Intern. J. Fuzzy Mathematical Archive
Vol. 4, No. 2, 2014, 88-95
ISSN: 2320 –3242 (P), 2320 –3250 (online)
Published on 20 June 2014
www.researchmathsci.org

International Journal of
Fuzzy Mathematical
Archive

A Note on Fuzzy Labeling

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Received 5 May 2014; accepted 15 May 2014

Abstract. The concept of fuzzy magic labeling is introduced.Fuzzy magic labeling for some graphs like path, cycle and star graph are defined. It is proved that, every fuzzy magic graph is a fuzzy labeling graph, but the converse is not true. And we show that the removal of a fuzzy bridge from a fuzzy magic graph G, such that G* is a cycle with odd number of nodes is a fuzzy magic graph.And also some properties related to fuzzy bridge and fuzzy cut node have been discussed.

Keywords: Fuzzy labelling, Fuzzy magic labelling, fuzzy bridge and fuzzy cut node

AMS mathematics Subject Classification (2010): 03E72, 05C72, 05C78

1. Introduction

Fuzzy is a newly emerging mathematical frame work to exemplify the phenomenon of uncertainty in real life tribulations. It was introduced by Zadeh in 1965, and the concepts were pioneered by various independent researches viz; Rosenfeld [10], Yeh and Bang [18] during 1970's. Bhattacharya has established the connectivity concepts between fuzzy cut nodes and fuzzy bridges titled "*Some remarks on fuzzy graphs*" [1]. Several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness were explored by them. There are many problems, which can be solved with the help of the fuzzy graphs.

Though it is very young, it has been growing fast and has numerous applications in various fields. Further, research on fuzzy graphs has been witnessing an exponential growth; both within mathematics and in its applications in science and Technology. A fuzzy graph is the generalisation of the crisp graph. Therefore it is natural that many properties are similar to crisp graph and also it deviates at many places.

In crisp graph, A bijection f: $V \cup E \rightarrow N$ that assigns to each vertex and/or edge if G= (V, E), a unique natural number is called a labeling. The concept of magic labeling in crisp graph was motivated by the notion of magic squares in number theory. The notion of magic graph was first introduced by J.Sedlacek [11] in 1964. He defined a graph to be magic if it has an edge-labeling, within the range of real numbers, such that the sum of the labels around any vertex equals some constant, independent of the choice of vertex. These labelingshave been studied by B.M.Stewart [13, 14] who called thelabeling assuper magic if the labels are consecutive integers, starting from 1. Several others have studied these labelings.

Kotzig and Rosa [3] defined a magic labeling to be a total labeling in which the labels are the integers from 1 to$|V(G)| + |E(G)|$. The sum of labels on an edge and its two endpoints is constant. Recently HikoeEnomoto [2] et al., introduced the name super edge magic for magic labeling in the sense of Kotzig and Rosa, with the added property that the *v* vertices receive the smaller labels. Many other researchers have investigated different forms of magic graphs. For example see Selvam Avadayappan [12] et al., A.A.G.Ngurah [9] et al.Trenkler [16].

In this paper, section 1.1 contains basic definitions and in section 1.2 a new concept of fuzzy magic labeling has been introduced and also fuzzystar graph is defined. In section 2 fuzzy magic labeling for some graphs like path, cycle and star are defined. In section 3, some properties and results with fuzzy bridge and fuzzy cut nodes are discussed. The graphs which are considered in this paper are finite and connected.

All basic definitions and symbols are followed as in [4, 6, 7,15, 17].

1.1. Preliminaries

Let U and V be two sets. Then ρ is said to be a *fuzzy relation* from U into V if ρ is a fuzzy set of U×V.A *fuzzy graph* G = (σ, μ) is a pair of functions σ: V → [0, 1] and μ: V×V → [0, 1], where for all $u, v \in V$, we have $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$.A *path*P in a fuzzy graph is a sequence of distinct nodes $v_1, v_2, ..., v_n$ such that$\mu(v_i, v_{i+1}) > 0$; $1 \leq i \leq n$; here $n \geq 1$ is called the *length of the path*P. The consecutive pairs (v_i, v_{i+1}) are called the *edge of the path*.A path P is called *a cycle* if $v_1 = v_n$ and$n \geq 3$. The *strength* of a path P is defined as $\bigwedge_{i=1}^{n} \mu(v_i, v_{i+1})$.Let G :(σ, μ) be a fuzzy graph. The *degree* of a vertex *v* is defined as $d(v) = \sum_{u \neq v, u \in V} \mu(v, u)$.Let G :(σ, μ) be a fuzzy graph. The *strong degree* of a node *v* is defined as the sum of membership values of all strong edges incident at *v*. It is denoted by $d_s(v)$. Also if $N_s(v)$ denote the set of all strong neighbours of *v*, then $d_s(v) = \sum_{u \in Ns(v)} \mu(v, u)$.An edge is called a *fuzzy bridge* of G if its removal reduces the strength of connectedness between some pair of nodes in G.A node is a *fuzzy cut node* of G = (σ, μ) if removal of it reduces the strength of connectedness between some other pair of nodes.

1.2. Fuzzy labeling

Definition 1.2.1. [8] A graph G = (σ, μ) is said to be a fuzzy labeling graph, if σ: V → [0, 1] and μ: V×V → [0, 1] is bijective such that the membership value of edges and vertices are distinct and $\mu(u, v) < \sigma(u) \wedge \sigma(v)$ for all u, v ∈V.

Example 1.

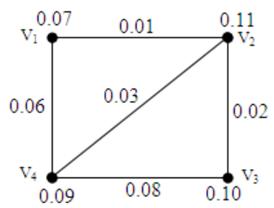

Figure1: A fuzzy labeling graph

In Fig.1, σ and μ are bijective, such that no vertices and edges ofG receives the same membership value.

Definition 1.2.2. A fuzzy labeling graph is said to be a fuzzy magic graph if σ (u) + μ (u, v) +σ (v) has a same value for all u, v∈V which is denoted as$m_o(G)$.

Example 2.

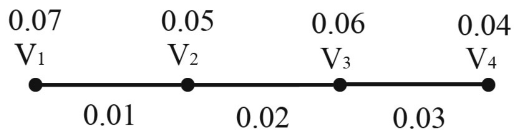

Figure 2: A fuzzy magic path graph.

$m_o(P) = 0.13$.
In Fig. 2 $\sigma(V_1) + \mu(V_1, V_2) + \sigma(V_2) = 0.07+0.01+0.05 = 0.13$, for all $V_1, V_2 \in V$.

Definition1 2.3. A *star* in a fuzzy graph consist of two node setsV and U with |V|= 1 and |U|> 1, such that$\mu(v, u_i) > 0$ and $\mu(u_i, u_{i+1}) = 0$, $1 \le i \le n$. It is denoted by $S_{1,n}$.

Example3.

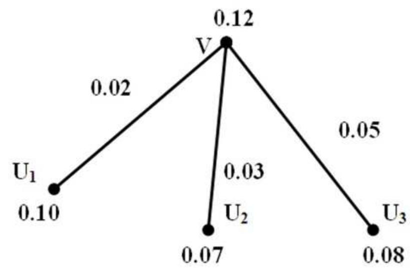

Figure 3: Afuzzy star graph

Definition 1.2.4. [8] The fuzzy labeling graph H= (τ, ρ) is called a fuzzy labeling subgraph of G= (σ, μ) if$\tau(u) \le \sigma(u)$ for all u$\in$ V and$\rho(u, v) \le \mu(u, v)$, for all u, v $\in$V.

2. Main Results

Proposition 2.1. EverypathP is a fuzzy magic graph.

Proof: Let P be a path with length $n \ge 1$ and $v_1, v_2, \ldots v_n$ & $v_1v_2, v_2v_3, \ldots, v_{n-1}v_n$ are the nodes and edges of P.Let $z \rightarrow (0, 1]$ such that one can choose z = 0.1 if n ≤ 4 and z = 0.01 if n ≥ 5. if the length of the path P is odd, then the fuzzy labeling is defined as follows.

$$\sigma(v_{2i-1}) = (2n+2-i)\, z,\ 1 \le i \le \frac{n+1}{2}$$

$$\sigma(v_{2i}) = \text{Min}\left\{\sigma(v_{2i-1}) \,/ 1 \le i \le \frac{n+1}{2}\right\} - i\,(z),\ 1 \le i \le \frac{n+1}{2}$$

$$\mu(v_{n-i+2}, v_{n+1-i}) = \text{Max}\{\sigma(v_i) \,/ 1 \le i \le n+1\} - \text{Min}\{\sigma(v_i) \,/ 1 \le i \le n+1\} - (i-1)z,\ 1 \le i \le n$$

Case (i) i is even

Then $i = 2x$ for any positive integer$x$

For each edge v_i, v_{i+1}

$m_o(P) = \sigma(v_i) + \mu(v_i, v_{i+1}) + \sigma(v_{i+1})$
$= \sigma(v_{2x}) + \mu(v_{2x}, v_{2x+1}) + \sigma(v_{2x+1})$
$= \text{Min}\left\{\sigma(v_{2i-1}) / 1 \le i \le \frac{n+1}{2}\right\} - x\,(z) + \text{Max}\,\{\sigma(v_i) / 1 \le i \le n+1\}$

$- \text{Min}\,\{\sigma(v_i) / 1 \le i \le n+1\} - (n-2x)\,z + (2n-x+1)\,z$

$= \text{Min}\left\{\sigma(v_{2i-1}) / 1 \le i \le \frac{n+1}{2}\right\} + \text{Max}\,\{\sigma(v_i) / 1 \le i \le n+1\} - \text{Min}\{\sigma(v_i) / 1 \le i \le n+1\}$
$+ (n+1)z$

Case (ii) *i* is odd
Then $i = 2x+1$for any positive integer x
For each edge v_i, v_{i+1}
$m_o(P) = \sigma(v_i) + \mu(v_i, v_{i+1}) + \sigma(v_{i+1})$
$= \sigma(v_{2x+1}) + \mu(v_{2x+1}, v_{2x+2}) + \sigma(v_{2x+2})$
$= (2n-x+1)z + \text{Max}\,\{\sigma(v_i) / 1 \le i \le n+1\} - \text{Min}\,\{\sigma(v_i) / 1 \le i \le n+1\} - (n-2x-1)$
$+ \text{Min}\left\{\sigma(v_{2i-1}) / 1 \le i \le \frac{n+1}{2}\right\} - (x+1)z$
$= \text{Min}\left\{\sigma(v_{2i-1}) / 1 \le i \le \frac{n+1}{2}\right\} + \text{Max}\,\{\sigma(v_i) / 1 \le i \le n+1\} - \text{Min}\,\{\sigma(v_i) / 1 \le i \le n+1\}$
$+ (n+1)z.$

If the length of the path P is even then it has the following labeling.

$\sigma(v_{2i}) = (2n+2-i)\,z,\ 1 \le i \le \frac{n}{2}$
$\sigma(v_{2i-1}) = \text{Min}\left\{\sigma(v_{2i}) / 1 \le i \le \frac{n}{2}\right\} - i\,(z),\ 1 \le i \le \frac{n+2}{2}$
$\mu(v_{n-i+2}, v_{n-i+1}) = \text{Max}\,\{\sigma(v_i) / 1 \le i \le n+1\}$
$- \text{Min}\,\{\sigma(v_i) / 1 \le i \le n+1\} - (i-1)z,\ 1 \le i \le n$

Case (iii) *i* is even
Then $i = 2x$ for any positive integer x
For each edge v_i, v_{i+1}
$m_o(P) = \sigma(v_i) + \mu(v_i, v_{i+1}) + \sigma(v_{i+1})$
$= \sigma(v_{2x}) + \mu(v_{2x}, v_{2x+1}) + \sigma(v_{2x+1})$
$= (2n+2-x)z + \text{Max}\,\{\sigma(v_i) / 1 \le i \le n+1\} - \text{Min}\,\{\sigma(v_i) / 1 \le i \le n+1\} -$
$(n-2x)\,z + \text{Min}\left\{\sigma(v_{2i}) / 1 \le i \le \frac{n}{2}\right\} - (x+1)z$
$= \text{Min}\left\{\sigma(v_{2i}) / 1 \le i \le \frac{n}{2}\right\} + \text{Max}\,\{\sigma(v_i) / 1 \le i \le n+1\} - \text{Min}\,\{\sigma(v_i) / 1 \le i \le n+1\}$
$+ (n+1)z$

Case (iv) *i* is odd
Then $i = 2x+1$ for any positive integer x
For each edge v_i, v_{i+1}
$m_o(P) = \sigma(v_i) + \mu(v_i, v_{i+1}) + \sigma(v_{i+1})$
$= \sigma(v_{2x+1}) + \mu(v_{2x+1}, v_{2x+2}) + \sigma(v_{2x+2})$
$= \text{Min}\left\{\sigma(v_{2i}) / 1 \le i \le \frac{n}{2}\right\} - (x+1)z - (n-2x-1)\,z$

$+ \text{Max}\,\{\sigma(v_i) / 1 \le i \le n+1\} - \text{Min}\,\{\sigma(v_i) / 1 \le i \le n+1\} + (2n-x+1)z$

$= \text{Min}\left\{\sigma(v_{2i}) / 1 \le i \le \frac{n}{2}\right\} + \text{Max}\,\{\sigma(v_i) / 1 \le i \le n+1\}$
$- \text{Min}\,\{\sigma(v_i) / 1 \le i \le n+1\} + (n+1)z.$

Therefore in both the cases the magic value $m_o(P)$ is same and unique. Thus P is fuzzy magic graph.

Proposition 2.2. Every graph G, such that G* is a cycle with odd number of nodes *is a* fuzzy magic graph.
Proof: LetG* be a cycle with odd number of nodes and $v_1, v_2, \ldots v_n$ & $v_1v_2, v_2v_3, \ldots, v_nv_1$ be the nodes and edges of C_n.Let $z \rightarrow (0, 1]$ such that one can choose z = 0.1 if n ≤ 3 and z = 0.01 if n ≥ 4. The fuzzy labeling is defined as follows.

$\sigma(v_{2i}) = (2n+1-i)\, z,\ 1 \le i \le \frac{n-1}{2}$

$\sigma(v_{2i-1}) = \text{Min}\left\{\sigma(v_{2i}) / 1 \le i \le \frac{n-1}{2}\right\} - i\,(z),\ 1 \le i \le \frac{n+1}{2}$

$\mu(v_1, v_n) = \frac{1}{2}\text{Max}\,\{\sigma(v_i) / 1 \le i \le n\}$

$\mu(v_{n-i+1}, v_{n-i}) = \mu(v_1, v_n) - i\,(z),\ 1 \le i \le n-1$

Case (i) *i* is even

Then $i = 2x$for any positive integer x

For each edge v_i, v_{i+1}

$m_o(C_n) = \sigma(v_i) + \mu(v_i, v_{i+1}) + \sigma(v_{i+1})$

$= \sigma(v_{2x}) + \mu(v_{2x}, v_{2x+1}) + \sigma(v_{2x+1})$

$= (2n+1-x)\, z + \frac{1}{2}\text{Max}\,\{\sigma(v_i) / 1 \le i \le n\} - (n-2x)\, z + \text{Min}\left\{\sigma(v_{2i}) / 1 \le i \le \frac{n-1}{2}\right\} - (x+1)z$

$= \frac{1}{2}\text{Max}\,\{\sigma(v_i) / 1 \le i \le n\} + \text{Min}\left\{\sigma(v_{2i}) / 1 \le i \le \frac{n-1}{2}\right\} + n\,(z)$

Case (ii) *i* is odd

Then $i = 2x+1$ for any positive integer x

For each edge v_i, v_{i+1}

$m_o(C_n) = \sigma(v_i) + \mu(v_i, v_{i+1}) + \sigma(v_{i+1})$

$= \sigma(v_{2x+1}) + \mu(v_{2x+1}, v_{2x+2}) + \sigma(v_{2x+2})$

$= \text{Min}\left\{\sigma(v_{2i}) / 1 \le i \le \frac{n-1}{2}\right\} - (x+1)z + \frac{1}{2}\text{Max}\,\{\sigma(v_i) / 1 \le i \le n\} - (n-2x-1)\, z + (2n-x)\, z$

$= \frac{1}{2}\text{Max}\,\{\sigma(v_i) / 1 \le i \le n\} + \text{Min}\left\{\sigma(v_{2i}) / 1 \le i \le \frac{n-1}{2}\right\} + n\,(z)$

Therefore from above cases G is a fuzzy magic graph if it has odd number of nodes.

Proposition 2.3. For any n ≥ 2, star $S_{1,n}$is a fuzzy magic graph.
Proof: Let $S_{1,n}$* be a star graph with $v, u_1, u_2, \ldots, u_n$ as nodes and $vu_1, vu_2, \ldots, vu_n$ as edges. Let $z \rightarrow (0, 1]$ such that one can choose z = 0.1 if n ≤ 4 and z = 0.01 if n ≥ 5. Such a fuzzy labeling is defined as follows.

$\sigma(u_i) = [2(n+1) - i]\, z,\ 1 \le i \le n$

$\sigma(v) = \text{Min}\,\{\sigma(u_i) / 1 \le i \le n\} - z$

$\mu(v, u_{n-i}) = \text{Max}\,\{\sigma(u_i), \sigma(v)/1 \le i \le n\} - \text{Min}\,\{\sigma(u_i), \sigma(v)/1 \le i \le n\}$
$- i\,(z),\ 0 \le i \le n-1.$

Case (i) *i* is even

Then $i = 2x$ for any positive integer x

For each edge v, u_i

$m_o(S_{1,n}) = \sigma(v) + \mu(v, u_i) + \sigma(u_i)$

$= \sigma(v) + \mu(v, u_{2x}) + \sigma(u_{2x})$

$= \text{Min}\,\{\sigma(u_i) / 1 \le i \le n\} - (z) + \text{Max}\,\{\sigma(u_i), \sigma(v)/1 \le i \le n\}$

$-$ Min $\{\sigma(u_i),\sigma(v)/1\leq i\leq n\}$–$(n$-$2x)\ z + [2(n+1) - 2x]z$

$= $ Min $\{\sigma(u_i)\ /1 \leq i \leq n\}$ + Max $\{\sigma(u_i),\ \sigma(v)/1\leq i \leq n\}$
- Min $\{\sigma(u_i),\ \sigma(v)/1\leq i \leq n\} + (n+1)\ z$

Case (ii) *i* is odd
Then $i = 2x+1$for any positive integer x
For each edge v,u_i
$m_o(S_{1,n}) = \sigma(v) + \mu\ (v,u_i) + \sigma(u_i)$
$= \sigma(v) + \mu\ (v, u_{2x+1}) + \sigma(u_{2x+2})$
$=$ Min $\{\sigma(u_i)\ /1 \leq i \leq n\}$- z + Max $\{\sigma(u_i),\ \sigma(v)/1\leq i \leq n\}$
- Min $\{\sigma(u_i),\ \sigma(v)/1\leq i \leq n\}$- $(n$-$2x$-$1)z + [2(n$-$x)]\ z$

$=$ Min $\{\sigma(u_i)\ /1 \leq i \leq n\}$ + Max $\{\sigma(u_i),\ \sigma(v)/1\leq i \leq n\}$
-Min $\{\sigma(u_i),\sigma(v)/1\leq i \leq n\} + (n+1)\ z.$

From the above cases one can easily verify that all star graphs are fuzzy magic graphs.

Remark 1. One can observe the same labeling holds good if we choose the value of z as 0.03, 0.05 etc for the proposition 2.3, 2.4 and 2.5.

Remark 2.
(i) If G is a fuzzy magic graph, then $d(u) \neq d(v)$ for any pair of nodes u and v.
(ii) For any fuzzy magic graph, $0 \leq d_s(v) \leq d(v)$.
(iii) Sum of the degree of all nodes in a fuzzy magic graph is equal to twice the sum of membership values of all edges.(i.e.) $\sum_{i=1}^{n} d\ (v_i) = 2\sum_{u\neq v} \mu\ (u, v)$.
(iv) Sum of strong degree of all nodes in a fuzzy magic graph is equal to twice the sum of the membership values of all strong arcs in G
i.e. $\sum_{i=1}^{n} d_s(v_i) = 2\sum_{u\in N_s(v)} \mu(v, u)$

3. Properties of fuzzy magic graphs
Proposition 3.1. Every fuzzy magic graph is a fuzzy labeling graph, but the converse is not true.
Proof: This is immediate from the definition 1.2.2.
Proposition 3.2. For every fuzzy magic graph G, there exists atleast one fuzzy bridge.
Proof: Let Gbe a fuzzy magic graph, such that there exist only one edge $\mu(x,y)$ with maximum value, since μ is bijective. Now we claim that $\mu(x,y)$ is a fuzzy bridge. If we remove the edge (x,y) fromG, then in its subgraph we'weμ'^{∞} (x, y) < μ (x,y), which implies (x, y) is a fuzzy bridge.

Proposition 3.3. Removal of a fuzzy cut node from a fuzzy magic graph G, such that G* is a path is also a fuzzy magic graph.
Proof: Since G* is a path, there exist at least one cut node. Now if we remove thatcut node from Gthen it either becomes a smaller path or disconnected path, anyway it remains to be a path with odd or even length, by proposition2.1, it is concluded that removal of a fuzzy cut node from a fuzzy magic graph is also a fuzzy magic graph.

Proposition 3.4. Removal of a fuzzy bridge from a fuzzy magic graph G, such that G* is a cycle with odd number of nodesis a fuzzy magic graph.
Proof: Since G is a fuzzy magic graph with odd number of nodes. We can choose any path, say (u, v) then there must be at least one fuzzy bridge, whose removal from Gwill result as a path of odd or even length. By proposition 2.1, the removal of a fuzzy bridge from a fuzzy magic graph G is also a fuzzy magic graph.

Remark 3.
(i) Removal of a fuzzy cut node from the fuzzy magic graph G such that G* is a cycle with odd number of nodesis also a fuzzy magic graph.
(ii) For all fuzzy magic graph G, such that G*is a cycle with odd number of nodes, there exist at least one pair of nodes u and v such that $d_s(u)= d_s(v)$.

4. Concluding remarks

In this paper, the concept of fuzzy labeling and fuzzy magic labeling has been introduced. As it is a new one, much more work could be done to find many fuzzy magic graphs and also many properties could be derived. The remaining work will be discussed in the forthcoming papers.

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