy subsemigroup and various kinds of interval-valued fuzzy ideals in semigroups. Kuroki [6] characterized regular semigroups, intraregular semigroups that are semilattices of left (right) simple semigroups in terms of fuzzy ideals, fuzzy bi-ideals and fuzzy generalized bi-ideals. In this paper, we characterized the regular and intraregular semigroups in terms of interval-valued fuzzy left, right, interior and bi-ideals.

2. Preliminaries

Let S be a semigroup.

A non-empty subset A of S is called a *subsemigroup* of S if $AA \subseteq A$ and is called a *left* (resp. *right*) *ideal* of S if $SA \subseteq A$ (resp. $AS \subseteq A$).

By *two sided ideal* or simply *ideal*, we mean a non-empty subset of S which is both a left and a right ideal of S.

A subsemigroup A of S is called a *bi-ideal* of S if $ASA \subseteq A$.

A non-empty subset A of S is called a *interiorideal* of S if $SAS \subseteq A$.

Definition 2.1. A semigroup S is called *regular* if for each element $a \in S$ there exists $x \in S$ such that $a = axa$. In other words S is regular if $a \in aSa \quad \forall a \in S$.

Definition 2.2. A semigroup S is called *intraregular* if for each element $a \in S$ there exist $x, y \in S$ such that $a = xa^2y$. In other words S is intraregular if $a \in Sa^2S \ \ \forall a \in S$.

We now review some fuzzy concepts.

A *fuzzy subset* A of a non-empty set X is a mapping from X to [0, 1].

Let S be a semigroup. A fuzzy subset A of S is called a *fuzzy subsemigroup* of S if $A(xy) \geq min\{A(x), A(y)\} \ \ \forall x, y \in S$. A fuzzy subset A of S is called a *fuzzy left* (*resp. right*) *ideal* of S if $A(xy) \geq A(y)\big(resp. A(xy) \geq A(x)\big) \forall x, y \in S$.

A fuzzy subset A of S is called a *fuzzy two-sided ideal* or simply *fuzzy ideal* of S if it is both a fuzzy left ideal and a fuzzy right ideal of S.

A fuzzy subsemigroup A of S is called a *fuzzy bi-ideal* of S if $A(xyz) \geq min\{A(x), A(z)\} \ \forall x, y, z \in S$.

An interval number on $[0, 1]$, say $\bar{a}$ is a closed subinterval of $[0, 1]$, that is $\bar{a} = [a^-, a^+]$ where $0 \leq a^- \leq a^+ \leq 1$. Let $D[0, 1]$ denote the family of all closed subintervals of $[0, 1]$, $\bar{0} = [0, 0]$ and $\bar{1} = [1, 1]$.

For any two elements $\bar{a} = [a^-, \ a^+]$ and $\bar{b} = [b^-, \ b^+]$ in $D[0, 1]$, we define

(i) $\bar{a} \leq \bar{b}$ if and only if $a^- \leq b^-$ and $a^+ \leq b^+$,

(ii) $\bar{a} = \bar{b}$ if and only if $a^- = b^-$ and $a^+ = b^+$,

(iii) $Min^i\{\bar{a}, \bar{b}\} = [\, min\{a^-, \ b^-\}, \ min\{\, a^+, \ b^+\}\,]$,

(iv) $Max^i\, \{\bar{a}, \bar{b}\} = [\, max\{a^-, b^-\}, \ max\{\, a^+, \ b^+\}\,]$.

Similarly we can define Inf^i and Sup^i in case of family of elements in D[0, 1].

A mapping $\bar{A}: X \longrightarrow D[0, 1]$ is called an *interval–valued fuzzy subset* (briefly, an *i-v fuzzy subset*) of X, where $\bar{A}(x) = [A^-(x), A^+(x)] \ \ \forall x \in X$, A^- and A^+ are fuzzy subsets in X such that $A^-(x) \leq A^+(x) \, \forall x \in X$.

Definition 2.3. Let $\bar{A}, \bar{B}$ be i-v fuzzy subsets of X. Then we have the following:

(i) $\bar{A} \leq \bar{B}$ if and only if $\bar{A}(x) \leq \bar{B}(x) \ \forall x$.

(ii) $\bar{A} = \bar{B}$ if and only if $\bar{A}(x) = \bar{B}(x) \ \forall x$.

(iii) $(\bar{A} \cup \bar{B})(x) = max^i\{\bar{A}(x), \bar{B}(x)\}$

(iv) $(\bar{A} \cap \bar{B})(x) = min^i\{\bar{A}(x), \bar{B}(x)\}$.

Definition 2.4. Let '.' be a binary composition in a set S. The product $\bar{A} \circ \bar{B}$ of any two i-v fuzzy subsets $\bar{A}, \bar{B}$ of S is defined by

$$(\bar{A} \circ \bar{B})(x) = \begin{cases} \underset{x = a \cdot b}{Sup^i} \left\{ Min^i\{\bar{A}(a), \bar{B}(b)\} \right\} & if\ x\ is\ expressible\ as\ x = a \cdot b \\ \bar{0} & otherwise \end{cases}$$

Since semigroup S is associative, the operation $\circ$ is associative. We denote xy instead of $x \cdot y$ and $\bar{A}\bar{B}$ for $\bar{A} \circ \bar{B}$.

Let B be a subset of a set X. Define a function $\overline{\chi_B}: X \to D[0,1]$ by

$$\overline{\chi_B}(x) = \begin{cases} \bar{1}\ if\ x \in B \\ \bar{0}\ otherwise \end{cases} \quad \forall x \in X.$$

Clearly $\overline{\chi_B}$ is an i-v fuzzy subset of X. Throughout this paper $\overline{\chi_S}$ is denoted by $\bar{S}$ and S will denote a semigroup unless otherwise mentioned.

An i-v fuzzy subset $\bar{A}$ of S is called an *interval–valued fuzzy subsemigroup* (briefly, *an i-v fuzzy subsemigroup*) of S if $\bar{A}(ab) \geq Min^i\{\bar{A}(a), \bar{A}(b)\}\ \forall a, b \in S$.

An i-v fuzzy subset $\bar{A}$ of S is called an *interval-valued fuzzy left* (resp. *right*) *ideal* (briefly, *an i-v fuzzy left* (*resp. right*) *ideal*) of S if $\bar{A}(ab) \geq \bar{A}(b)$(resp. $\bar{A}(ab) \geq \bar{A}(a)$) for all $a, b \in S$.

Every i-v fuzzy right (left, two sided) ideal of S is an i-v fuzzy subsemigroup of S. However the converse is not true.

An i-v fuzzy subset $\bar{A}$ of S is called an *interval-valued fuzzy two-sided ideal* or simply *i-v fuzzy ideal* of S if it is both an i-v fuzzy left ideal and an i-v fuzzy right ideal of S.

An i-v fuzzy subsemigroup $\bar{A}$ of S is called an *i-v fuzzy bi-ideal* of S if $\bar{A}(xyz) \geq Min^i\{\bar{A}(x), \bar{A}(z)\}\ \forall x, y, z \in S$.

An i-v fuzzy subset $\bar{A}$ of S is called an *i-v fuzzy interior ideal* of S if $\bar{A}(xay) \geq \bar{A}(a)$.

3. Results

In this section, we obtained the structure of i-v fuzzy interior ideal of an intraregular semigroup and obtained equivalent conditions for a semigroup to be intraregular and showed that in an intraregular semigroup the concept of an i-v fuzzy ideal and an i-v fuzzy interior ideal are identical.

Theorem 3.1. Let S be an intraregular semigroup. Then $\bar{A} = \bar{S}\bar{A}\bar{S}$ for every i-v fuzzy interior ideal $\bar{A}$ of S.

Proof: Let $\bar{A}$ be an i-v fuzzy interior ideal of an intraregular semigroup S.

$$\begin{aligned} \bar{S}\bar{A}\bar{S}(a) &= \underset{a = xy}{Sup^i} \left\{ Min^i\{(\bar{S}\bar{A})(x), \bar{S}(y)\} \right\} \\ &= \underset{a = xy}{Sup^i} \{(\bar{S}\bar{A})(x)\} \\ &= \underset{a = xy}{Sup^i} \left\{ \underset{x = uv}{Sup^i} \left\{ Min^i\{\bar{S}(u), \bar{A}(v)\} \right\} \right\} \\ &= \underset{a = xy}{Sup^i} \left\{ \underset{x = uv}{Sup^i} \{\bar{A}(v)\} \right\} \end{aligned}$$

$$= \underset{a=uvy}{Sup^i}\left\{\{\bar{A}(v)\}\right\}$$
$$\leq \underset{a=uvy}{Sup^i}\left\{\{\bar{A}(uvy)\}\right\}$$
$$= Sup^i\{\bar{A}(a)\}$$
$$= \bar{A}(a).$$

Therefore, $\bar{S}\bar{A}\bar{S} \leq \bar{A}$.

$$\bar{S}\bar{A}\bar{S}(a) = \underset{a=pq}{Sup^i}\left\{Min^i\{\bar{S}(p), (\bar{A}\bar{S})(q)\}\right\}$$
$$\geq Min^i\{\bar{S}(xa), (\bar{A}\bar{S})(ay)\}$$
$$= (\bar{A}\bar{S})(ay)$$
$$= \underset{ay=uv}{Sup^i}\left\{Min^i\{\bar{A}(u), \bar{S}(v)\}\right\}$$
$$\geq Min^i\{\bar{A}(xaa), \bar{S}(y^2)\}$$
$$= \bar{A}(xaa)$$
$$= \bar{A}(a), \text{ since } \bar{A} \text{ is an interior ideal.}$$

Thus $\bar{A} = \bar{S}\bar{A}\bar{S}$.

Lemma 3.2. [5] For a semigroup S, the following conditions are equivalent:

(i) S is intraregular

(ii) $A \cap B \subseteq AB$ holds for every left ideal A and every right ideal B of S.

Theorem 3.3. For a semigroup S, the following conditions are equivalent

(i) S is intraregular

(ii) $\bar{A} \cap \bar{B} \subseteq \bar{A} \circ \bar{B}$ holds for every i-v fuzzy left ideal $\bar{A}$ and every i-v fuzzy right ideal $\bar{B}$ of S.

Proof: Assume that S is intraregular. Therefore $\forall a \in S, \exists x, y \in S$ such that $a = xa^2y$.

Then we have $(\bar{A} \circ \bar{B})(a) = \underset{a=uv}{Sup^i}\left\{Min^i\{\bar{A}(u), \bar{A}(v)\}\right\}$

$$\geq Min^i\{\bar{A}(xa), \bar{B}(ay)\}$$
$$\geq Min^i\{\bar{A}(a), \bar{B}(a)\}$$
$$= (\bar{A} \cap \bar{B})(a) \quad \forall a \in S$$

Hence by Definition 2.3(i) $\bar{A} \circ \bar{B} \geq \bar{A} \cap \bar{B}$.

Conversely assume that $\bar{A} \cap \bar{B} \subseteq \bar{A} \circ \bar{B}$ for every left ideal $\bar{A}$ and every right ideal $\bar{B}$ of S.

Let R be a right ideal and L be a left ideal of S respectively.

Then $\overline{\chi_R}$ is an i-v fuzzy right ideal of S and $\overline{\chi_L}$ is an i-v fuzzy left ideal of S.

By our assumption, $\overline{\chi_L} \cap \overline{\chi_R} \subseteq \overline{\chi_L} \circ \overline{\chi_R}$.

Since $\overline{\chi_L} \cap \overline{\chi_R} = \overline{\chi_{L\cap R}}$, (lemma 2.3.12[2]) we have $\overline{\chi_{L\cap R}} \subseteq \overline{\chi_L} \circ \overline{\chi_R}$.

Now, let $a \in L \cap R$.

Therefore $\overline{\chi_{L\cap R}}(a) = \bar{1}$ and by our assumption $\overline{\chi_L} \circ \overline{\chi_R}(a) = \bar{1}$ that is

$\underset{a=uv}{Sup^i}\left\{Min^i\{\overline{\chi_L}(u), \overline{\chi_R}(v)\}\right\} = \bar{1}$.

Therefore $\exists x \in L$ and $\exists y \in R$ such that $a = xy$ which implies that $a \in LR$.

Therefore $L \cap R \subseteq LR$ for every left ideal L and every right ideal R of S.

By Lemma 3.2, we have S is intraregular.

Theorem 3.4. A semigroup S is intraregular if and only if $(\forall a \in S)\ \bar{A}(a) = \bar{A}(a^2)$ for every i-v fuzzy ideal $\bar{A}$ of S.

Proof: $\Longrightarrow$ Let $\bar{A}$ be an i-v fuzzy ideal of S. And $a \in S$.
Then by hypothesis, as S is intraregular $\exists x, y \in S$ such that $a = xa^2y$.
And $\bar{A}(a) = \bar{A}(xa^2y) \geq \bar{A}(a^2y) \geq \bar{A}(a^2) \geq \bar{A}(aa) \geq \bar{A}(a)$.
This implies that $\bar{A}(a) = \bar{A}(a^2)$.
$\Longleftarrow$ $I(a^2)$ is an ideal of S generated by a^2.
And $I(a^2) = \{a^2\} \cup \{Sa^2\} \cup \{a^2S\}$.
$\overline{\chi_{I(a^2)}}$ is an i-v fuzzy ideal of S.
By our assumption $\overline{\chi_{I(a^2)}}(a^2) = \overline{\chi_{I(a^2)}}(a)$.
We have $\overline{\chi_{I(a^2)}}(a) = \bar{1}$.
Hence $a \in I(a^2) = \{a^2\} \cup \{Sa^2\} \cup \{a^2S\}$.
Suppose $a \in \{a^2\}$
Then $a = a^2 = aa = a^2a^2 = aa^2a$.
Therefore $a \in Sa^2S$.
If $a \in \{Sa^2\}$, then $\exists x \in S$ such that $a = xa^2 = xaa = x(xa^2)a = x^2a^2a$.
Therefore $a \in Sa^2S$.
If $a \in \{a^2S\}$, then $\exists x \in S$ such that $a = a^2x = aax = a(a^2x)x = aa^2x^2$.
Therefore $a \in Sa^2S$.
Thus in all the cases, by Definition 2.2, S is intraregular.
This completes the proof.

Theorem 3.5. Let S be an intraregular semigroup. Then for any i-v fuzzy ideal $\bar{A}$ of S, we have, $\bar{A}(ab) = \bar{A}(ba), \forall\, a, b \in S$

Proof: Let $\bar{A}$ be any i-v fuzzy ideal of S and $a, b \in S$.
Then by theorem 3.4 and hypothesis, we have

$$\bar{A}(ab) = \bar{A}((ab)^2) = \bar{A}(a(ba)b) \geq \bar{A}(ba) = \bar{A}((ba)^2) = \bar{A}(b(ab)a) \geq \bar{A}(ab)$$

Thus we have $\bar{A}(ab) = \bar{A}(ba)$.

Lemma 3.6. A semigroup S is regular and intraregular if and only if every bi-ideal of S is idempotent, that is $B = B^2$ for every bi-ideal B of S.

Proof: Let S be both regular and intraregular semigroup and B be a bi-ideal of S.
Since $B$is a bi-ideal, we have $BSB \subseteq B$and since S is both regular and intraregular, we have $B \subseteq BSB$ and $B \subseteq SB^2S$

$$\begin{aligned} \text{Thus } B &\subseteq BSB \\ &\subseteq BSBSB \\ &\subseteq BS(SB^2S)SB \\ &\subseteq BS^2B^2S^2B \\ &\subseteq BSB^2SB \\ &\subseteq BSB\ BSB \\ &\subseteq B\ B \\ &= B^2 \end{aligned}$$

That is $B \subseteq B^2$.
On the other hand, since B is a bi-ideal of S we have $B^2 \subseteq B$.
Hence we have $B = B^2$.

Conversely, let $B = B^2$ for every bi-ideal of S and let $a \in S$.
But $B(a) = \{a \cup a^2 \cup aSa\}$ is a biideal.
Since $a \in B(a)$, by our assumption $B(a) = B^2(a)$, $a \in B^2(a) = \{a^2 \cup a^2Sa \cup aSa^2\}$.
Therefore either $a = a^2$ or $a \in a^2Sa$ or $a \in aSa^2$.
In all the cases it can easily seen that $a \in aSa$ and $a \in SaS$.
This is true for any $a \in S$.
Therefore S is both regular and intraregular.

Theorem 3.7. Let S be an intraregular semigroup. Then the following are equivalent.
(i) $\bar{A}$ is an i-v fuzzy ideal of S.
(ii) $\bar{A}$ is an i-v fuzzy interior ideal of S.

Proof: $(i) \Rightarrow (ii)$. By Lemma 2.4.9 [2] every i-v fuzzy ideal of a semigroup S is an i-v fuzzy interior ideal of S.
$(ii) \Rightarrow (i)$. Assume that $\bar{A}$ is an i-v fuzzy interior ideal of a intraregular semigroup S.
Let $a, b \in S$. Then since S is intraregular $\exists\ x, y, x', y' \in S$ such that $a = xa^2y$ and $b = x'b^2y'$.
Thus $\bar{A}(ab) = \bar{A}(xa^2yb) = \bar{A}\big((xa)a(yb)\big) \geq \bar{A}(a)$ and
$$\bar{A}(ab) = \bar{A}(ax'b^2y') = \bar{A}\left(\left(ax'\right)b(by')\right) \geq \bar{A}(b).$$
This implies that $\bar{A}$ is an i-v fuzzy ideal of S.

Definition 3.8. An i-v fuzzy subset $\bar{A}$ of a semigroup S is called idempotent if $(\bar{A}\bar{A})(x) = \bar{A}(x) \quad \forall x \in S$.

Theorem 3.9. Let S be a semigroup. Then the following are equivalent.
(i) S is both regular and intraregular semigroup
(ii) Every i-v fuzzy bi-ideal $\bar{A}$ is idempotent.

Proof: Let $\bar{A}$ be an i-v fuzzy bi-ideal of an regular and intraregular semigroup S.
Since $\bar{A}$ is an i-v fuzzy bi-ideal $\bar{A}(xy) \geq Min^i\{\bar{A}(x), \bar{A}(y)\}$ and
$\bar{A}(xyz) \geq Min^i\{\bar{A}(x), \bar{A}(z)\}$ ………(1)
First we will prove that $\bar{A} \circ \bar{A} \subseteq \bar{A}$
Let $a \in S$. Since S is regular $\exists x \in S$ such that $a = axa$
$$(\bar{A} \circ \bar{A})(a) = \underset{a=uv}{Sup^i}\left\{Min^i\{\bar{A}(u), \bar{A}(v)\}\right\}$$
$$\leq \underset{a=uv}{Sup^i}\{\bar{A}(uv)\} \qquad \text{by (1)}$$
$$= Sup^i\{\bar{A}(a)\}$$
$$= \bar{A}(a)$$
That is $\bar{A} \circ \bar{A} \subseteq \bar{A}$.
Now we will prove that $\bar{A} \subseteq \bar{A} \circ \bar{A}$. Since S is regular $\forall a \in S, \exists x \in S$ such that $a = axa$.
And since S is intraregular $\forall a \in S, \exists x', y' \in S$ such that $a = x'a^2y'$.
Therefore $a = axa$
$$= axaxa$$
$$= (ax)(x'a^2y')(xa)$$
$$= \left(axx'a\right)(ay'xa).$$

Now $(\bar{A}\circ\bar{A})(a) = \underset{a=uv}{Sup^i}\left\{Min^i\{\bar{A}(u),\bar{A}(v)\}\right\}$
$\geq Min^i\{\bar{A}(axx'a),\bar{A}(ay'xa)\}$
$\geq Min^i\left\{Min^i\{\bar{A}(a),\bar{A}(a)\},Min^i\{\bar{A}(a),\bar{A}(a)\}\right\}$ by (1)
$= Min^i\{\bar{A}(a),\bar{A}(a)\}$
$= \bar{A}(a)$

$$\bar{A}\circ\bar{A}\supseteq\bar{A}$$

Therefore $\bar{A}$ is idempotent.
Conversely assume that any i-v fuzzy bi-ideal $\bar{A}$ of S is idempotent. That is $\bar{A}\circ\bar{A}=\bar{A}$.
Now let B be any bi-ideal of S. Therefore $B^2\subseteq B$ and $\overline{\chi_B}$ is an i-v fuzzy bi-ideal of S.
By our assumption $\overline{\chi_B}\circ\overline{\chi_B}=\overline{\chi_B}$.
Let $a\in B$. Therefore $\overline{\chi_B}(a)=\bar{1}$ which implies $(\overline{\chi_B}\circ\overline{\chi_B})(a)=\bar{1}$ and therefore
$\underset{a=uv}{Sup^i}\left\{Min^i\{\overline{\chi_B}(u),\overline{\chi_B}(v)\}\right\}=\bar{1}$.
Thus there exist $b,c\in B$ such that $a=bc$. Therefore $a\in B^2$
Hence $B\subseteq B^2$ and hence $B=B^2$.
Then by lemma 3.6 S is both regular and intraregular.

Lemma 3.10. (Theorem 2.7.2 [2]) A semigroup is regular if and only if for every i-v fuzzy right ideal $\bar{A}$ and every i-v fuzzy left ideal $\bar{B}$ of S, we have $\bar{A}\circ\bar{B}=\bar{A}\cap\bar{B}$.

Theorem 3.11. Let S be an ordered semigroup. Then the following are equivalent.

(i) S is regular and intraregular
(ii) $\bar{A}\cap\bar{B}\subseteq(\bar{A}\circ\bar{B})\cap(\bar{B}\circ\bar{A})$ for any i-v fuzzy bi-ideals $\bar{A}$ and $\bar{B}$ of S
(iii) $\bar{A}\cap\bar{B}\subseteq(\bar{A}\circ\bar{B})\cap(\bar{B}\circ\bar{A})$ for every i-v fuzzy bi-ideal $\bar{A}$ and every i-v fuzzy left ideal $\bar{B}$ of S.
(iv) $\bar{A}\cap\bar{B}\subseteq(\bar{A}\circ\bar{B})\cap(\bar{B}\circ\bar{A})$ for every i-v fuzzy right ideal $\bar{A}$ and every i-v fuzzy bi-ideal $\bar{B}$ of S.
(v) $\bar{A}\cap\bar{B}\subseteq(\bar{A}\circ\bar{B})\cap(\bar{B}\circ\bar{A})$ for every i-v fuzzy right ideal $\bar{A}$ and every i-v fuzzy left ideal $\bar{B}$ of S.

Proof: $(i)\Rightarrow(ii)$. let $\bar{A}$ and $\bar{B}$ are i-v fuzzy bi-ideals of S. And $a\in S$.
Then since S is both regular and intraregular, there exists $x\in S$ such that $a=axa=axaxa$
And there exist $y,z\in S$ such that $a=ya^2z$.
Thus $a=axa=axaxa=ax(ya^2z)xa=(axya)(azxa)$.
Since $\bar{A}$ and $\bar{B}$ are i-v fuzzy bi-ideals of S, we have
$\bar{A}(axya)\geq Min^i\{\bar{A}(a),\bar{A}(a)\}=\bar{A}(a)$ and
$\bar{B}(azxa)\geq Min^i\{\bar{B}(a),\bar{B}(a)\}=\bar{B}(a)$.
Then $(\bar{A}\circ\bar{B})(a) = \underset{a=uv}{Sup^i}\left\{Min^i\{\bar{A}(u),\bar{B}(v)\}\right\}$
$\geq Min^i\{\bar{A}(axya),\bar{B}(azxa)\}$
$\geq Min^i\{\bar{A}(a),\bar{B}(a)\}$
$=(\bar{A}\cap\bar{B})(a)$
which means that $\bar{A}\cap\bar{B}\subseteq\bar{A}\circ\bar{B}$.

In the same way we can show that $\bar{A} \cap \bar{B} \subseteq \bar{B} \circ \bar{A}$.
Hence $\bar{A} \cap \bar{B} \subseteq (\bar{A} \circ \bar{B}) \cap (\bar{B} \circ \bar{A})$.
Since every i-v fuzzy left (right) ideal of S is a i-v fuzzy bi-ideal of S, we have $(ii) \Rightarrow (iii), (ii) \Rightarrow (iv),\ (ii) \Rightarrow (v), (iii) \Rightarrow (v)$ and $(iv) \Rightarrow (v)$ are clear.
$(v) \Rightarrow (i)$. Let $\bar{A}$ and $\bar{B}$ are i-v fuzzy right ideal and a i-v fuzzy left ideal of S respectively.
By hypothesis, $\bar{A} \cap \bar{B} \subseteq (\bar{A} \circ \bar{B}) \cap (\bar{B} \circ \bar{A}) \subseteq \bar{B} \circ \bar{A}$
By Theorem 3.3 S is intraregular.
On the other hand, $\bar{A} \cap \bar{B} \subseteq (\bar{A} \circ \bar{B}) \cap (\bar{B} \circ \bar{A}) \subseteq \bar{A} \circ \bar{B}$
But $\bar{A} \circ \bar{B} \subseteq \bar{A} \circ \bar{S} \subseteq \bar{A}$ and $\bar{A} \circ \bar{B} \subseteq \bar{S} \circ \bar{B} \subseteq \bar{B}$ implies $\bar{A} \circ \bar{B} \subseteq \bar{A} \cap \bar{B}$
Thus $\bar{A} \circ \bar{B} = \bar{A} \cap \bar{B}$
By Lemma 3.10 S is regular.
Thus S is both regular and intraregular.
This completes the proof.

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