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Optimal Policy for Vendor-Buyer Integrated Inventory Model within Just In Time in Fuzzy Environment

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Abstract. Normally, fuzzy inventory problems for the vendor and the buyer are treated separately. The integration of vendor-buyer inventory system is an important issue. This co-operative approach to inventory management contributes to the success of supply chain management by minimizing the joint inventory cost. The inclusion of the JIT (Just In Time) concept in this model contributes significantly to a joint inventory cost reduction. All costs are taken as triangular fuzzy numbers. A numerical example and sensitivity analysis are determined. Graded mean integration representation method is used to defuzzify the results. The derived results show an impressive cost reduction when compared with Goyal's model.

Keywords: Integrated deteriorating model, JIT delivery, Lot sizing, triangular fuzzy number, defuzzification

AMS Mathematics Subject Classification (2010): 03E72, 90B05

1. Introduction

In past, economic order quantity (EOQ) and economic production quantity (EPQ) were treated independently from the viewpoints of the buyer or the vendor. In most cases, the optimal solution for one player was non-optimal to the other player. Inventoried goods can be broadly classified into three meta-categories based on (a) obsolescence, (b) deterioration, (c) no obsolescence/deterioration. Obsolescence refers to items that lose their value through time because of rapid changes of technology or the introduction of a new product by competitor. Deterioration refers to the damage, spoilage, dryness, vaporization, etc. of the products. The products like green vegetables, human blood, foodstuffs, photographic film, etc. Also, the shelf-life of some products can be indefinite and they would fall under the no obsolescence/deterioration category.

The players (vendors, retailers, distributors, etc.) in a supply chain may belong to different corporate entities and be more interested in minimizing their own cost rather than that of the chain as a whole. This kind of single-sided optimal strategy is not suitable for today's global competitive environment. In real life, there are many examples in

which a manufacturer (vendor) has own set of direct outlets (buyers) to route the produced item to end customers. With the growing focus on supply chain management, firms realize that inventories across the entire supply chain can be more efficiently managed through better coordination to reduce cost and lead time without sacrificing customer service.

A supply chain consists of number of distinct entities (e.g. raw material supplier, manufacturer, transporter, retailers, etc.) who are responsible for converting the raw material into finished product and make them available to ultimate customers to satisfy their demand in time at least possible cost.

In today's competitive markets, close cooperation between the vendor and the buyer is necessary to reduce the joint inventory cost and the response time of the vendor– buyer system. The successful experiences of National Semiconductor, Wal-Mart, and Procter and Gamble have demonstrated that integrating the supply chain has significantly influenced the company's performance and market share (Simchi-Levi et al. [10]). For inventory Management and production planning one may refer Silver et al. [9].

Banerjee [1] developed a joint economic lot size model for a single vendor with finite production rate. Goyal [2] extended Banerjee's model by relaxing the lot-for-lot production assumption. Ha and Kim [4] derived the JIT system between the vendor and the buyer using geometric programming. Wee and Jong [12] developed the JIT system between multiple parts and the finished product. A comprehensive review of vendor–buyer integration models was done by Goyal and Gupta [3] and Thomas and Griffin [11]. An integrated approach on economic ordering policy of deteriorated item for vendor and buyer was done by Yang and Wee [13]. In this paper, incorporated the JIT concept, and modified Goyal's model to develop a significant cost reduction approach to the problem.

Nagoor Gani and Maheswari [5] investigated the effects of product quality and transportation flow in uncertain environment. Nagoor Gani and Maheswari [6] presented a Possibilistic Linear Programming (PLP) method of solving integrated Manufacturing and Distribution Planning Decisions (MDPD) problems with multiple imprecise goals in supply chains under uncertain environment. Nagoor Gani and Palaniammal [7] made on a continuous fuzzy production inventory model for deteriorating items without shortages. Nagoor Gani et al. [8] investigates just in time (JIT) single buyer single supplier integrated inventory model with deteriorating items with multiple deliveries.

2. Methodology

2.1. Fuzzy Numbers

Any fuzzy subset of the real line R, whose membership function μ_A satisfied the following conditions is a generalized fuzzy number \tilde{A} .

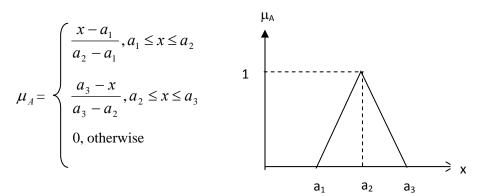
- (i) μ_A is a continuous mapping from R to the closed interval [0,1],
- $(ii) \qquad \mu_A \qquad = 0, \text{ } \infty < \ x \ \leq a_1,$
- (iii) $\mu_A = L(x)$ is strictly increasing on $[a_1,a_2]$
- $(iv) \qquad \mu_A \qquad = w_A, \, a_2 \leq x \leq a_3$
- (v) $\mu_A = R(x)$ is strictly decreasing on $[a_3, a_4]$
- $(vi) \qquad \mu_A \qquad = 0, \, a_4 \leq x < \infty$

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where $0 < w_A \le 1$ and a_1 , a_2 , a_3 and a_4 are real numbers. Also this type of generalized fuzzy number be denoted as $\widetilde{A} = (a_1, a_2, a_3, a_4, w_A)_{LR}$; when $w_A=1$, it can be simplified as $\widetilde{A} = (a_1, a_2, a_3, a_4)_{LR}$.

2.2. Triangular Fuzzy Number

The fuzzy set $\check{A} = (a_1, a_2, a_3)$ where $a_1 < a_2 < a_3$ and defined on R, is called the triangular fuzzy number, if the membership function of \tilde{A} is given by



2.3. The Function Principle

The function principle was introduced by Chen to treat fuzzy arithmetical operations. This principle is used for the operation of addition, subtraction, multiplication and division of fuzzy numbers.

Suppose $\widetilde{A} = (a_1, a_2, a_3)$ and $\widetilde{B} = (b_1, b_2, b_3)$ are two triangular fuzzy numbers. Then

The addition of \widetilde{A} and \widetilde{B} is (i) $\widetilde{A} + \widetilde{B} = (a_1+b_1, a_2+b_2, a_3+b_3)$ where $a_1, a_2, a_3, b_1, b_2, b_3$ are any real numbers. The multiplication of \widetilde{A} and \widetilde{B} is $\widetilde{A} \times \widetilde{B} = (c_1, c_2, c_3)$ (ii) where $T = \{a_1b_1, a_1b_3, a_3b_1, a_3b_3\}$ $c_1 = \min T, c_2 = a_2b_2, c_3 = Max T$ If $a_1, a_2, a_3, b_1, b_2, b_3$ are all non zero positive real numbers, then $\widetilde{A} \times \widetilde{B} = (a_1b_1, a_2b_2, a_3b_3)$ - $\widetilde{B} = (-b_3, -b_2, -b_1)$ then the subtraction of \widetilde{A} and \widetilde{B} is (iii) \widetilde{A} - \widetilde{B} = (a₁-b₃, a₁-b₂, a₃-b₁) where $a_1, a_2, a_3, b_1, b_2, b_3$ are any real numbers (iv) $\frac{1}{\widetilde{R}} = \widetilde{B}^{-1} = (1/b_3, 1/b_2, 1/b_1)$ where b_1, b_2, b_3 are all non zero positive real number, then the division of \tilde{A} and \tilde{B} is $\tilde{A}/\tilde{B} = (a_1/b_3, a_2/b_2, a_3/b_1)$ $K \tilde{A} = (Ka_1, Ka_2, Ka_3) \text{ if } K > 0$ (v) For any real number K, $K \tilde{A} = (Ka_3, Ka_2, Ka_1)$ if K < 0

2.4. Graded Mean Integration Representation Method

If $\overline{A} = (a_1, a_2, a_3)$ is a triangular fuzzy number then the graded mean integration

representation of \tilde{A} is given by $P(\tilde{A}) = a_1 + 4a_2 + a_3/6$

2.5. Assumptions

- (a) The integrated system of single-vendor and single-buyer is considered.
- (b) Both the production and demand rates are constant.
- (c) The vendor and the buyer have complete knowledge of each other's information.
- (d) Shortage is not allowed.

2.6. Notations

- Q Buyer's lot size per delivery
- n Number of deliveries from the vendor to the buyer
- S Vendor's setup cost per setup
- A Buyer's ordering cost per order
- C_v Vendor's unit production cost
- C_b Unit purchase cost paid by the buyer
- r Annual inventory carrying cost
- P Annual production rate
- D Annual demand rate
- ITC Integrated total cost of the vendor and the buyer
- \tilde{Q} Buyer's lot size per delivery
- \tilde{n} Number of deliveries from the vendor to the buyer
- \tilde{S} Vendor's fuzzy setup cost per setup
- \tilde{A} Buyer's fuzzy ordering cost per order
- \tilde{C}_{v} Vendor's fuzzy unit production cost
- \tilde{C}_b Fuzzy Unit purchase cost paid by the buyer
- \tilde{r} Annual inventory carrying cost
- \widetilde{ITC} Integrated fuzzy total cost of the vendor and the buyer

3. Fuzzy Mathematical Model

The buyer's replenishment interval is \tilde{Q}/D . The vendor's replenishment interval is $\tilde{n}\tilde{Q}/D$. The buyer's average inventory level is $\tilde{Q}/2$.

The vendor's average inventory level \tilde{I}_{v} is derived as follows:

$$\tilde{I}_v = \frac{vendor's time - weighted inventory}{vendor's replenishment interval}$$

$$=\frac{\tilde{n}\tilde{Q}^2}{2P}+\tilde{Q}^2\left(\frac{1}{D}-\frac{1}{P}\right)+2\tilde{Q}^2\left(\frac{1}{D}-\frac{1}{P}\right)+\dots+(\tilde{n}-1)\left(\frac{1}{D}-\frac{1}{P}\right)}{\tilde{n}\tilde{Q}!/D!}$$

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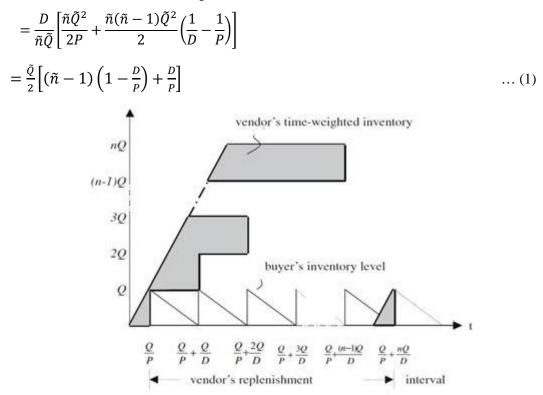


Figure 1: Buyer's inventory level and vendor.s time-weighted inventory

The result in (1) is different from that of Goyal's model [02] due to different modelling strategy. Goyal's model supplies goods to the buyer only after the termination of the production period, while our method supplies goods to the buyer as soon as there is enough to make up the batch-size, thus reducing the inventory cost during the production period.

Buyer's fuzzy annual ordering $\cot = \frac{D\tilde{A}}{\tilde{Q}}$ Buyer's fuzzy annual carrying $\cot = \frac{\tilde{r}\tilde{Q}\tilde{C}_b}{2}$ Then, Buyer's fuzzy annual $\cot = \frac{D\tilde{A}}{\tilde{Q}} + \frac{\tilde{r}\tilde{Q}\tilde{C}_b}{2}$ Vendor's fuzzy annual setup $\cot = \frac{D\tilde{S}}{\tilde{n}\tilde{Q}}$ Vendor's fuzzy annual carrying $\cot = \frac{\tilde{r}\tilde{Q}\tilde{C}_v}{2} \left[(\tilde{n} - 1)\left(1 - \frac{D}{P}\right) + \frac{D}{P} \right]$

Then, Vendor's fuzzy annual cost
$$= \frac{D\tilde{S}}{\tilde{n}\tilde{Q}} + \frac{\tilde{r}\tilde{Q}\tilde{C}_{v}}{2} \left[(\tilde{n}-1)\left(1-\frac{D}{P}\right) + \frac{D}{P} \right]$$

Using (1), the fuzzy integrated total cost of the vendor and the buyer per year is

$$\widetilde{ITC} = \frac{D\tilde{A}}{\tilde{Q}} + \frac{\tilde{r}\tilde{Q}\tilde{C}_{b}}{2} + \frac{D\tilde{S}}{\tilde{n}\tilde{Q}} + \frac{\tilde{r}\tilde{Q}\tilde{C}_{v}}{2} \left[(\tilde{n} - 1)\left(1 - \frac{D}{p}\right) + \frac{D}{p} \right] \qquad \dots (2)$$

In the right-side of (2), the first two terms are the buyer's annual fuzzy ordering cost, and fuzzy carrying cost, respectively, and the last two terms are the vendor's fuzzy annual setup cost and fuzzy carrying cost, respectively.

Equating the first derivative of (2) with regard to Q to zero and solving the equation, the buyer's economic lot size is,

$$\tilde{Q}^* = \sqrt{\frac{2D\left(\tilde{A} + \frac{\tilde{S}}{\tilde{n}}\right)}{\tilde{r}\left\{\tilde{c}_b + \tilde{c}_v\left[(\tilde{n} - 1)\left(1 - \frac{D}{p}\right) + \frac{D}{p}\right]\right\}}} \qquad \dots (3)$$

After substituting (3) into (2), the optimal integrated total cost is,

$$\widetilde{ITC}^* = \sqrt{2D\tilde{r}\left(\tilde{A} + \frac{\tilde{s}}{\tilde{n}}\right)\left\{\tilde{C}_b + \tilde{C}_v\left[\left(\tilde{n} - 1\right)\left(1 - \frac{D}{p}\right) + \frac{D}{p}\right]\right\}} \qquad \dots (4)$$

Substituting (3) into (2), equating the first derivatives with respect to n to zero and solving the equation, the value of n that minimizes the integrated total cost is

$$\tilde{n} = \sqrt{\frac{\tilde{S}\left[\tilde{C}_{b} - \tilde{C}_{\nu}\left(1 - \frac{2D}{p}\right)\right]}{\tilde{A}\tilde{C}_{\nu}\left(1 - \frac{D}{p}\right)}} \qquad \dots (5)$$

Since the value of n is positive integer, the optimal value of n, denoted by n^* , is

$$\tilde{n}_{1}^{*} = \left[\sqrt{\frac{\tilde{S}\left[\tilde{C}_{b} - \tilde{C}_{\nu}\left(1 - \frac{2D}{p}\right)\right]}{\tilde{A}\tilde{C}_{\nu}\left(1 - \frac{D}{p}\right)}} \right] \text{, when } \tilde{ITC}(\tilde{n}_{1}^{*}) \leq \tilde{ITC}(\tilde{n}_{1}^{*} + 1) \qquad \dots (6)$$

or

$$\tilde{n}_{2}^{*} = \left[\sqrt{\frac{\tilde{S}\left[\tilde{C}_{b} - \tilde{C}_{\nu}\left(1 - \frac{2D}{p}\right)\right]}{\tilde{A}\tilde{C}_{\nu}\left(1 - \frac{D}{p}\right)}} \right] + 1 \text{, when } \tilde{ITC}(\tilde{n}_{2}^{*}) \le \tilde{ITC}(\tilde{n}_{2}^{*} - 1) \qquad \dots (7)$$

where [x] is the greatest integer less than or equal to x, and ITC (n) represents that the ITC is a function of n.

From (3), the economic lot size of the lot-for-lot condition (i.e., when n=1) is

$$\tilde{Q}^* = \sqrt{\frac{2D(\tilde{A}+\tilde{S})}{\tilde{r}\left\{\tilde{C}_b + \tilde{C}_v\left(\frac{D}{P}\right)\right\}}} \qquad \dots (8)$$

From (3), when the production rate is very large, the economic lot size becomes

$$\tilde{Q}^* = \sqrt{\frac{2D\left(\tilde{A} + \frac{\tilde{S}}{\tilde{n}}\right)}{\tilde{r}\{\tilde{C}_b + (\tilde{n} - 1)\tilde{C}_v\}}} \qquad \dots (9)$$

4. Numerical Example

The numerical example can be illustrated by numerical example of Goyal [02] and yang et. al [14].

Vendor's fuzzy setup cost per setup $\tilde{S} = (350,400,450)$ Buyer's fuzzy ordering cost per order $\tilde{A} = (20,25,30)$ Vendor's fuzzy unit production cost $\tilde{C}_{\nu} = (19,20,21)$ Fuzzy Unit purchase cost paid by the buyer $\tilde{C}_b = (24,25,26)$ Annual inventory carrying cost $\tilde{r} = (0.1,0.2,0.3)$ Annual Demand rate D = 1000 units Annual Production rate P = 3200 units

4.1. Number of Deliveries ñ

$$\begin{split} \tilde{n} &= \sqrt{\frac{\tilde{S}\left[\tilde{C}_{b} - \tilde{C}_{v}\left(1 - \frac{2D}{P}\right)\right]}{\tilde{A}\tilde{C}_{v}\left(1 - \frac{D}{P}\right)}}}\\ \tilde{n} &= \sqrt{\frac{\left(350,400,450\right)\left[\left(24,25,26\right) - \left(19,20,21\right)\left(1 - \frac{2(3200)}{1000}\right)\right]}{\left(20,25,30\right)\left(19,20,21\right)\left(1 - \frac{3200}{1000}\right)}}\\ \tilde{n} &= \left(3.61,4.51,5.70\right)\\ \tilde{n} &= \left(4,5,6\right) \end{split}$$

4.2. Buyer's lot size per delivery \tilde{Q} :

$$\tilde{Q} = \sqrt{\frac{2D\left(\tilde{A} + \frac{\tilde{S}}{\tilde{n}}\right)}{\tilde{r}\left\{\tilde{C}_{b} + \tilde{C}_{v}\left[\left(\tilde{n} - 1\right)\left(1 - \frac{D}{P}\right) + \frac{D}{P}\right]\right\}}}$$

$$\begin{split} \tilde{Q} &= \sqrt{\frac{2000 \left[(20,25,30) + \frac{(350,400,450)}{(4,5,6)} \right]}{(0.1,0.2,0.3) \{ (24,25,26) + (19,20,21) [(3,4,5) (0.6875) + (0.3125)] \}}} \\ \tilde{Q} &= (70.61,110.33,203.05) \\ \tilde{Q} &= (71,110,203) \end{split}$$

4.3. Fuzzy Integrated Total Cost (\widetilde{ITC})

Buyer's fuzzy annual ordering cost $\frac{D\tilde{A}}{\tilde{Q}} = \frac{1000(20,25,30)}{(71,110,203)} = (98.52,227.27,422.54)$

Buyer's fuzzy annual carrying cost

$$\frac{\tilde{r}QC_b}{2} = \frac{1}{2}(0.1, 0.2, 0.3)(71, 110, 203)(24, 25, 26) = (85.2, 275, 791.7)$$

Buyer's fuzzy annual cost $\frac{D\tilde{A}}{\tilde{Q}} + \frac{\tilde{r}\tilde{Q}\tilde{C}_b}{2} = (183.72, 502.27, 1214.24)$

Vendor's fuzzy annual setup cost $\frac{D\tilde{S}}{\tilde{n}\tilde{Q}} = \frac{1000(350,400,450)}{(4,5,6)(71,110,203)} = (287.36,727.27,1584.51)$

Vendor's fuzzy annual carrying cost

$$\frac{\tilde{r}QC_{v}}{2} \left[(\tilde{n}-1)\left(1-\frac{D}{P}\right) + \frac{D}{P} \right]$$

= $\frac{1}{2}(0.1,0.2,0.3)(71,110,203)(19,20,21)[(3,4,5)(0.6875) + (0.3125)]$
= (160.19, 673.75, 2397.9375)

Vendor's fuzzy annual cost

$$\frac{D\tilde{S}}{\tilde{n}\tilde{Q}} + \frac{\tilde{r}\tilde{Q}\tilde{C}_{v}}{2} \left[(\tilde{n} - 1)\left(1 - \frac{D}{P}\right) + \frac{D}{P} \right] = (447.55, 1401.02, 3982.44)$$

The fuzzy integrated total cost of the vendor and the buyer per year is $I\widetilde{T}C = \frac{D\widetilde{A}}{\widetilde{Q}} + \frac{\widetilde{r}\widetilde{Q}\widetilde{C}_{b}}{2} + \frac{D\widetilde{S}}{\widetilde{n}\widetilde{Q}} + \frac{\widetilde{r}\widetilde{Q}\widetilde{C}_{v}}{2} \left[(\widetilde{n} - 1)\left(1 - \frac{D}{P}\right) + \frac{D}{P} \right]$ = (631.27, 1903.29, 5196.68)

Using Graded mean integration method, to get crisp values,

Number of Deliveries n = 5Buyer's Order size per delivery Q = 119 units Vendor's lot size = 595 units Buyer's annual ordering cost = Rs. 238.36 Buyer's annual carrying cost = Rs. 329.48 Buyer's annual cost = Rs. 567.84 Vendor's annual setup cost = Rs. 796.83 Vendor's annual carrying cost = Rs. 875.52

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Vendor's annual cost = Rs. 1672.34

Integrated total cost of the vendor and the buyer = Rs.2240.19

4.4. Sensitivity Analysis

	Q	ITC
n=1	386 Units	Rs. 2465.82
n=2	234 Units	Rs. 2155.83
n=3	172 Units	Rs. 2067.49
n=4	137 Units	Rs. 2043.35

4.5. Comparison of Models

Here, compare the results to Goyal's model [2].

	Goyal's Model [2]	Our model
ITC (if n=1)	Rs. 2304.09	Rs. 2465.82
ITC (if n=2)	Rs. 2274.9	Rs. 2178.78
ITC (if n=3)	Rs. 2303.1	Rs. 2067.49
ITC (if n=4)	Rs. 2345.2	Rs. 2043.35

The decrease in the vendor's average inventory using this model. Thus, our model is better than Goyal's model. The ITC of this model is lower than Goyal's model.

5. Conclusion

In this paper, we developed an economic lot size policy for an integrated vendor-buyer system. The derived results show an impressive cost reduction when compared with Goyal's model. The ITC of our model is shown to be lower than that of Goyal's model. Goyal's model supplies goods to the buyer only after the termination of the production period, while our method supplies goods to the buyer as soon as there is enough to make up the batch-size, thus reducing the inventory cost during the production period. Here we find out the Buyer's Order size, Vendor's lot size and integrated total cost of the vendor and the buyer.

REFERENCES

- 1. A. Banerjee, A joint economic lot size model for purchaser and vendor, *Decis. Sci.*, 17 (1986), 292–311.
- 2. S.K.Goyal, A joint economic lot size model for purchaser and vendor: A comment, *Decis. Sci.*, 19 (1988), 236–241.
- 3. S.K.Goyal and Y.P Gupta, Integrated inventory models: the buyer-vendor coordination, *Euro. J. Oper. Res.*, 41 (1989), 261–269.
- 4. D.Ha, S.L.Kim, Implementation of JIT purchasing: an integrated approach, *Prod. Plan. Control*, 8 (1997), 152–157.
- 5. A.Nagoor Gani and S.Maheswari, Integrated profit oriented supply chain of perfect quality items with multiple imprecise goals in an uncertain environment. *J. Math. Computer Science*, 3(2) (2013), 482-504.

- 6. A.Nagoor Gani and S.Maheswari, Inspection cost and imperfect quality items with multiple imprecise goals in supply chains in an uncertain environment, *International Journal of Fuzzy Mathematical Archive*, 1 (2013), 23-26.
- 7. A.Nagoor Gani and P.Palaniammal, A fuzzy production inventory system with deterioration, *Applied Mathematical Sciences*, 5(5) (2011), 233 241.
- 8. A.Nagoor Gani and G.Sabarinathan, Fuzzy approach on a near optimal solution for production integrated model under JIT delivery with deteriorating items, *Inter. J. Mathematical Sciences and Engineering Applications*, 7(II) (2013). 99-116.
- 9. E.A.Silver, D.F.Pyke and R.Peterson, Inventory management and production planning and scheduling, Wiley Inc, New York (1998).
- 10. D.Simchi-Levi, P.Kaminsky and E.Simchi-Levi, Designing and managing the supply chain, McGraw- Hill Companies, Singapore (2000).
- 11. D.J.Thomas and P.M.Griffin, Coordinated supply chain management, *European J. Oper. Res.*, 94 (1996), 1–15.
- 12. H.M.Wee and J.F.Jong, An integrated multi-lot-size production inventory model for deteriorating items, *Manage. Syst.*, 5 (1998), 97–114.
- 13. P.C.Yang and H.M.Wee, Economic ordering policy of deteriorated item for vendor and buyer: an integrated approach, *Prod. Plan. Control*, 11(5) (2000), 474–480.
- 14. P.C.Yang, H.M.Wee and H.J.Yang, Global optimal policy for vendor-buyer integrated inventory system within just in time environment, *Journal of Glob. Optim.* 37 (2007), 505-511.