On Fuzzy Dot SU-subalgebras

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Received 9 January 2013; accepted 24 January 2013

Abstract. This paper introduces the notion of fuzzy Dot SU-subalgebra and deals with some of their basic but interesting properties by applying the idea of fuzzy subset.

Keywords: SU-algebra, subalgebra, fuzzy Subset, fuzzy Dot SU-subalgebra.

AMS Mathematics Subject Classification (2010): 06F35, 08A72, 03E72, 03F55, 03G2

1. Introduction
The concept of a fuzzy set was introduced by Zadeh [6]. Since then many research work have been carried out in the fuzzy settings. Y.Imai and K.Iseki[3] introduced two classes of abstract algebras; BCK-algebras and BCI-algebra. It is known that the class of BCK-algebras is a proper sub class of the class of BCI-algebras. Recently, Supawadee Keawrahun and Utsanee Leerawat[5] introduced a new algebraic structure named SU-Algebra. In this paper we investigate the fuzzy Dot SU-subalgebra and establish some of their basic properties.

2. Preliminaries
In this section basic definitions of a SU-algebra, fuzzy subset and fuzzy subalgebra are recalled. We start with,

Definition 2.1.[5] A SU-algebra is a non-empty set X with a constant 0 and a single binary operation * satisfying the following axioms for any x, y, z ∈ X

i. y * z = (y * z) * z = 0 ;
ii. x * 0 = x ;
iii. If x * y = 0 ⇒ x = y .

Example 2.2. The following table with X = {0, 1, 2, 3} is a SU-algebra.

<table>
<thead>
<tr>
<th>*</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
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<tr>
<td>1</td>
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<tr>
<td>3</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Definition 2.3. A binary relation “≤” on X can be defined as x ≤ y if and only if x * y = 0 .

Definition 2.4. A non-empty subset S of a SU-algebra X is said to be a subalgebra if x * y ∈ S for all x, y ∈ S.
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Definition 2.5. A function \( f : X \to Y \) of SU-algebras \( X \) and \( Y \) is called homomorphism if 
\[
f(x * y) = f(x) * f(y) \quad \forall x, y \in X.
\]
And \( f : X \to Y \) is called anti-homomorphism if 
\[
f(x * y) = f(y) * f(x) \quad \forall x, y \in X.
\]

Remark 2.6. If \( f : X \to Y \) is a homomorphism on SU-algebras then \( f(0_X) = 0_Y \).

Definition 2.7. A fuzzy subset \( \mu \) in a non-empty set \( X \) is a function \( \mu : X \to [0, 1] \).

Definition 2.8. A fuzzy Subset \( \mu \) in a SU-algebra \( X \) is said to have Sup-property if for any subset \( T \subseteq X \) there exists \( x_0 \in T \) such that \( \mu(x_0) = \text{Sup}_T \mu(t) \)

Definition 2.9. Let \( f : X \to Y \) be a function and \( \mu \) be a fuzzy subset of \( X \). Then the image of \( \mu \) under \( f \) is a fuzzy subset \( \nu \) on \( Y \) defined by
\[
\nu(y) = \begin{cases} 
\sup_{z \in f^{-1}(y)} \mu(z) & \text{if } f^{-1}(y) \neq \emptyset; f(x) = y \not\exists \phi \quad \forall y \in Y \\
0 & \text{otherwise}
\end{cases}
\]

Definition 2.10. Let \( f : X \to Y \) be a function and \( \nu \) be the fuzzy subset of \( Y \). Then the inverse image of \( \nu \) under \( f \) is a fuzzy subset \( \mu \) of \( X \) defined by \( \nu(f(x)) = \mu(x) \quad \forall x \in X \).

Definition 2.11.[4] A fuzzy Subset \( \mu \) in \( X \) is said to be a fuzzy SU-subalgebra of \( X \) if 
\[
\mu(x * y) \geq \min \mu(x), \mu(y) \quad \forall x, y \in X.
\]

Example 2.12. Consider the SU-algebra \( X = \{1, 2, 3 \} \) in Example 2.2. and \( \mu \) is the fuzzy Subset of \( X \) defined by 
\[
\mu = \begin{cases} 
0.3 & x = 1, 2 \\
0.6 & x = 0, 3
\end{cases}
\]
is fuzzy SU-subalgebra of \( X \).

3. Fuzzy Dot SU-Subalgebra

Here we introduce the notion of fuzzy Dot SU-subalgebra in a SU-algebra \( X \). Here after unless otherwise specified \( X \) denotes a SU-algebra.

Definition 3.1. A fuzzy Subset \( \mu \) in \( X \) is said to be a fuzzy Dot SU-subalgebra of \( X \) if 
\[
\mu(x * y) \geq \mu(x) \cdot \mu(y) \quad \forall x, y \in X
\]

Example 3.2. Consider the SU-algebra \( X = \{1, 2, 3 \} \) in Example 2.2. and \( \mu \) is the fuzzy Subset of \( X \) defined by 
\[
\mu = \begin{cases} 
0.3; & x = 0 \\
0.4; & x = 1 \\
0.5; & x = 2 \\
0.6; & x = 3
\end{cases}
\]
is fuzzy Dot SU-subalgebra of \( X \).

Remark 3.3. Every fuzzy SU-subalgebra is fuzzy Dot SU-subalgebra but the converse is not true. In the Example 3.2, \( \mu(3 * 3) \geq \min \{3, 3\} \) is not valid, since we have 
\[
\mu(3 * 3) = \mu(0) = 0.3 < \min \{3, 3\} = 0.6,
\]

Theorem 3.4. If \( \mu \) is a fuzzy Dot SU-subalgebra of \( X \), then \( \mu(x) \geq \mu(x) = 0 \) and 
\[
\mu(x^n) \geq \mu(x)^n \quad \forall x \in X \text{ and } n \in \mathbb{N} \quad \text{where } 0^n * x = 0 * x \geq 0 \text{ occurs n times.}
\]

Proof. Since \( x * x = 0 \quad \forall x \in X \), we have 
\[
\mu(0) = \mu(x^n) \geq \mu(x) \cdot \mu(x) = \mu(x) = 0
\]
and by induction 
\[
\mu(x^n) \geq \mu(x)^n \quad \forall x \in X \text{ and } n \in \mathbb{N}
\]

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Theorem 3.5. If \( \mu \) is a fuzzy Dot SU-subalgebra of \( X \), then \( K \left[ \mu \right] ; \frac{1}{1} \subseteq \mathcal{A} \subseteq X ; \mu(x) = 1 \) is a subalgebra of \( X \).

**Proof.** Let \( x, y \in K \left[ \mu \right] ; \frac{1}{1} \). Then \( \mu(x) \geq 1 \). Therefore, \( x \cdot y \in \mathcal{A} \) and \( \mu(x) \geq 1 \).

Theorem 3.6. Intersection of two fuzzy Dot SU-subalgebras of \( X \) is also a fuzzy Dot SU-subalgebra of \( X \).

**Proof.** Let \( \mu \) and \( \sigma \) be any two fuzzy Dot SU-subalgebras of \( X \) and \( \lambda = \mu \cap \sigma \) where \( \lambda(x) = \min \{ \mu(x), \sigma(x) \} \). We want to prove \( \lambda(x) \cdot \lambda(y) \geq \lambda(x) \cdot \lambda(y) \) for all \( x, y \in X \).

For \( \lambda(x \cdot y) = \min \{ \mu(x \cdot y), \sigma(x \cdot y) \} \),

\[
\geq \min \{ \mu(x), \sigma(x) \} \cdot \min \{ \mu(y), \sigma(y) \}
\]

\[
= \lambda(x) \cdot \lambda(y)
\]

The above theorem can be generalized as follows.

Theorem 3.7. The intersection of a family of fuzzy Dot SU-subalgebras of \( X \) is also a fuzzy Dot SU-subalgebra of \( X \).

Theorem 3.8. Let \( f \) be a homomorphism from SU-algebras \( X \) onto \( Y \) and \( \mu \) be a fuzzy Dot SU-subalgebra of \( X \) with Sup-property. Then the image of \( \mu \) is also fuzzy Dot SU-subalgebra of \( Y \).

**Proof.** Let \( a, b \in Y \) with \( x_0 \in f^{-1}(a) \) and \( y_0 \in f^{-1}(b) \) such that

\[
\mu(x_0) = \sup_{t \in f^{-1}(a)} \mu(t) \quad \text{and} \quad \mu(y_0) = \sup_{t \in f^{-1}(b)} \mu(t)
\]

Let \( \nu \) be the image of \( \mu \).

Now by definitions 2.8 and 2.9,

\[
\nu(a \cdot b) = \sup_{t \in f^{-1}(a \cdot b)} \mu(t) \geq \mu(x_0 \cdot y_0) \geq \mu(x_0) \cdot \mu(y_0)
\]

Hence the image \( \nu \) is a fuzzy Dot SU-subalgebra of \( Y \).

Theorem 3.9. Let \( f \) be a homomorphism from SU-algebras \( X \) to \( Y \) and \( \nu \) be a fuzzy Dot SU-subalgebra of \( Y \). Then the inverse image of \( \nu \), is also fuzzy Dot SU-subalgebra of \( X \).

**Proof.** Let \( x, y \in X \) and \( \mu \) be the inverse image of \( \nu \). Then

\[
\mu(x \cdot y) = \nu(\mu(x \cdot y)) = \nu(\mu(x) \cdot f(y)) \geq \nu(\mu(x)) \cdot \nu(f(y)) = \mu(x) \cdot \mu(y)
\]

Hence the inverse image of \( \nu \), is fuzzy Dot SU-subalgebra of \( X \).

Remark 3.10. One can verify the theorem 3.8 and 3.9 for an anti-homomorphism on SU-algebras.

**4. Conclusion**

In this article we have extended the notions of fuzzy Dot SU-subalgebras of SU-algebras and verified some of their basic properties and the characteristic of their homomorphic(anti-
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homomorphic) image(pre image). Also in [5] Supawadee Keawrahun and Utsanee Leerawat says that the structure SU-algebra becomes a TM-algebra, QS-algebra and a BF-algebra and Andrzej Walendziak[1] proved that a BF-algebra is BG-algebra. Hence we conclude that, the results we have proved for SU-algebras are obviously true in TM-algebras, QS-algebras, BF-algebras and BG-algebras.

REFERENCES