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The Degree of an Edge in Union and Join of Two Fuzzy Graphs

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Abstract. A fuzzy graph can be obtained from two given fuzzy graphs using union and join. In this paper, we find the degree of an edge in fuzzy graphs formed by these operations in terms of the degree of edges in the given fuzzy graphs in some particular cases.

Keywords: Degree of a vertex, degree of an edge, union and join.

AMS Mathematics Subject Classification (2010): 03E72, 05C72

1. Introduction

Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975 [4]. Though it is very young, it has been growing fast and has numerous applications in various fields. During the same time Yeh and Bang have also introduced various concepts in connectedness in fuzzy graphs [9]. Mordeson and Peng introduced the concept of operations on fuzzy graphs. Sunitha and Vijayakumar discussed about complementary of the operations of union, join, Cartesian product and composition on two fuzzy graphs [8]. The degree of a vertex in fuzzy graphs which are obtained from two given fuzzy graphs using these operations were discussed by Nagoor Gani and Radha [3]. Radha and Kumaravel introduced the concept of degree of an edge and total degree of an edge in fuzzy graphs [5]. We study about the degree of an edge in fuzzy graphs which are obtained from two given fuzzy graphs using the operations of union and join. In general, the degree of an edge in union and join of two fuzzy graphs G_1 and G_2 cannot be expressed in terms of these in G_1 and G_2. In this paper, we find the degree of an edge in union and join of two fuzzy graphs G_1 and G_2 in terms of the degree of edges of G_1 and G_2 in some particular cases.

First we go through some basic concepts.

Definition 1.1. [3] A fuzzy subset of a set V is a mapping σ from V to [0, 1]. A fuzzy graph G is a pair of functions G: (σ, μ) where σ is a fuzzy subset of a non-empty set V and μ is a symmetric fuzzy relation on σ, (i.e.) $\mu(x, y) \le \sigma(x) \wedge \sigma(y)$ for all $x, y \in V$. The underlying crisp graph of G: (σ, μ) is denoted by G^*: (V, E) where $E \subseteq V\times V$.

Throughout this paper, $G_1: (\sigma_1, \mu_1)$ and $G_2: (\sigma_2, \mu_2)$ denote two fuzzy graphs with underlying crisp graphs $G_1^*: (V_1, E_1)$ and $G_2^*: (V_2, E_2)$ with $|V_i| = p_i$, $i = 1, 2$. Also $d_{G_i^*}(u_i)$ denotes the degree of u_i in G_i^*.

Definition 1.2. [3] Let $G: (\sigma, \mu)$ be a fuzzy graph on $G^*: (V, E)$. The degree of a vertex u is $d_G(u) = \sum_{u \neq v} \mu(uv) = \sum_{uv \in E} \mu(uv)$.
The minimum degree of G is $\delta(G) = \wedge\{d_G(v) : v \in V\}$.
The maximum degree of G is $\Delta(G) = \vee\{d_G(v) : v \in V\}$.

Definition 1.3. [3] The total degree of a vertex $u \in V$ is defined by
$td_G(u) = \sum_{u \neq v} \mu(uv) + \sigma(u) = d_G(u) + \sigma(u)$.

Definition 1.4. [1] Let $G^*: (V, E)$ be a graph and let $e = uv$ be an edge in G^*. Then the degree of an edge $e = uv \in E$ is defined by $d_{G^*}(uv) = d_{G^*}(u) + d_{G^*}(v) - 2$.

Definition 1.5. [3] Let $G_1: (\sigma_1, \mu_1)$ and $G_2: (\sigma_2, \mu_2)$ be two fuzzy graphs with underlying graphs $G_1^*: (V_1, E_1)$ and $G_2^*: (V_2, E_2)$ and let $G^* = G_1^* \cup G_2^* = (V_1 \cup V_2, E_1 \cup E_2)$ be the union of G_1^* and G_2^*. Then the union of two fuzzy graphs G_1 and G_2 is a fuzzy graph $G = G_1 \cup G_2: (\sigma_1 \cup \sigma_2, \mu_1 \cup \mu_2)$ defined by

$$(\sigma_1 \cup \sigma_2)(u) = \begin{cases} \sigma_1(u), if u \in V_1 - V_2 \\ \sigma_2(u), if u \in V_2 - V_1 \\ \sigma_1(u) \vee \sigma_2(u), if u \in V_1 \cap V_2 \end{cases}$$

$$(\mu_1 \cup \mu_2)(uv) = \begin{cases} \mu_1(uv), if uv \in E_1 - E_2 \\ \mu_2(uv), if uv \in E_2 - E_1 \\ \mu_1(uv) \vee \mu_2(uv), if uv \in E_1 \cap E_2 \end{cases}.$$

Definition 1.6. [3] Let $G_1: (\sigma_1, \mu_1)$ and $G_2: (\sigma_2, \mu_2)$ be two fuzzy graphs with underlying graphs $G_1^*: (V_1, E_1)$ and $G_2^*: (V_2, E_2)$ with $V_1 \cap V_2 = \phi$ and let $G^* = G_1^* + G_2^* = (V_1 \cup V_2, E_1 \cup E_2 \cup E')$ be the join of G_1^* and G_2^*, where E' is the set of all edges joining the vertices of V_1 and V_2. Then the join(sum) of two fuzzy graphs G_1 and G_2 is a fuzzy graph $G = G_1 + G_2: (\sigma_1 + \sigma_2, \mu_1 + \mu_2)$ defined by

$(\sigma_1 + \sigma_2)(u) = (\sigma_1 \cup \sigma_2)(u), \forall u \in V_1 \cup V_2$ and

$$(\mu_1 + \mu_2)(uv) = \begin{cases} (\mu_1 \cup \mu_2)(uv), if uv \in E_1 \cup E_2 \\ \sigma_1(u) \wedge \sigma_2(v), if uv \in E' \end{cases}.$$

Definition 1.7. [2] The order of a fuzzy graph G is defined by $O(G) = \sum_{u \in V} \sigma(u)$.

Definition 1.8. [2] The size of a fuzzy graph G is defined by $S(G) = \sum_{uv\in E}\mu(uv)$.

Definition 1.9. [5] Let G: (σ, μ) be a fuzzy graph on G^*: (V, E). The degree of an edge uv is $d_G(uv) = d_G(u) + d_G(v) - 2\mu(uv) = \sum_{\substack{uw\in E\\ w\neq v}}\mu(uw) + \sum_{\substack{wv\in E\\ w\neq u}}\mu(wv)$.

The minimum degree of G is $\delta_E(G) = \wedge\{d_G(uv) : uv\in E\}$.
The maximum degree of G is $\Delta_E(G) = \vee\{d_G(uv) : uv\in E\}$.

Definition 1.10. [5] Let G: (σ, μ) be a fuzzy graph on G^*: (V, E). The total degree of an edge uv∈ E is defined by $td_G(uv) = d_G(uv) + \mu(uv) = d_G(u) + d_G(v) - \mu(uv)$.

Example 1.11.

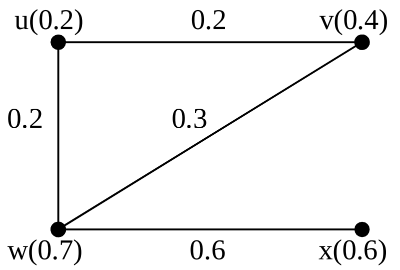

Figure 1.1. Fuzzy graph G: (σ, μ)

$d_G(u) = \mu(uv) + \mu(uw) = 0.2 + 0.2 = 0.4$,
$td_G(u) = d_G(u) + \sigma(u) = 0.4 + 0.2 = 0.6$.
$\delta(G) = \wedge\{d_G(v), \forall\, v\in V\} = \wedge\{0.4, 0.5, 1.1, 0.6\} = 0.4 = d_G(u)$.
$\Delta(G) = \vee\{d_G(v), \forall\, v\in V\} = \vee\{0.4, 0.5, 1.1, 0.6\} = 1.1 = d_G(w)$.
$d_G(uv) = \sum_{\substack{uw\in E\\ w\neq v}}\mu(uw) + \sum_{\substack{wv\in E\\ w\neq u}}\mu(wv) = 0.2 + 0.3 = 0.5$.
$td_G(uv) = d_G(uv) + \mu(uv) = 0.5 + 0.2 = 0.7$.
$\delta_E(G) = \wedge\{d_G(uv), \forall uv\in E\} = \wedge\{0.5, 1.1, 1.0, 0.5\} = 0.5 = d_G(uv) = d_G(wx)$.
$\Delta_E(G) = \vee\{d_G(uv), \forall uv\in E\} = \vee\{0.5, 1.1, 1.0, 0.5\} = 1.1 = d_G(uw)$.

2. Degree of an edge in union

For any $uv\in E_1\cup E_2$, fix $u\in V_1\cup V_2$. We have three cases to consider.

Case 1: $V_1\cap V_2 = \phi$.
Let $uv \in E_1 \cup E_2$ be any edge.
Hence $E_1\cap E_2= \phi$.
Therefore $uv\in E_1$ or $uv\in E_2$, but not both.

So $(\mu_1 \cup \mu_2)(uv) = \begin{cases} \mu_1(uv), if\, uv \in E_1 - E_2 \\ \mu_2(uv), if\, uv \in E_2 - E_1 \end{cases}$.

By definition, $d_{G_1 \cup G_2}(uv) = \sum_{uw \in E_1 \cup E_2, w \neq v} (\mu_1 \cup \mu_2)(uw) + \sum_{wv \in E_1 \cup E_2, w \neq u} (\mu_1 \cup \mu_2)(wv)$.

If $uv \in E_1$, $d_{G_1 \cup G_2}(uv) = \sum_{uw \in E_1, w \neq v} \mu_1(uw) + \sum_{wv \in E_1, w \neq u} \mu_1(wv)$.

$\therefore d_{G_1 \cup G_2}(uv) = d_{G_1}(uv)$.

Similarly, if $uv \in E_2$, $d_{G_1 \cup G_2}(uv) = d_{G_2}(uv)$.

Example 2.1.

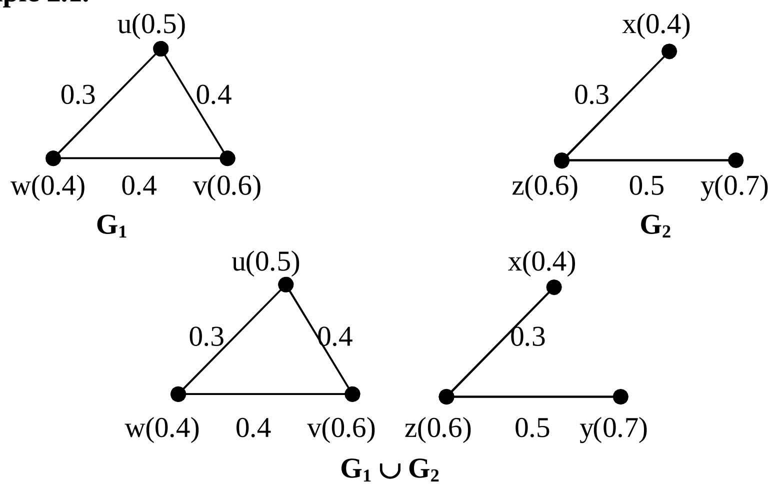

Figure 2.1:

Here $uv \in E_1$. Then $d_{G_1 \cup G_2}(uv) = d_{G_1}(uv) = 0.3 + 0.4 = 0.7$.

Case 2: $V_1 \cap V_2 \neq \phi$, $E_1 \cap E_2 = \phi$.

Then $uv \in E_1$ or $uv \in E_2$. Choose $uv \in E_1$. Then $uv \notin E_2$.

Also, if both $u, v \notin V_1 \cap V_2$, then it is of case 1.

So, consider either $u \in V_1 \cap V_2$ or $v \in V_1 \cap V_2$ or both $u, v \in V_1 \cap V_2$.

Subcase 1: $u \in V_1 \cap V_2$ or $v \in V_1 \cap V_2$.

When $u \in V_1 \cap V_2$,

By definition, $d_{G_1 \cup G_2}(uv) = d_{G_1 \cup G_2}(u) + d_{G_1 \cup G_2}(v) - 2(\mu_1 \cup \mu_2)(uv)$.

$= [d_{G_1}(u) + d_{G_2}(u)] + d_{G_1}(v) - 2\mu_1(uv)$.

$= [d_{G_1}(u) + d_{G_1}(v) - 2\mu_1(uv)] + d_{G_2}(u)$.

$\therefore d_{G_1 \cup G_2}(uv) = d_{G_1}(uv) + d_{G_2}(u)$, if $u \in V_1 \cap V_2$.

Similarly, $d_{G_1\cup G_2}(uv) = d_{G_1}(uv) + d_{G_2}(v)$, if v ∈ $V_1 \cap V_2$.

Thus, $$d_{G_1\cup G_2}(uv) = \begin{cases} d_{G_1}(uv) + d_{G_2}(u), & if\ u \in V_1 \cap V_2 \\ d_{G_1}(uv) + d_{G_2}(v), & if\ v \in V_1 \cap V_2 \end{cases}.$$

In a similar way, if $uv \in E_2$, then $d_{G_1\cup G_2}(uv) = \begin{cases} d_{G_2}(uv) + d_{G_1}(u), & if\ u \in V_1 \cap V_2 \\ d_{G_2}(uv) + d_{G_1}(v), & if\ v \in V_1 \cap V_2 \end{cases}$.

Example 2.2.

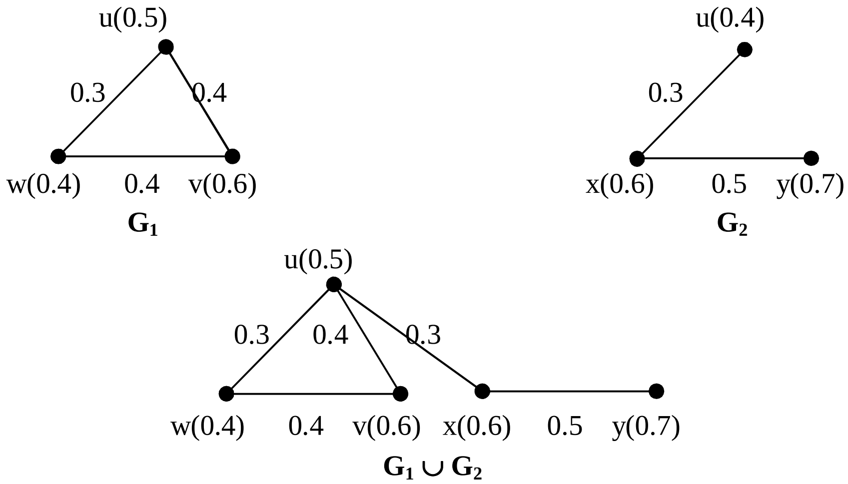

Figure 2.2:

Here $uv \in E_1$ and $u \in V_1 \cap V_2$. Then $d_{G_1\cup G_2}(uv) = d_{G_1}(uv) + d_{G_2}(u)$
= 0.7 + 0.3 = 1.0.

Subcase 2: $u, v \in V_1 \cap V_2$, $uv \in E_1$.

Since $E_1 \cap E_2 = \phi$, no edge incident at u or v is in $E_1 \cap E_2$.

Since the edges incident at u & v in G_1 and also in G_2 appear with the same membership values in $G_1 \cup G_2$.

By definition, $d_{G_1\cup G_2}(uv) = d_{G_1\cup G_2}(u) + d_{G_1\cup G_2}(v) - 2(\mu_1 \cup \mu_2)(uv)$.

$= d_{G_1}(u) + d_{G_2}(u) + d_{G_1}(v) + d_{G_2}(v) - 2\mu_1(uv)$.

$= [d_{G_1}(u) + d_{G_1}(v) - 2\mu_1(uv)] + d_{G_2}(u) + d_{G_2}(v)$.

$\therefore d_{G_1\cup G_2}(uv) = d_{G_1}(uv) + d_{G_2}(u) + d_{G_2}(v)$.

In a similar way, if $uv \in E_2$, then $d_{G_1\cup G_2}(uv) = d_{G_2}(uv) + d_{G_1}(u) + d_{G_1}(v)$.

Example 2.3.

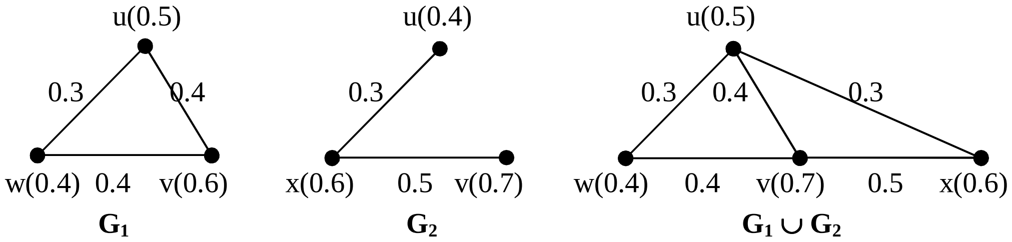

Figure 2.3:

Here $uv \in E_1$ and $u, v \in V_1 \cap V_2$. Then $d_{G_1 \cup G_2}(uv) = d_{G_1}(uv) + d_{G_2}(u) + d_{G_2}(v)$

= 0.7 + 0.3 + 0.5 = 1.5.

Case 3: $V_1 \cap V_2 \neq \phi$, $E_1 \cap E_2 \neq \phi$.

Then $uv \in E_1 \cap E_2$. Therefore $uv \in E_1$ and $uv \in E_2$.

So, consider either no edge incident at u & v other than uv is in $E_1 \cap E_2$ or some of the edges incident at u & v other than uv are in $E_1 \cap E_2$.

Subcase 1: No edge incident at u and v other than uv is in $E_1 \cap E_2$.

Then the edge incident at u or v is in either E_1 or in E_2, those edges appear with the same membership value in $G_1 \cap G_2$.

By definition,

$$d_{G_1 \cup G_2}(uv) = \sum_{uw \in E_1 \cup E_2, w \neq v} (\mu_1 \cup \mu_2)(uw) + \sum_{wv \in E_1 \cup E_2, w \neq u} (\mu_1 \cup \mu_2)(wv).$$

$$= \sum_{uw \in E_1 - E_2, w \neq v} \mu_1(uw) + \sum_{uw \in E_2 - E_1, w \neq v} \mu_2(uw) + \sum_{wv \in E_1 - E_2, w \neq u} \mu_1(wv) + \sum_{wv \in E_2 - E_1, w \neq u} \mu_2(wv).$$

$$= \sum_{uw \in E_1 - E_2, w \neq v} \mu_1(uw) + \sum_{wv \in E_1 - E_2, w \neq u} \mu_1(wv) + \sum_{uw \in E_2 - E_1, w \neq v} \mu_2(uw) + \sum_{wv \in E_2 - E_1, w \neq u} \mu_2(wv).$$

$\therefore\ d_{G_1 \cup G_2}(uv) = d_{G_1}(uv) + d_{G_2}(uv)$.

Example 2.4.

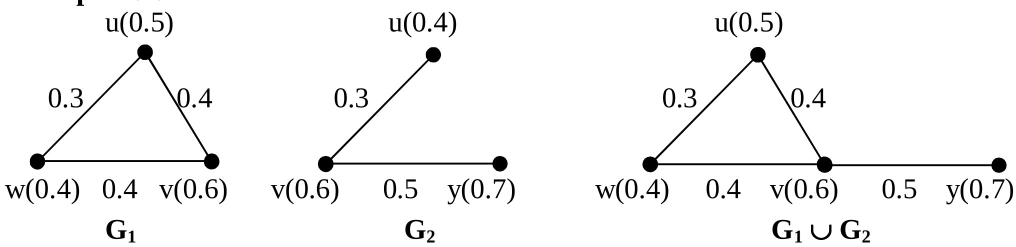

Figure 2.4:

Here $uv \in E_1 \cap E_2$.

Then $d_{G_1 \cup G_2}(uv) = d_{G_1}(uv) + d_{G_2}(uv)$ = 0.7 + 0.5 = 1.2.

Subcase 2: Some of the edges incident at u & v other than uv are in $E_1 \cap E_2$.

By definition, $d_{G_1\cup G_2}(uv) = \sum_{uw\in E_1\cup E_2, w\neq v}(\mu_1\cup\mu_2)(uw) + \sum_{wv\in E_1\cup E_2, w\neq u}(\mu_1\cup\mu_2)(wv)$.

$$= \sum_{uw\in E_1-E_2, w\neq v}\mu_1(uw) + \sum_{uw\in E_2-E_1, w\neq v}\mu_2(uw) + \sum_{wv\in E_1-E_2, w\neq u}\mu_1(wv) + \sum_{wv\in E_2-E_1, w\neq u}\mu_2(wv) + \sum_{uw\in E_1\cap E_2, w\neq v}\mu_1(uw)\vee\mu_2(uw) + \sum_{wv\in E_1\cap E_2, w\neq u}\mu_1(wv)\vee\mu_2(wv).$$

$$= \sum_{uw\in E_1-E_2, w\neq v}\mu_1(uw) + \sum_{wv\in E_1-E_2, w\neq u}\mu_1(wv) + \sum_{uw\in E_2-E_1, w\neq v}\mu_2(uw) + \sum_{wv\in E_2-E_1, w\neq u}\mu_2(wv) + \sum_{uw\in E_1\cap E_2, w\neq v}\mu_1(uw)\vee\mu_2(uw) + \sum_{wv\in E_1\cap E_2, w\neq u}\mu_1(wv)\vee\mu_2(wv) + \sum_{uw\in E_1\cap E_2, w\neq v}\mu_1(uw)\wedge\mu_2(uw) + \sum_{wv\in E_1\cap E_2, w\neq u}\mu_1(wv)\wedge\mu_2(wv) - \sum_{uw\in E_1\cap E_2, w\neq v}\mu_1(uw)\wedge\mu_2(uw) - \sum_{wv\in E_1\cap E_2, w\neq u}\mu_1(wv)\wedge\mu_2(wv)$$

$$= \sum_{uw\in E_1, w\neq v}\mu_1(uw) + \sum_{wv\in E_1, w\neq u}\mu_1(wv) + \sum_{uw\in E_2, w\neq v}\mu_2(uw) + \sum_{wv\in E_2, w\neq u}\mu_2(wv) - \sum_{uw\in E_1\cap E_2, w\neq v}\mu_1(uw)\wedge\mu_2(uw) - \sum_{wv\in E_1\cap E_2, w\neq u}\mu_1(wv)\wedge\mu_2(wv).$$

$$\therefore\ d_{G_1\cup G_2}(uv) = d_{G_1}(uv) + d_{G_2}(uv) - \sum_{uw\in E_1\cap E_2, w\neq v}\mu_1(uw)\wedge\mu_2(uw) - \sum_{wv\in E_1\cap E_2, w\neq u}\mu_1(wv)\wedge\mu_2(wv).$$

Example 2.5.

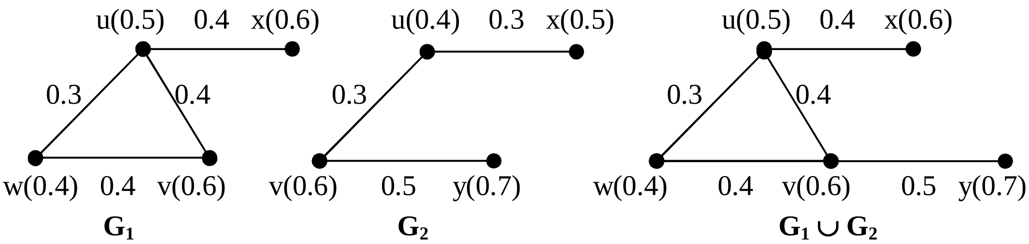

Figure 2.5:

Here uv $\in E_1 \cap E_2$.

Then $d_{G_1\cup G_2}(uv) = d_{G_1}(uv) + d_{G_2}(uv) - \sum_{uw\in E_1\cap E_2, w\neq v}\mu_1(uw)\wedge\mu_2(uw) - \sum_{wv\in E_1\cap E_2, w\neq u}\mu_1(wv)\wedge\mu_2(wv)$.

$\therefore\ d_{G_1 \cup G_2}(uv)$ = 1.1 + 0.8 – 0.3 – 0 = 1.6.

3. Degree of an edge in join

Here $V_1 \cap V_2 = \phi$. Hence $E_1 \cap E_2 = \phi$.

$$(\mu_1 + \mu_2)(uv) = \begin{cases} \mu_1(uv), if\, uv \in E_1 \\ \mu_2(uv), if\, uv \in E_2 \\ \sigma_1(u) \wedge \sigma_2(v), if\, uv \in E' \end{cases}$$

By definition,

$$d_{G_1+G_2}(uv) = \sum_{uw \in E_1 \cup E_2 \cup E', w \neq v} \mu(uw) + \sum_{wv \in E_1 \cup E_2 \cup E', w \neq u} \mu(wv).$$

i.e., $d_{G_1+G_2}(uv) = \sum_{uw \in E_1 \cup E_2, w \neq v} \mu(uw) + \sum_{wv \in E_1 \cup E_2, w \neq u} \mu(wv) + \sum_{uw \in E', w \neq v} \mu(uw) + \sum_{wv \in E', w \neq u} \mu(wv)$.

For any $uv \in E_1$,

$$d_{G_1+G_2}(uv) = \sum_{uw \in E_1, w \neq v} \mu_1(uw) + \sum_{wv \in E_1, w \neq u} \mu_1(wv) + \sum_{uw \in E'} \sigma_1(u) \wedge \sigma_2(w) + \sum_{vw \in E'} \sigma_1(v) \wedge \sigma_2(w).$$

$$\therefore d_{G_1+G_2}(uv) = d_{G_1}(uv) + \sum_{uw \in E'} \sigma_1(u) \wedge \sigma_2(w) + \sum_{vw \in E'} \sigma_1(v) \wedge \sigma_2(w) \qquad (4.1)$$

Similarly, for any $uv \in E_2$,

$$d_{G_1+G_2}(uv) = d_{G_2}(uv) + \sum_{wu \in E'} \sigma_1(w) \wedge \sigma_2(u) + \sum_{wv \in E'} \sigma_1(w) \wedge \sigma_2(v) \qquad (4.2)$$

and for any $uv \in E'$ with $u \in V_1$, $v \in V_2$,

$$d_{G_1+G_2}(uv) = \sum_{uw \in E_1} \mu_1(uw) + \sum_{wv \in E_2} \mu_2(wv) + \sum_{uw \in E', w \neq v} \sigma_1(u) \wedge \sigma_2(w) + \sum_{wv \in E', w \neq u} \sigma_1(w) \wedge \sigma_2(v)$$

$$= d_{G_1}(u) + d_{G_2}(v) + \sum_{uw \in E', w \neq v} \sigma_1(u) \wedge \sigma_2(w) + \sum_{wv \in E', w \neq u} \sigma_1(w) \wedge \sigma_2(v) \qquad (4.3)$$

Definition 3.1. [3] The relation $\sigma_1 \geq \sigma_2$ means that $\sigma_1(u) \geq \sigma_2(v)$, for every $u \in V_1$ and for every $v \in V_2$, where σ_I is a fuzzy subset of V_i, i = 1, 2.

Theorem 3.2. Let $G_1: (\sigma_1, \mu_1)$ and $G_2: (\sigma_2, \mu_2)$ be two fuzzy graphs.

1. If $\sigma_1 \geq \sigma_2$, then

$$d_{G_1+G_2}(uv) = \begin{cases} d_{G_1}(uv) + 2O(G_2), & if\ uv \in E_1 \\ d_{G_2}(uv) + p_1(\sigma_2(u) + \sigma_2(v)), & if\ uv \in E_2 \\ d_{G_1}(u) + d_{G_2}(v) + O(G_2) + (p_1 - 2)\sigma_2(v), & if\ uv \in E' \end{cases}.$$

2. If $\sigma_2 \geq \sigma_1$, then

$$d_{G_1+G_2}(uv) = \begin{cases} d_{G_1}(uv) + p_2(\sigma_1(u) + \sigma_1(v)), & \text{if } uv \in E_1 \\ d_{G_2}(uv) + 2O(G_1), & \text{if } uv \in E_2 \\ d_{G_1}(u) + d_{G_2}(v) + O(G_1) + (p_2 - 2)\sigma_1(u), & \text{if } uv \in E' \end{cases}.$$

Proof. We have $\sigma_1 \geq \sigma_2$.

From (4.1), for any $uv \in E_1$,

$$d_{G_1+G_2}(uv) = d_{G_1}(uv) + \sum_{uw \in E'} \sigma_1(u) \wedge \sigma_2(w) + \sum_{vw \in E'} \sigma_1(v) \wedge \sigma_2(w)$$

$$= d_{G_1}(uv) + \sum_{w \in V_2} \sigma_2(w) + \sum_{w \in V_2} \sigma_2(w)$$

$$= d_{G_1}(uv) + 2O(G_2).$$

From (4.2), for any $uv \in E_2$,

$$d_{G_1+G_2}(uv) = d_{G_2}(uv) + \sum_{wu \in E'} \sigma_1(w) \wedge \sigma_2(u) + \sum_{wv \in E'} \sigma_1(w) \wedge \sigma_2(v)$$

$$= d_{G_2}(uv) + \sum_{w \in V_1} \sigma_2(u) + \sum_{w \in V_1} \sigma_2(v)$$

$$= d_{G_2}(uv) + p_1\sigma_2(u) + p_1\sigma_2(v)$$

$$= d_{G_2}(uv) + p_1(\sigma_2(u) + \sigma_2(v))$$

From (4.3), for any $uv \in E'$,

$$d_{G_1+G_2}(uv) = d_{G_1}(u) + d_{G_2}(v) + \sum_{uw \in E', w \neq v} \sigma_1(u) \wedge \sigma_2(w) + \sum_{wv \in E', w \neq u} \sigma_1(w) \wedge \sigma_2(v)$$

$$= d_{G_1}(u) + d_{G_2}(v) + \sum_{w \in V_2} \sigma_2(w) + \sum_{w \in V_1} \sigma_2(v) - 2\sigma_2(v)$$

$$= d_{G_1}(u) + d_{G_2}(v) + O(G_2) + (p_1 - 2)\sigma_2(v).$$

Proof of (2) is similar to the proof of (1).

Example 3.3. Consider G_1 and G_2 in figure 3.1.

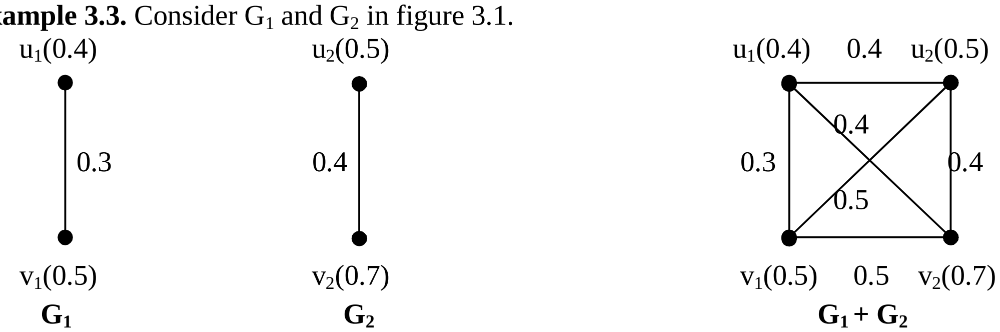

Figure 3.1:

We have $\sigma_2 \geq \sigma_1$. So by (2) of theorem 3.2,

$d_{G_1+G_2}(u_1v_1) = d_{G_1}(u_1v_1) + p_2(\sigma_1(u_1) + \sigma_1(v_1))$ = 0 + 2(0.4 + 0.5) = 1.8

The degree of an edge in union and join of two fuzzy graphs

$d_{G_1+G_2}(u_2v_2) = d_{G_2}(u_2v_2) + 2O(G_1)$ = 0 + 2(0.9) = 1.8

$d_{G_1+G_2}(u_1v_2) = d_{G_1}(u_1) + d_{G_2}(v_2) + O(G_1) + (p_2 - 2)\sigma_1(u_1)$

= 0.3 + 0.4 + 0.9 + (2 – 2)(0.4) = 1.6.

The above degrees can be verified in the figure of $G_1 + G_2$ given in figure 3.1.

Theorem 3.4. Let G_1: (σ_1, μ_1) and G_2: (σ_2, μ_2) be two fuzzy graphs such that $\sigma_1 \wedge \sigma_2$ is a constant function.

Then $d_{G_1+G_2}(uv) = \begin{cases} d_{G_1}(uv) + 2cp_2, & \text{if } uv \in E_1 \\ d_{G_2}(uv) + 2cp_1, & \text{if } uv \in E_2 \\ d_{G_1}(u) + d_{G_2}(v) + c(p_1 + p_2 - 2), & \text{if } uv \in E' \end{cases}$.

Where $\sigma_1(u) \wedge \sigma_2(v) = c$ is a constant, for all u ∈ V_1 and v ∈ V_2.

Proof. Let $\sigma_1(u) \wedge \sigma_2(v) = c$, for all u ∈ V_1 and v ∈ V_2, where c is a constant.

From (4.1), for any uv∈ E_1,

$$\begin{aligned} d_{G_1+G_2}(uv) &= d_{G_1}(uv) + \sum_{uw\in E'} \sigma_1(u) \wedge \sigma_2(w) + \sum_{vw\in E'} \sigma_1(v) \wedge \sigma_2(w) \\ &= d_{G_1}(uv) + \sum_{w\in V_2} \sigma_1(u) \wedge \sigma_2(w) + \sum_{w\in V_2} \sigma_1(v) \wedge \sigma_2(w) \\ &= d_{G_1}(uv) + \sum_{w\in V_2} c + \sum_{w\in V_2} c \\ &= d_{G_1}(uv) + cp_2 + cp_2 \\ &= d_{G_1}(uv) + 2cp_2. \end{aligned}$$

From (4.2), for any uv∈ E_2,

$$\begin{aligned} d_{G_1+G_2}(uv) &= d_{G_2}(uv) + \sum_{wu\in E'} \sigma_1(w) \wedge \sigma_2(u) + \sum_{wv\in E'} \sigma_1(w) \wedge \sigma_2(v) \\ &= d_{G_2}(uv) + \sum_{w\in V_1} c + \sum_{w\in V_1} c \\ &= d_{G_2}(uv) + cp_1 + cp_1 \\ &= d_{G_2}(uv) + 2cp_1. \end{aligned}$$

From (4.3), for any uv∈ E′ with u ∈ V_1 and v ∈ V_2.

$$\begin{aligned} d_{G_1+G_2}(uv) &= d_{G_1}(u) + d_{G_2}(v) + \sum_{uw\in E', w\neq v} \sigma_1(u) \wedge \sigma_2(w) + \sum_{wv\in E', w\neq u} \sigma_1(w) \wedge \sigma_2(v) \\ &= d_{G_1}(u) + d_{G_2}(v) + \sum_{w\in V_2} c - \sigma_1(u) \wedge \sigma_2(v) + \sum_{w\in V_1} c - \sigma_1(u) \wedge \sigma_2(v) \\ &= d_{G_1}(u) + d_{G_2}(v) + p_2c + p_1c - 2c \\ &= d_{G_1}(u) + d_{G_2}(v) + c(p_1 + p_2 - 2). \end{aligned}$$

Example 3.5. The following figure illustrating the above theorem,

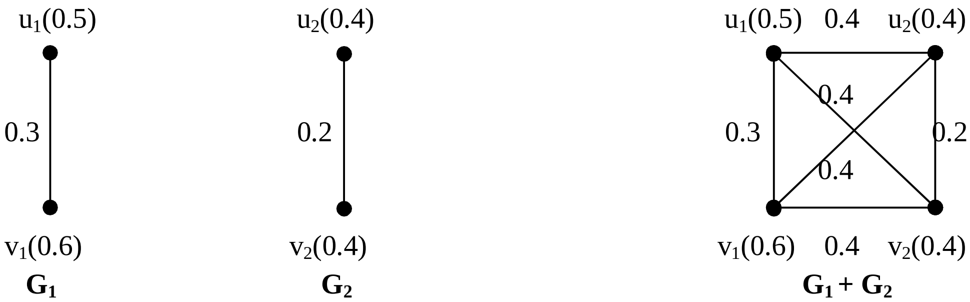

Figure 3.2:

By theorem 3.4,

(1). $d_{G_1+G_2}(u_1v_1) = d_{G_1}(u_1v_1) + 2cp_2$ = 0 + 2(0.4)(2) = 1.6

(2). $d_{G_1+G_2}(u_2v_2) = d_{G_2}(u_2v_2) + 2cp_1$ = 0 + 2(0.4)(2) = 1.6

(3). $d_{G_1+G_2}(u_1v_2) = d_{G_1}(u_1) + d_{G_2}(v_2) + c(p_1 + p_2 - 2)$

= 0.3 + 0.2 + 0.4(2 + 2 – 2) = 1.3

4. Conclusion

In this paper, we have found the degree of edges in $G_1 \cup G_2$ in terms of G_1 and G_2, the degree of edges in $G_1 + G_2$ in terms of the degree of vertices and edges in G_1 and G_2 and also in terms of the degree of vertices in G_1^* and G_2^* under some conditions and illustrated them through examples. They will be more helpful especially when the graphs are very large. Also they will be useful in studying various conditions, properties of union and join of two fuzzy graphs.

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