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On Characteristic Polynomial of Directed Divisor Graphs

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Abstract. Let N be the set of all positive integers and let S be the finite subset of N. A divisor graph G(S) is a graph whose vertex set is labeled from the set V=S and any two vertices, u,v of v are adjacent if either u divides v or v divides u. The divisor graph is a directed divisor graph if all the edges are directed from low to high levels. In this paper the characteristic polynomials of directed divisor graphs cycle C_n, path P_n& Bistar graphs are obtained from the corresponding skew adjacency matrix.

Keywords: Divisor graph, directed divisor graph, skew adjacency matrix.

AMS Mathematics Subject Classification (2010): 05C62

1. Introduction

Let DD be a directed divisor graph of order n with vertex set V(DD) = {v_1,v_2, …v_n} and arc set Γ(DD) ⊂ V(DD) x V(DD). Throughout this paper the graph DD has no loops and multiple arcs. The skew adjacency matrix of DD is the n x n matrix.

$$(DD)=\left[S_{ij}\right] where\, s_{ij}=\begin{cases}1 & if \quad (v_i,v_j)\in\Gamma(DD)\\ -1 & if \quad (v_j,v_i)\in\Gamma(DD)\\ 0 & otherwise\end{cases}$$

Because of the assumptions on DD, S(DD) is indeed a skew-symmetric matrix. Hence the eigenvalue $\{\lambda_1,\lambda_2,.....\lambda_n\}$ of S(DD) are all purely imaginary numbers, and the singular values of S(DD) coincide with the absolute values $\{|\lambda|_1,|\lambda_2|,.....|\lambda_n|\}$ of its eigenvalues. Consequently, the energy of S(DD), which is defined as the sum of the absolute values of its eigenvalues. The energy or the skew energy of S(DD) is denoted by E_s(DD) and is defined by $\sum_{i=1}^{n}|\lambda_i|$.

Definition 1.1. Let N be the set of all positive integers and S be the finite subset of N. A divisor graph G(S) is a graph whose vertex set is labeled from the set V=S and any two vertices u, v of V are adjacent if either u divides v (or) v divides u. The divisor graph is a directed divisor graph if all the edges are directed from low to high.

Note: Thecharacteristic polynomial of any graph is denoted by
$\phi(DD;\lambda)=\lambda^n+C_1\lambda^{n-1}+C_2\lambda^{n-2}+.....+C_n.$

Definition 1.1. The pathP_n is a tree with 2 nodes of vertex degree 1 and the other n-2 nodes of vertex degree 2.
Definition 1.2:
The bistar graph $B_{m,n}$is the graph obtained from K_2 by joining m pendant edges to one end and n pendant edges to the other end of K_2.

2. Characteristic polynomial of directed cycle graph

Theorem 2.1. The characteristic polynomial of skew adjacency matrix of directed divisor cycle graph C_n(n is even) is

$$\phi(C_n;\lambda)=\lambda^n+n\lambda^{n-2}+\frac{n(n-3)}{2!}\lambda^{n-4}+\frac{n(n-4)(n-5)}{3!}\lambda^{n-6}$$
$$+\frac{n(n-5)(n-6)(n-7)}{4!}\lambda^{n-8}+.....+C_n$$

where $C_n=\begin{cases}\left(\frac{n}{2}\right)^2\lambda^2 & if\ \ n\equiv 0(\bmod 4)\\ 1 & if\ \ n\equiv 2(\bmod 4).\end{cases}$

Proof: Let V(C_n)={v_1,v_2,….v_n} be the vertex set of C_n. Let E(C_n)={v_iv_{i+1}, v_nv_i; } be the edge set ofC_n, When n is even.
Let v_1 and $v_n$of V(C_n) be the start and end vertices respectively.
The cycle graph $C_n$becomes the divisor graph with vertex labeling as below.

$$v_n=p_1p_{\frac{n}{2}},\ v_{2i-1}=p_i\ for 1\le i\le\frac{n}{2},\ \ v_{2i}=p_ip_{i+1}\ \text{for} 1\le i\le\frac{n}{2}-1.$$

The divisor cycle graph $C_n$becomes a directed cycle graph if the direction of edges is from low to high levels.
For example, the directed divisor cycle graph C_6 is given in the Fig 1.

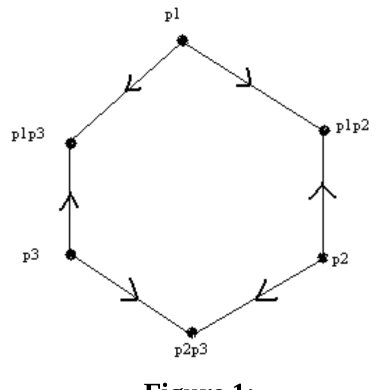

Figure 1:

Also $\phi(c_4;\lambda) = \lambda^4 + 4\lambda^2$

$\phi(c_6;\lambda) = \lambda^6 + 6\lambda^2 + 9\lambda^2 + 4$

$\phi(c_8;\lambda) = \lambda^8 + 8\lambda^2 + 20\lambda^4 + 16\lambda^2$

$$\phi(c_n;\lambda) = \lambda^n + 8\lambda^{n-2} + \frac{n(n-3)}{2!}\lambda^{n-4} + \frac{n(n-4)(n-5)}{3!}\lambda^{n-6}$$

$$+\frac{n(n-5)(n-6)(n-7)}{4!}\lambda^{n-8} + ... + c_n$$

where $c_n = \begin{cases} \left(\frac{n}{2}\right)^2 \lambda^2, if\ n \equiv 0(\text{mod}\,4) \\ 1,\ \ if\ \ n \equiv 2\,(\text{mod}\,4). \end{cases}$

3. Characteristic polynomial of directed path graph and directed Bistar

Theorem 3.1. The characteristic polynomial of skew adjacency matrix of directed divisor path graph P_n is

$$\phi(P_n;\lambda) = \lambda^{n+1} + n\lambda^{n-1} + \frac{1}{2!}(n-1)(n-2)\lambda^{n-3} + \frac{1}{3!}(n-2)(n-3)(n-4)\lambda^{n-5} + ... + c_n.$$

where $c_n = \begin{cases} \frac{n+2}{2}\ if\ \ n \geq 2\ \&\, n\, is\, even \\ 1\ \ if\ \ n \geq 3\ \&\, n\ is\ odd. \end{cases}$

Proof: Let $V(P_n) = \{v_1, v_2, \ldots v_{n+1}\}$ be the vertex set of P_n.

Let $E(P_n) = \{v_i\, v_{i+1},\ 1 \leq i \leq n\}$be the edge set of P_n.

The path graph P_n becomes the divisor graph with vertex labeling as below.

$v_{2i} = P_i, 1 \leq i \leq \left[\frac{n}{2}\right]$

$v_{2i+1} = P_i P_{i+1}, 1 \leq i \leq \left[\frac{n}{2}\right]$

$v_1 = P_i P_{\left[\frac{n}{2}\right]}.$

The divisor path graph $P_n$becomes a directed path graph if the direction of edges is from low to high labels. For example, directed path graph P_5 is given in the Fig.2.

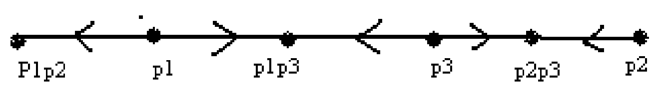

Figure 2:

The skew adjacency matrix S(DD) is

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ -1 & 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Also,

$$\phi(P_2;\lambda)=\lambda^3+2\lambda$$

$$\phi(P_3;\lambda)=\lambda^4+3\lambda^2+1$$

$$\phi(P_4;\lambda)=\lambda^5+4\lambda^3+3\lambda.$$

$$\phi(P_5;\lambda)=\lambda^6+5\lambda^4+6\lambda^2+1.$$

$$\phi(P_5;\lambda)=\lambda^{n+1}+n\lambda^{n-1}+\frac{1}{2!}(n-1)(n-2)\lambda^{n-3}+\frac{1}{3!}(n-2)(n-3)(n-4)\lambda^{n-5}+....+c_n$$

where $c_n=\begin{cases}\dfrac{n+2}{2}, & if\ \ n\geq 2\, and\ n\ is\ even\\ 1, & if\ \ n\geq 3\, and\ n\ is\ odd.\end{cases}$

Theorem 3.2. The characteristic polynomial of skew adjacency matrix of directed divisor bistat $B_{n,\,n}$ is

$$\phi\left(B_{n,n};\lambda\right)=\lambda^n+\left(n-1\right)\lambda^{n-2}+\left(\frac{n}{2}-1\right)^2\lambda^{n-4}.$$

Proof: Let $V(B_{n,n}) = \{v_1,v_2,\ldots..v_n, v_{11},v_{12},\ldots,v_{1n}\}$ be the vertex set of $B_{n,n}$.

E $(B_{n,n}) =\{ v_iv_{i+1},v_{1i}\, v_{1(i+1)},\ 1\leq i \leq n\}$ be the edge set of $B_{n,n}$.

The bistar graph $B_{n,n}$becomes the divisor graph with vertex labeling as below.

The skew adjacency matrix S(DD) is

$V_1 = p_1$

$V_{2i} = p_1p_{2i+1}$ for $1\leq i \leq n-2$

$V_{2i+1} = p_1p_{2i+2}$ for $1\leq i \leq n-2$.

$V_{11} = p_1p_2$

$V_{1(2i)} =p_1p_2p_{2i+4}$ for $1\leq i\leq n-2$

$V_{1(2i+1)} = p_1p_2\, p_{2i+5}$ for $1\leq i \leq n-2$

The skew adjacency matrix s (DD) is

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Also

$\phi(B_{4,4};\lambda)=\lambda^8+7\lambda^6+9\lambda^4$

$\phi(B_{5,5};\lambda)=\lambda^{10}+9\lambda^8+16\lambda^6.$

$\phi(B_{6,6};\lambda)=\lambda^{12}+11\lambda^{10}+25\lambda^8.$

$$\phi(B_{n,n};\lambda)=\lambda^{2n}+\left(2n-1\right)\lambda^{2n-2}+\left(\frac{2n}{2}-1\right)^2\lambda^{n-4}.$$

3. Conclusion

In this paper, a new family of directed graphs are constructed from the divisor graph. In particular the characteristic polynomial of directed divisor graph C_n, $P_n$and $B_{n,n}$ are studied.

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