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Anti Q-Fuzzy R-Closed KU-Ideals in KU-Algebras and its Lower Level Cuts

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Abstract. In this paper, we introduce the concept of anti Q-fuzzy R-closed KU-ideals of KU-algebras, lower level cuts of a fuzzy set, lower level R-closed KU-ideals and prove some results . We show that a Q-fuzzy set of a KU-algebra is a R- closed KU-ideal if and only if the complement of this Q-fuzzy set is an anti Q-fuzzy R-closed KU-ideal. Also we discussed few results of R-closed KU-ideals of KU-algebras under homomorphism and anti homomorphism. Cartesian Product of Anti Q-fuzzy KU-ideals of KU-algebras are also discussed.

Keywords: KU-algebra, fuzzy KU- ideal, fuzzy R-closed KU- ideal, Anti Q-fuzzy R-closed KU-ideal, lower level cuts

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1. Introduction

Imai and Iseki [6,7] introduced two classes of abstract algebras: BCK-algebras and BCI–algebras. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. Hu and Li [4,5] introduced a wide class of abstract BCH-algebras. They have shown that the class of BCI-algebras. Neggers, Ahn [1] and Kim introduced Q-algebras which is a generalization of BCK/BCI algebras and obtained several results. Fuzzification of ideals in BCC algebras are studied in [3]. Recently, Senapathi et al. [12] have studied on Fuzzy closed ideals of B-algebras with interval valued membership function. Prabpayak and Leerawat [8] introduced a new algebraic structure which is called KU-algebras and investigated some properties. Samy Mostafa and Abdel Naby [13] introduced fuzzy KU-ideals in KU-algebras. Biswas [2] introduced the concept of Anti fuzzy subgroups of groups. Modifying his idea, in this paper we apply the idea in KU-algebras. We introduce the notion of anti Q-fuzzy R-closed KU-ideals of KU-algebras and investigate some of its properties.

2. Preliminaries

In this section we site the fundamental definitions that will be used in the sequel.

Definition 2.1. [13] A non empty set X with a constant 0 and a binary operation * is called a KU-algebra if it satisfies the following axioms.

1. $(x * y) *[(y * z) * (x * z)] = 0$
2. $0 * x = x$
3. $x * 0 = 0$
4. $x * y = 0 = y * x$ implies $x = y$, for all $x, y, z \in X$.

In X we can define a binary operation ≤ by $x \le y$ if and only if $y * x = 0$. Then (X,*,0) is a KU-algebra if and only if it satisfies that

i. $(y * z) * (x * z) \le (x * y)$
ii. $0 \le x$
iii. $x \le y$, $y \le x$ implies $x = y$.
iv. $x \le y$ if and only if $y * x = 0$, for all $x, y, z \in X$.

In a KU-algebra, the following identities are true [13]:

1. $z * z = 0$.
2. $z * (x * z) = 0$
3. $x \le y \Rightarrow y * z \le x * z$
4. $z * (y * x) = y * (z * x)$
5. $y * [(y * x) * x] = 0$, for all $x, y, z \in X$.

Example 2.1. Let X = { 0 , a, b, c,d } be a set with a binary operation * defined by the following table

*	0	a	b	c	d
0	0	a	b	c	d
a	0	0	b	c	d
b	0	a	0	c	c
c	0	0	b	0	b
d	0	0	0	0	0

Then clearly (X , * , 0) is a KU-algebra.

Definition 2.2. [13] Let (X,*, 0) be a KU-algebra. A non empty set I of X is called an ideal of X if it satisfies

i) $0 \in I$
ii) $x * y \in I$ and $y \in I$ imply $x \in I$, for all $x, y \in X$.

Definition 2.3. [13] Let (X , * , 0) be a KU-algebra. A non empty subset I of X is called KU ideal of X if it satisfies the following conditions

(1) $0 \in I$
(2) $x * (y * z) \in I$ and $y \in I \Rightarrow x * z \in I$ for all $x, y, z \in X$.

Definition 2.4. [15] Let X be a non-empty set. A fuzzy subset μ of the set X is a mapping $\mu : X \to [0, 1]$.

Definition 2.5. [10] Let Q and G be any two sets.A mapping β: G x Q $\to$[0, 1] is called a Q –fuzzy set in G.

Definition 2.6. [10] A Q- fuzzy set μ in X is called a Q-fuzzy KU- ideal of X if

(i) $\mu(0,q) \geq \mu(x,q)$

(ii) $\mu(x * z, q) \geq \min\{\mu(x * (y * z), q), \mu(y,q)\}$,for all x, y, z$\in$ X and q $\in$ Q.

Definition 2.7. [10] A Q-fuzzy set μ of a KU-algebra X is called an anti Q-fuzzy KU-ideal of X, if

(i) $\mu(0,q) \leq \mu(x,q)$

(ii) $\mu(x * z, q) \leq \max\{\mu((x * (y * z)), q), \mu(y, q)\}$, for all x,y,z$\in$ X and q $\in$ Q.

Example 2.2. Let X = { 0 , 1 , 2, 3,4 } be a set with a binary operation * defined by the following table

*	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	2	4
2	0	0	0	1	4
3	0	0	0	0	4
4	0	0	0	1	0

Then Clearly (X , * , 0) is a KU-algebra. Let $t_0, t_1, t_2 \in [0, 1]$ be such that $t_0 < t_1 < t_2$. Define a Q-fuzzy set $\mu : X \times Q \to [0, 1]$ by $\mu(0,q) = t_0$, $\mu(1,q) = t_1 = \mu(2,q)$, $\mu(3,q) = t_2 = \mu(4,q)$, By routine calculations μ is an anti Q-fuzzy KU- ideal of X and q$\in$Q.

Definition 2.8. [10] If μ is a Q-fuzzy set in set X then the complement denoted by μ^c is the Q-fuzzy subset of X given by $\mu^c(x,q) = 1 - \mu(x,q)$, for all x,y$\in$ X and q $\in$ Q.

3. Anti Q-fuzzy R-closed KU- ideal

Definition 3.1. An ideal A of a KU-algebra X is said to be R-closed if $x * 0 \in A$ for all $x \in A$.

Definition 3.2. Let (X , * , 0) be a KU-algebra. A non empty subset I of X is called R-closed KU ideal of X if

(1) $x * 0 \in I$

(2) $x * (y * z) \in I$ and $y \in I \Rightarrow x * z \in I$ for all x , y, z $\in$ X .

Remark: From Example 2.1, It is clear that $A_1 = \{0,a\}$ and $A_2 = \{0,a,b\}$ are R- closed KU-ideals of X.

Definition 3.3. A Q- fuzzy set μ in X is called a Q-fuzzy R-closed KU- ideal of X if

(i) $\mu(x*0,q) \geq \mu(x,q)$
(ii) $\mu(x*z,q) \geq \min\{\mu(x*(y*z),q), \mu(y,q)\}$, for all $x, y, z \in X$ and $q \in Q$.

Definition 3.4. A Q-fuzzy set μ of a KU-algebra X is called an anti Q-fuzzy R-closed KU-ideal of X, if

(i) $\mu(x*0,q) \leq \mu(x,q)$
(ii) $\mu(x*z,q) \leq \max\{\mu((x*(y*z)),q), \mu(y,q)\}$, for all $x,y,z \in X$ and $q \in Q$.

Theorem 3.1. Every Anti Q-fuzzy R-closed KU- ideal μ of a KU-algebra X is order preserving.

Proof: Let μ be an anti Q-fuzzy R-closed KU- ideal of a KU-algebra X and let $x, y \in X$ and $q \in Q$.

$$\begin{aligned} \text{Then } \mu(x,q) &= \mu(0*x,q) \\ &\leq \max\{\mu(0*(y*x),q), \mu(y,q)\} \\ &= \max\{\mu(0*0,q), \mu(y,q)\} \\ &= \max\{\mu(0*(y*0),q), \mu(y,q)\} \\ &= \max\{\mu(y*0,q), \mu(y,q)\} \\ &= \mu(y,q) \end{aligned}$$

$$\mu(x,q) \leq \mu(y,q).$$

Theorem 3.2. μ is a Q-fuzzy R-closed KU-ideal of a KU-algebra X if and only if μ^c is an anti Q-fuzzy R-closed KU-ideal of X.

Proof: Let μ be a Q-fuzzy closed KU- ideal of X and let $x, y, z \in X$ and $q \in Q$.

(i) $\mu(x*0,q) \geq \mu(x,q)$

$1 - \mu^c(x*0,q) \geq 1 - \mu^c(x,q)$

$\mu^c(x*0,q) \leq \mu^c(x,q)$

That is $\mu^c(x*0,q) \leq \mu^c(x,q)$.

(ii)
$$\begin{aligned} \mu^c(x*z,q) &= 1 - \mu(x*z,q) \\ &\leq 1 - \min\{\mu(x*(y*z),q), \mu(y,q)\} \\ &= 1 - \min\{1 - \mu^c(x*(y*z),q), 1 - \mu^c(y,q)\} \\ &= \max\{\mu^c(x*(y*z),q), \mu^c(y,q)\} \end{aligned}$$

That is $\mu^c(x*z,q) \leq \max\{\mu^c(x*(y*z),q), \mu^c(y,q)\}$.

Thus μ^c is an anti Q-fuzzy R-closed KU-ideal of X. The converse also can be proved similarly.

Theorem 3.3. Let μ be an anti Q-fuzzy R-closed KU-ideal of KU – algebra X.If the inequality $x*y \leq z$, then $\mu(y,q) \leq \max\{\mu(x,q),\mu(z,q)\}$ for all $x,y,z \in X$ and $q \in Q$.

Proof: Assume that the inequality $x*y \leq z$ for all $x,y,z \in X$ and $q \in Q$.

Then $z*(x*y) = 0$.

$$\begin{aligned} \text{Now, } \mu(y,q) &= \mu(0*y,q] \\ &\leq \max\{\mu(0*(x*y),q), \mu(x,q)\} \\ &= \max\{\mu(x*y,q), \mu(x,q)\} \\ &\leq \max\{\max\{\mu(x*(z*y),q), \mu(z,q)\}, \mu(x,q)\} \\ &= \max\{\max\{\mu(z*(x*y),q), \mu(z,q)\}, \mu(x,q)\} \end{aligned}$$

$= \max\{ \max\{ \mu(0, q), \mu(z, q)\}, \mu(x,q)\}$
$= \max\{ \max\{ \mu(z*0, q), \mu(z, q)\}, \mu(x,q)\}$
$= \max\{ \mu(z, q)\}, \mu(x,q)\}$

$\therefore \mu(y,q) \leq \max\{\mu(x,q),\mu(z,q)\}$.

Theorem 3.4. If μ is an anti Q-fuzzy R-closed KU-ideal of KU– algebra X, then for all $x,y \in X$ and $q \in Q$, $\mu(x*(x*y), q) \leq \mu(y, q)$.

Proof: Let $x,y \in X$ and $q \in Q$.

$\mu(x*(x*y), q) \leq \max\{ \mu(x*(y*(x*y)), q), \mu(y, q)\}$
$= \max\{ \mu(x*(x*(y*y)), q), \mu(y, q)\}$
$= \max\{ \mu(x*(x*0), q), \mu(y, q)\}$
$= \max\{ \mu(x*0, q), \mu(y, q)\}$
$= \max\{ \mu(0, q), \mu(y, q)\}$
$= \max\{ \mu(y*0, q), \mu(y, q)\}$
$= \mu(y,q)$

$\therefore \mu(x*(x*y), q) \leq \mu(y, q)$

4. Lower level cuts in anti Q-fuzzy R-closed KU-ideals of KU-algebra

Definition 4.1.[10] Let μ be a Q-fuzzy set of X. For a fixed $t \in [0, 1]$, the set $\mu^t = \{x \in X | \mu(x,q) \leq t$ for all $q \in Q\}$ is called the lower level subset of μ.
Clearly $\mu^t \cup \mu_t = X$ for $t\in[0,1]$ if $t_1 < t_2$, then $\mu^{t1} \subseteq \mu^{t2}$.

Theorem 4.1. If μ is an anti Q-fuzzy R-closed KU-ideal of KU-algebra X,then μ^t is a R-closed KU-ideal of X for every $t\in [0,1]$.

Proof: Let μ be an anti Q-fuzzy R-closed KU-ideal of KU-algebra X.

(i) Let $y \in \mu^t \Rightarrow \mu(y, q) \leq t$.

$\mu(x*0, q) \leq \max\{ \mu(x*(y*0),q), \mu(y,q)\}$
$= \max\{ \mu(y*(x*0)),q), \mu(y,q)\}$
$= \max\{ \mu(y*0),q), \mu(y,q)\}$
$= \mu(y,q) \leq t$.

$\Rightarrow x*0 \in \mu^t$.

(ii) Let $x*(y*z) \in \mu^t$ and $y \in \mu^t$, for all $x,y,z \in X$ and $q \in Q$.

$\Rightarrow \mu(x*(y*z), q) \leq t$ and $\mu(y,q) \leq t$.

$\mu((x*z),q) \leq \max\{ \mu((x*(y*z)), q), \mu(y, q)\} \leq \max\{t,t\} = t$.

$\Rightarrow x*z \in \mu^t$.

Hence, μ^t is an R-closed KU- ideal of X for every $t\in [0,1]$.

Theorem 4.2. Let μ be a Q-fuzzy set of KU- algebra X.If for each $t \in [0,1]$, the lower level cut μ^t is a R-closed KU-ideal of X, then μ is an anti Q- fuzzy R-closed KU-ideal of X.

Proof: Let μ^t be a R-closed KU-ideal of X.

If $\mu(x*0,q) > \mu(x,q)$ for some $x\in X$ and $q \in Q$.Then $\mu(x*0,q) > t_0 > \mu(x,q)$ by taking $t_0 = \frac{1}{2}\{ \mu(x*0,q) + \mu(x,q)\}$.Hence $x*0 \notin \mu^{t0}$ and $x \in \mu^{t0}$, which is a contradiction.

Therefore, $\mu(x*0,q) \leq \mu(x,q)$.

Let x,y,z ∈ X and q ∈ Q be such that μ((x * z) , q) > max{μ (x * (y *z) , q), μ(y,q)}
Taking $t_1 = \frac{1}{2}$ { μ((x * z) , q) + max {μ (x * (y * z) , q),μ(y,q)}} and
μ((x * z) , q) > t_1 > max{μ (x * (y * z) , q), μ(y,q)} .
It follows that (x * (y * z)),y ∈ μ^{t1} and x * z ∉ μ^{t1}. This is a contradiction.
Hence μ((x * z) , q) ≤ max{μ (x * (y * z) , q), μ(y,q)}
Therefore μ is an anti Q-fuzzy R-closed KU-ideal of X.

Definition 4.2.[11] Let X be an KU- algebra and a,b ∈ X.We can define an set A(a,b) by
A(a,b) = { x ∈ X / a * (b * x) = 0 }.
It is easy to see that 0,a,b ∈ A(a,b) for all a,b ∈ X.

Theorem 4.3. Let μ be a Q-fuzzy set in KU-algebra X.Then μ is an anti Q-fuzzy R-closed KU- ideal of X iff μ satisfies the following condition:
(for all $(\forall a,b\in X),(\forall t\in[0,1])\,(a,b)\in \mu^t \Rightarrow A(a,b)\subseteq \mu^t$.

Proof: Assume that μ is an anti Q-fuzzy R-closed KU-ideal of X.
Let a,b ∈ μ^t. Then μ (a,q) ≤ t and μ (b,q) ≤ t.
Let x ∈ A (a,b). Then a * (b * x) = 0.
Now,
μ (x,q) = μ (0 * x , q)
≤ max { μ (0 * (b * x) , q) , μ (b,q)}
= max { μ ((b * x) , q) , μ (b,q)}
≤ max { max {μ (b * (a * x) , q) , μ (a,q)} , μ(b,q)}
= max { max {μ (a * (b * x) , q) , μ (a,q)} , μ(b,q) }
= max { max {μ (0,q) , μ (a,q)} , μ(b,q) }
= max { max {μ (a * 0,q) , μ (a,q)} , μ(b,q) }
= max { μ (a,q)} , μ(b,q) }
≤ max { t , t} = t
⇒ μ (x,q) ≤ t .
⇒ x ∈ μ^t.
Therefore, A(a,b) ⊆ μ^t.
Conversely, suppose that A(a,b) ⊆ μ^t .
Obviously x*0 = 0 ∈ A(a,b) ⊆ μ^t for all a,b ∈ X.
Let x,y,z ∈ X be such that x * (y * z) ∈ μ^t and y ∈ μ^t .
Since (x * (y * z)) * (y * (x * z)) = (x * (y * z)) * (x * (y * z)) = 0.
We have x * z ∈ A (x * (y * z) , y) ⊆ μ^t .
∴ μ^t is a R-closed KU- ideal of X.
Hence, by Theorem 4.2, μ is an anti Q-fuzzy R-closed KU-ideal of X.

Theorem 4.4. Let μ be a Q-fuzzy set in KU-algebra X.If μ is an anti Q-fuzzy R-closed KU-ideal of X then

$$(\forall t\in[0,1])\,\mu^t \neq \phi \Rightarrow \mu^t = \bigcup_{a,b\in\mu^t} A(a,b).$$

Proof: Let t ∈ [0,1] be such that $\mu^t \neq \phi$. Since x*0 = 0 ∈ μ^t, we have

$$\mu^t \subseteq \bigcup_{a \in \mu^t} A(a,0) \subseteq \bigcup_{a,b \in \mu^t} A(a,b).$$

Now, let $x \in \bigcup_{a,b \in \mu^t} A(a,b).$

Then there exists $(u,v) \in \mu^t$ such that $x \in A(u,v) \subseteq \mu^t$ by theorem 3.3. Thus $\bigcup_{a,b \in \mu^t} A(a,b) \subseteq \mu^t.$

$\therefore\ \mu^t = \bigcup_{a,b \in \mu^t} A(a,b).$

5. Homomorphism and anti Homomorphism on anti Q-fuzzy R-closed KU-algebras

In this section, we discussed about ideals in KU-algebra under homomorphism and anti homomorphism and some of its properties.

Definition 5.1.[10] Let $(X,*,0)$ and $(Y,\Delta,0`)$ be KU– algebras. A mapping $f: X \to Y$ is said to be a homomorphism if

$f(x * y) = f(x) \Delta f(y)$ for all $x,y \in X$.

Definition 5.2. [10] Let $(X,*,0)$ and $(Y,\Delta,0`)$ be KU–algebras. A mapping $f: X \to Y$ is said to be an anti homomorphism if $f(x * y) = f(y) \Delta f(x)$ for all $x,y \in X$.

Definition 5.3. [10] Let $f: X \to X$ be an endomorphism and μ be a fuzzy set in X. We define a new fuzzy set in X by μ_f in X as $\mu_f(x) = \mu(f(x))$ for all x in X.

Definition 5.4. [10] For any homomorphism $f: X \to Y$, the set $\{x \in X \,/\, f(x) = 0'\}$ is called the kernel of f, denoted by Ker(f) and the set $\{f(x) \,/\, x \in X\}$ is called the image of f, denoted by Im(f).

Theorem 5.1. Let f be an endomorphism of a KU- algebra X. If μ is an anti Q- fuzzy R-closed KU-ideal of X, then so is μ_f.

Proof: Let μ be an anti Q-fuzzy R-closed KU-ideal of X.

Now, $\mu_f(x * 0,q) = \mu(f(x * 0,q))$

$\leq \mu(f(x,q)) = \mu_f(x,q)$, for all $x,y \in X$ and $q \in Q$.

Let $x,y,z \in X$ and $q \in Q$.

Then $\mu_f(x * z,q) = \mu(f(x * z,q))$

$= \mu(f(x,q)) * f(z,q))$

$\leq \max\{\mu(f(x,q) * (f(y,q) * f(z,q))), \mu(f(y,q))\}$

$= \max\{\mu(f(x,q) * f(y * z,q)), \mu(f(y,q))\}$

$= \max\{\mu(f(x * (y * z), q)), \mu(f(y,q))\}$

$= \max\{\mu_f(x * (y * z), q), \mu_f(y,q)\}$

$\therefore\ \mu_f(x * z,q) \leq \max\{\mu_f(x * (y * z), q), \mu_f(y,q)\}$

Hence μ_f is an anti Q -fuzzy R-closed KU-ideal of X.

Theorem 5.2. Let $f: X \to Y$ be an epimorphism of KU- algebra. If μ_f is an anti Q-fuzzy R-closed KU-ideal of X, then μ is an anti Q-fuzzy R-closed KU-ideal of Y.

Proof: Let μ_f be an anti Q-fuzzy R-closed KU-ideal of X.
Let $y \in Y$ and $q \in Q$. Then there exists $x \in X$ such that $f(x, q) = (y, q)$.
Now,

$$\begin{aligned}\mu(y * 0,q) &= \mu((y,q)\,\Delta\,(0,q))\\ &= \mu(f(x,q)\,\Delta\, f(0,q))\\ &= \mu(f((x,q) * (0,q)))\\ &= \mu_f((x,q) * (0,q))\\ &\leq \mu_f(x,q) = \mu(f(x,q)) = \mu(y,q)\end{aligned}$$

$\therefore\ \mu(y * 0,q) \leq \mu(y,q)$

Let $y_1,y_2 \in Y$ and $q \in Q$.

$$\begin{aligned}\mu((y_1,q)\,\Delta\,(y_2,q)) &= \mu(f(x_1,q)\,\Delta\, f(x_2,q))\\ &= \mu(f((x_1,q) * (x_2,q)))\\ &= \mu_f((x_1,q) * (x_2,q))\\ &\leq \max\{\mu_f((x_1,q) * ((x_3,q) * (x_2,q))), \mu_f(x_3,q)\}\\ &= \max\{\mu[f((x_1,q) * ((x_3,q) * (x_2,q)))], \mu(f(x_3,q))\}\\ &= \max\{\mu[f(x_1,q)\,\Delta\, f((x_3,q) * (x_2,q))], \mu(f(x_3,q))\}\\ &= \max\{\mu[f(x_1,q)\,\Delta\,(f(x_3,q)\,\Delta\, f(x_2,q))], \mu(f(x_3,q))\}\\ &= \max\{\mu[(y_1,q)\,\Delta\,((y_3,q)\,\Delta\,(y_2,q))], \mu(y_3,q)\}\end{aligned}$$

$\therefore\ \mu((y_1,q)\,\Delta\,(y_2,q)) \leq \max\{\mu[(y_1,q)\,\Delta\,((y_3,q)\,\Delta\,(y_2,q))], \mu(y_3,q)\}$

$\Rightarrow\ \mu$ is an anti Q-fuzzy R-closed KU-ideal of Y.

Theorem 5.3. Let $f: X \to Y$ be a homomorphism of KU- algebra. If μ is an anti Q-fuzzy R-closed KU-ideal of Y then μ_f is an anti Q-fuzzy R-closed KU-ideal of X.
Proof: Let μ be an anti Q-fuzzy R-closed KU-ideal of Y.
Let $x,y \in X$ and $q \in Q$.

$$\begin{aligned}\mu_f(x*0,q) &= \mu[f(x*0,q)]\\ &\leq \mu[f(x,q)] = \mu_f(x,q)\end{aligned}$$

$\Rightarrow\ \mu_f(0,q) \leq \mu_f(x,q)$.

$$\begin{aligned}\mu_f(x * z, q) &= \mu(f((x * z),q))\\ &= \mu(f(x,q)\,\Delta\, f(z,q))\\ &\leq \max\{\mu[f(x,q)\,\Delta\,(f(y,q)\,\Delta\, f(z,q))], \mu(f(y,q))\}\\ &= \max\{\mu[f(x,q)\,\Delta\,(f((y * z),q)], \mu(f(y,q))\}\\ &= \max\{\mu[f(x * (y * z),q)], \mu(f(y,q))\}\\ &= \max\{\mu_f(x * (y * z),q), \mu_f(y,q)\}\end{aligned}$$

$\therefore\ \mu_f(x * z, q) \leq \max\{\mu_f(x * (y * z),q), \mu_f(y,q)\}$

Hence, μ_f is an anti Q-fuzzy R-closed KU-ideal of X.

6. Cartesian product of anti Q-fuzzy KU-ideals of KU–algebras

In this section, we introduce the concept of Cartesian product of anti Q-fuzzy R-closed KU-ideals of KU-algebra.

Definition 6.1. [9] Let μ and δ be the fuzzy sets in X. The Cartesian product $\mu \times \delta : X \times X \to [0,1]$ is defined by $(\mu \times \delta)(x, y) = \min\{\mu(x),\delta(y)\}$, for all $x, y \in X$.

Definition 6.2. [9] Let μ and δ be the anti fuzzy sets in X. The Cartesian product $\mu \times \delta$: $X \times X \rightarrow [0,1]$ is defined by $(\mu \times \delta)(x, y) = \max\{\mu(x), \delta(y)\}$, for all $x, y \in X$.

Definition 6.3. [9] Let μ and δ be the anti Q-fuzzy sets in X. The Cartesian product $\mu \times \delta$: $X \times X \rightarrow [0,1]$ is defined by $(\mu \times \delta)((x, y),q) = \max\{\mu(x, q), \delta(y, q)\}$, for all $x, y \in X$ and $q \in Q$.

Theorem 6.1. If μ and δ are anti Q-fuzzy R-closed KU-ideals in a KU – algebra X, then $\mu \times \delta$ is an anti Q-fuzzy R-closed KU-ideal in X x X.

Proof: Let $(x_1, x_2) \in X \times X$ and $q \in Q$.

$$\begin{aligned}(\mu \times \delta)((x_1 * 0, x_2 * 0),q) &= \max\{\mu(x_1 * 0,q), \delta(x_2 * 0,q)\} \\ &\leq \max\{\mu(x_1,q), \delta(x_2,q)\} \\ &= (\mu \times \delta)((x_1,x_2),q)\end{aligned}$$

$\therefore (\mu \times \delta)((x_1 * 0, x_2 * 0),q) \leq (\mu \times \delta)((x_1,x_2),q)$

Let $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$ and $q \in Q$.

Now,

$$\begin{aligned}(\mu \times \delta)(((x_1, x_2),q) * ((z_1, z_2),q)) &= (\mu \times \delta)((x_1 * z_1,q), (x_2 * z_2,q)) \\ &= \max\{\mu(x_1 * z_1,q), \delta(x_2 * z_2,q)\}\end{aligned}$$

$\leq \max\{\max\{\mu(x_1 * (y_1 * z_1),q), \mu(y_1,q)\}, \max\{\delta(x_2 * (y_2 * z_2),q), \delta(y_2,q)\}\}$

$= \max\{\max\{\mu(x_1 * (y_1 * z_1),q), \delta(x_2 * (y_2 * z_2),q)\}, \max\{\mu(y_1,q), \delta(y_2,q)\}\}$

$= \max\{(\mu \times \delta)(((x_1, x_2),q) * (((y_1 * y_2),q) * ((z_1, z_2),q))), (\mu \times \delta)((y_1, y_2),q)\}$.

$\therefore (\mu \times \delta)(((x_1, x_2),q) * ((z_1, z_2),q)) \leq \max\{(\mu \times \delta)(((x_1, x_2),q) * (((y_1 * y_2),q) * ((z_1, z_2),q))), (\mu \times \delta)((y_1, y_2),q)\}$.

Hence, $\mu \times \delta$ is an anti Q-fuzzy R-closed KU- ideal in X x X.

Theorem 6.2. Let μ and δ be fuzzy sets in a KU-algebra X such that $\mu \times \delta$ is an Anti Q-fuzzy R-closed KU-ideal of X x X. Then

(i) Either $\mu(x * 0,q) \leq \mu(x, q)$ (or) $\delta(x * 0,q) \leq \delta(x,q)$ for all $x \in X$ and $q \in Q$.

(ii) If $\mu(x * 0,q) \leq \mu(x,q)$ for all $x \in X$ and $q \in Q$, then either $\delta(x * 0,q) \leq \mu(x, q)$ (or) $\delta(x * 0,q) \leq \delta(x, q)$

(iii) If $\delta(x * 0,q) \leq \delta(x,q)$ for all $x \in X$ and $q \in Q$, then either $\mu(x * 0,q) \leq \mu(x,q)$ (or) $\mu(x * 0,q) \leq \delta(x,q)$.

Proof: Let $\mu \times \delta$ be an anti Q- fuzzy R-closed KU-ideal of X x X .

(i) Suppose that $\mu(x * 0,q) > \mu(x,q)$ and $\delta(x*0,q) > \delta(x,q)$ for some $x, y \in X$ and $q \in Q$.

$$\begin{aligned}\text{Then } (\mu \times \delta)((x,y),q) &= \max\{\mu(x,q), \delta(y,q)\} \\ &< \max\{\mu(x * 0,q), \delta(y*0,q)\} \\ &= (\mu \times \delta)((x*0,y*0),q)\end{aligned}$$
, Which is a contradiction.

Therefore $\mu(x*0,q) \leq \mu(x,q)$ or $\delta(x*0,q) \leq \delta(x,q)$ for all $x \in X$ and $q \in Q$.

(ii) Assume that there exists $x, y \in X$ and $q \in Q$ such that

$\delta(x*0,q) > \mu(x,q)$ and $\delta(x*0,q) > \delta(x,q)$.

Then $(\mu \times \delta)((x*0,y*0),q) = \max\{\mu(x*0,q), \delta(y*0,q)\} = \delta(y*0,q)$ and hence

$(\mu \times \delta)((x, y),q) = \max\{\mu(x,q), \delta(y,q)\} < \delta(y*0,q) = (\mu \times \delta)((x*0,y*0),q)$ which is a contradiction.

Hence, if $\mu(x*0,q) \le \mu(x,q)$ for all $x \in X$ and $q \in Q$, then either $\delta(x*0,q) \le \mu(x,q)$ (or) $\delta(x*0,q) \le \delta(x,q)$.

Similarly, we can prove that if $\delta(x*0,q) \le \delta(x,q)$ for all $x \in X$ and $q \in Q$,then either $\mu(x*0,q) \le \mu(x,q)$ (or) $\mu(x*0,q) \le \delta(y,q)$, which yields (iii).

Theorem 6.3. Let μ and δ be fuzzy sets in a KU-algebra X such that $\mu \times \delta$ is an Anti Q-fuzzy R-closed KU-ideal of X x X. Then either μ or δ is an anti Q-fuzzy R-closed KU-ideal of X.

Proof: First we prove that δ is an anti Q- fuzzy R-closed KU-ideal of X.
Since by 6.2(i) either $\mu(x*0,q) \le \mu(x,q)$ or $\delta(x*0,q) \le \delta(x,q)$ for all $x \in X$ and $q \in Q$.
Assume that $\delta(x*0,q) \le \delta(x,q)$ for all $x \in X$ and $q \in Q$. It follows from 6.2(iii) that either $\mu(x*0,q) \le \mu(x,q)$ (or) $\mu(x*0,q) \le \delta(x,q)$.
If $\mu(x*0,q) \le \delta(x,q)$, for any $x \in X$ and $q \in Q$,then $\delta(x,q) = \max\{\mu(x*0,q), \delta(x,q)\} = \max\{\mu(0,q), \delta(x,q)\} = (\mu \times \delta)((0, x),q)$

$$
\begin{aligned}
\delta((x * z),q) &= \max\{\mu(0,q) , \delta((x * z),q)\}.\\
&= (\mu \times \delta)((0, x*z),q)\\
&= (\mu \times \delta)(((0*0),q),((x*z),q))\\
&= (\mu \times \delta)(((0,x),q) * ((0,z),q))\\
&\le \max\{(\mu \times \delta)[((0, x),q) * (((0,y),q) * ((0,z),q))], (\mu \times \delta)((0, y),q)\}\\
&= \max\{(\mu \times \delta)[((0, x),q) * ((0*0,y*z),q)], (\mu \times \delta)((0, y),q)\}\\
&= \max\{(\mu \times \delta)[(0* (0*0),x *(y*z)),q)], (\mu \times \delta)((0, y),q)\}\\
&= \max\{(\mu \times \delta)[((0,x *(y*z)),q)], (\mu \times \delta)((0, y),q)\}\\
&= \max\{\delta(x *(y*z),q), \delta(y,q)\}
\end{aligned}
$$

Hence δ is an anti Q- fuzzy R-closed KU-ideal of X.
Next we will prove that μ is an anti Q- fuzzy R-closed KU-ideal of X.
Let $\mu(x*0,q) \le \mu(x, q)$.
Since by theorem 6.2(ii), either $\delta(x*0,q) \le \mu(x, q)$ or $\delta(x*0,q) \le \delta(x, q)$.
Assume that $\delta(x*0,q) \le \mu(x,q)$,then

$\mu(x,q) = \max\{\mu(x, q) , \delta(x*0,q)\} = \max\{\mu(x, q) , \delta(0,q)\} = (\mu \times \delta)((x,0),q)$

$$
\begin{aligned}
\mu(x * z, q) &= \max\{\mu(x * z,q), \delta(0,q)\}\\
&= (\mu \times \delta)((x * z,0), q)\\
&= (\mu \times \delta)((x * z ,q),(0 * 0,q))\\
&= (\mu \times \delta)(((x,0),q) * ((z,0),q))\\
&\le \max\{(\mu \times \delta)[((x,0),q) * (((y,0),q) * ((z,0),q))], (\mu \times \delta)((y,0),q)\}\\
&= \max\{(\mu \times \delta)[((x,0),q) * ((y*z,0*0),q)], (\mu \times \delta)((y,0),q)\}\\
&= \max\{(\mu \times \delta)[((x *(y*z),0*(0*0)),q)], (\mu \times \delta)((y,0),q)\}\\
&= \max\{(\mu \times \delta)[((x *(y*z),0),q)], (\mu \times \delta)((y,0),q)\}\\
&= \max\{\mu(x *(y*z),q), \mu(y,q)\}
\end{aligned}
$$

Hence μ is an anti Q- fuzzy R-closed KU-ideal of X.

7. Conclusion

In this article we have discussed anti Q-fuzzy R-closed KU- ideal of KU-algebras and its lower level cuts in detail. In our aspect this R-closed definition and main results can be similarly extended to some other algebraic systems such as BG-algebras,TM-algebras etc. We hope that this work would other foundations for further study of the theory of KU-algebras. In our future study of fuzzy structure of KU-algebra, may be the topics, Intuitionistic fuzzy set, interval valued fuzzy sets, should be considered .

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