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Solving Fully Fuzzy Linear Systems with Trapezoidal Fuzzy Number Matrices by Singular Value Decomposition

N.Jayanth Karthik[1]* and *E.Chandrasekaran[2]

[1] *Department of Mathematics, D.G.VaishnavCollege ,Chennai 600106 ,India.*
Email : n.jayanthkarthik@yahoo.com
[2] *Department of Mathematics, Presidency College , Chennai 600005, India.*
Email:e_chandrasekaran@yahoo.com

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Abstract. In this paper, a*n x n* fully fuzzy linear system $\tilde{A} \otimes \tilde{x} = \tilde{b}$ where $\tilde{A}$ is a fuzzy matrix$\tilde{x}$ and $\tilde{b}$ are fuzzy vectors with trapezoidal fuzzy numbers is solved by singular value decomposition method. The method is illustrated with numerical examples.

Keywords: Fully fuzzy linear system, trapezoidal fuzzy number matrices, singular value decomposition

AMS Mathematics Subject Classification (2010): 08A72, 15B15

1. Introduction

Linear system of equation has applications in many areas of science, engineering, finance and economics. Fuzzy linear system whose coefficient matrix is crisp and right hand side column is an arbitrary fuzzy number was first proposed by Friedman et al [5]. Several methods based on numerical algorithms and generalized inverse were used for solving fuzzy linear systems have been introduced by many authors [1,3,6,7,10].
A linear system is called a fully fuzzy linear system (FFLS) if all coefficients in the system are all fuzzy numbers. Nasseri [11] investigated linear system of equations with trapezoidal fuzzy numbers using embedding approach. Amitkumar [2] solved FFLS with trapezoidal fuzzy numbers using row reduced echelon form. In [8] and [9] FFLS with trapezoidal fuzzy number matrices was solved by partitioning them into sub matrices using Schur complement and QR decomposition method respectively.

In this paper a*n x n* FFLS $\tilde{A} \otimes \tilde{x} = \tilde{b}$ where $\tilde{A}$ is a fuzzy matrix $\tilde{x}$ and $\tilde{b}$ are fuzzy vectors with trapezoidal fuzzy numbers is solved by using Singular Value Decomposition method (SVD). Most direct methods fail when dealing with system of linear equations that are singular because matrix inversion is not possible. SVD is very powerful and efficient in dealing with such systems.

This paper is organized as follows. Some basic definitions and results on fuzzy sets and trapezoidal fuzzy numbers are given in section 2.In section 3, method to solve FFLS with trapezoidal fuzzy numbers matrices by SVD is given. Illustrations with numerical examples are given in section 4. Section 5 ends this paper with conclusion

2. Preliminaries

1. A fuzzy subset Ã of R is defined by its membership function by its membership function $\mu_{\tilde{A}}: R \to [0,1]$ which assigns a real number $\mu_{\tilde{A}}$ in the interval $[0,1]$ to each element $x \in R$ where the value of $\mu_{\tilde{A}}$ shows grade membership of x in Ã.

2. A trapezoidal fuzzy number denoted by Ã=(m, n, α, β)has the membership function

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x \leq \alpha \\ \dfrac{x-\alpha}{m-\alpha} & \alpha \leq x \leq m \\ 1 & m \leq x \leq n \\ \dfrac{\beta - x}{\beta - n} & n \leq x \leq \beta \\ 0 & x \geq \beta \end{cases}$$

3. A fuzzy number Ã is called positive (negative) denoted by Ã>0 (Ã<0) if its membership function $\mu_{\tilde{A}}$(x) satisfies $\mu_{\tilde{A}}=0$, $\forall x \leq 0 (\forall x \geq 0)$ using its mean value, left and right spreads such a fuzzy number Ã is symbolically written as Ã =(m, n, α, β) is positive if and only if $m - \alpha \geq 0$ and $n - \beta \geq 0$.

4. A Trapezoidal fuzzy number Ã = (m, n, α, β) is said to be zero trapezoidal fuzzy number if and only if m= 0, n =0, α =0, β=0.

5. Two fuzzy numbers $M = (m, n, \alpha, \beta)$and N =$(x, y, \gamma, \delta)$are equal if and only if . $m = x, n = y, \alpha = \gamma, \beta = \delta$.

6. For two fuzzy numbers $M = (m, n, \alpha, \beta)$and N =$(x, y, \gamma, \delta)$ the operations extended addition, extended opposite and extended multiplication are

$(m, n, \alpha, \beta) \oplus (x, y, \gamma, \delta) = (m + x, n + y, \alpha + \gamma, \beta + \delta)$

$$-M = -(m, n, \alpha, \beta) = (-m, -n, \beta, \alpha)$$

If M>0,N>0 then $(m, n, \alpha, \beta) \otimes (x, y, \gamma, \delta) = (mx, ny, m\gamma + x\alpha, n\delta + y\beta)$

For scalar multiplication

$$\lambda \otimes (m, n, \alpha, \beta) = \begin{cases} \lambda m, \lambda n, \lambda\alpha, \lambda\beta & \lambda \geq 0 \\ \lambda m, \lambda n, -\lambda\alpha, -\lambda\beta & \lambda < 0 \end{cases}$$

7**.** A matrix $\tilde{A} = (\tilde{a}_{ij})$ is called a fuzzy matrix if each element of $\tilde{A}$ is a fuzzy number. A fuzzy matrix $\tilde{A}$ is positive denoted by $\tilde{A}$>0 if each element of $\tilde{A}$ is positive. Fuzzy matrix $\tilde{A} = (\tilde{a}_{ij})$ which is n x n matrix can be represented such that$\tilde{a}_{ij} = (a_{ij}, b_{ij}, m_{ij}, n_{ij})$ where $\tilde{A} = (A, B, M, N)$ where $A = (a_{ij})$ $B = (b_{ij})$ $M = (m_{ij})$ $N = (n_{ij})$are $n\ x\ n$ crisp matrices.

8. A square matrix $\tilde{A} = (\tilde{a}_{ij})$ is symmetric matrix if $\tilde{a}_{ij} = \tilde{a}_{ji}$, ∀ i,j

9. Consider n x n fuzzy linear system of equations

$$(\tilde{a}_{11}\otimes\tilde{x}_1)\oplus(\tilde{a}_{12}\otimes\tilde{x}_2)\oplus\ldots\ldots\ldots\oplus(\tilde{a}_{1n}\otimes\tilde{x}_n) = \tilde{b}_1$$
$$(\tilde{a}_{21}\otimes\tilde{x}_1)\oplus(\tilde{a}_{22}\otimes\tilde{x}_2)\oplus\ldots\ldots\ldots\oplus(\tilde{a}_{2n}\otimes\tilde{x}_n) = \tilde{b}_2$$
$$\ldots\ldots\ldots\ldots..$$
$$(\tilde{a}_{n1}\otimes\tilde{x}_1)\oplus(\tilde{a}_{n2}\otimes\tilde{x}_2)\oplus\ldots\ldots\ldots\oplus(\tilde{a}_{nn}\otimes\tilde{x}_n) = \tilde{b}_n$$

The matrix of the above equation is $\tilde{A} \otimes \tilde{x} = \tilde{b}$ where coefficient matrix $\tilde{A} = (\tilde{a}_{ij})$ where $1 \le i, j \le n$ is a n x n fuzzy matrix and $\tilde{x}_j$, $\tilde{b}_j \in F(R)$. This system is called fully fuzzy linear system. (FFLS).

10. For solving n x n FFLS $\tilde{A} \otimes \tilde{x} = \tilde{b}$ where $\tilde{A} = (A,B,M,N)$, $\tilde{x} = (x, y,z,w)$ and $\tilde{b} = (b,g,h,k)$

$(A,B,M,N) \otimes (x,y,z,w) = (b,g,h,k)$

$(Ax,By,Az+Mx,Bw+Ny) = (b,g,h,k)$

$$Ax = b \quad (2.1)$$
$$By = g \quad (2.2)$$
$$Az + Mx = h (2.3)$$
$$Bw + Ny = k (2.4)$$

3. Solving FFLS using trapezoidal fuzzy number matrices by singular value decomposition

Any real *m x n* matrix *A* can be decomposed as $A = UDV^T$ where *U* is *m x m* orthogonal matrix and its columns are eigenvectors of AA^T, *D* is a diagonal matrix having the positive square roots of eigenvalues of A^TA or AA^T as its main diagonal called the singular values of *A* arranged in descending order and *V* is *n x n* orthogonal matrix its columns are eigenvectors of A^TA.

$A^TA = (VDU^T)(UDV^T) = VD^2V^T$.

$AA^T = (UDV^T)(VDU^T) = UD^2U^T$.

The product UDV^T is called the singular value decomposition (SVD) of *A*.

The SVD construction is based on the following result:

Singular value decomposition Theorem:
For every *m x n* real matrix *A* there exists an orthogonal *m x m* matrix *U*, an orthogonal *n x n* matrix *V* anda *m x n* diagonal matrix *D* such that $A = UDV^T$.
Proof: Refer [12].

Moore-Penrose Generalized inverse

Moore-Penrose generalized inverse or pseudo inverse can be computed using SVD. When A is singular taking $A = UDV^T$ in $Ax = b$ the solution is $x = A^+b$ where $A^+ = VD^+U^T$.

The solution is not exact but is closest in the least squares sense. If *A* is non singular then $A^+ = A^{-1}$

3.1. Solving FFLS by singular value decomposition method

Consider the FFLS $\tilde{A} \otimes \tilde{x} = \tilde{b}$

Taking trapezoidal fuzzy number matrices $(A,B,M,N) \otimes (x,y,z,w) = (b,g,h,k)$.

Let $A = U_1 D_1 V_1^T$ and $B = U_2 D_2 V_2^T$
$(U_1 D_1 V_1^T, U_2 D_2 V_2^T, M, N) \otimes (x, y, z, w) = (b, g, h, k)$
$(U_1 D_1 V_1^T x, U_2 D_2 V_2^T y, U_1 D_1 V_1^T z + Mx, U_2 D_2 V_2^T w + Ny) = (b, g, h, k)$
$U_1 D_1 V_1^T x = b \implies x = V_1 D_1^+ U_1^T b$ (3.1)
where D_1^+ is the Moore–Penrose generalized inverse of D_1 and is obtained by taking the reciprocal of each non-zero entry on the diagonal of D_1, leaving zeros in place.
$U_2 D_2 V_2^T y = g \implies y = V_2 D_2^+ U_2^T g$ (3.2)
where D_2^+ is the Moore–Penrose generalized inverse of D_2 and is obtained by taking the reciprocal of each non-zero entry on the diagonal of D_2, leaving zeros in place.
$U_1 D_1 V_1^T z + Mx = h \implies z = V_1 D_1^+ U_1^T (h - Mx)$ (3.3)
$U_2 D_2 V_2^T w + Ny = k \implies w = V_2 D_2^+ U_2^T (k - Ny)$ (3.4)
If A and B are square , symmetric and positive definite then $U = V$.

3.2. Algorithm for solving FFLS by singular value decomposition method

1. For the crisp linear system $Ax = b$ find the eigenvalues of AA^T and $A^T A$. The eigenvectors of AA^T make up columns of U_1 and eigenvectors of $A^T A$ make up columns of V_1. Square roots of eigenvalues from AA^T or $A^T A$ called singular values are the diagonal entries of D_1 in descending order. Compute $A = U_1 D_1 V_1^T$.

2. Compute $x = V_1 D_1^+ U_1^T b$

3. For the crisp linear system $By = g$ find the eigenvalues of BB^T and $B^T B$ .The eigenvectors of BB^T make up columns of U_2 and eigenvectors of $B^T B$ make up columns of V_2. Square roots of eigenvalues from BB^T or $B^T B$ called singular values are the diagonal entries of D_2 in descending order. Compute $B = U_2 D_2 V_2^T$.

4. Compute $y = V_2 D_2^+ U_2^T g$

5. Compute $z = V_1 D_1^+ U_1^T (h - Mx)$

6. Compute $w = V_2 D_2^+ U_2^T (k - Ny)$

4. Numerical examples

In this section we apply the algorithm for solving FFLS of fuzzy trapezoidal numbers.

Example 4.1. Consider the FFLS

$$\begin{bmatrix} (1,1,1,5) & (8,5,9,1) \\ (6,4,4,2) & (2,2,2,6) \end{bmatrix} \begin{bmatrix} (x_1, y_1, z_1, w_1) \\ (x_2, y_2, z_2, w_2) \end{bmatrix} = \begin{bmatrix} (23,33,62,45) \\ (46,24,70,66) \end{bmatrix}$$

$$A = \begin{pmatrix} 1 & 8 \\ 6 & 2 \end{pmatrix}, B = \begin{pmatrix} 1 & 5 \\ 4 & 2 \end{pmatrix}, M = \begin{pmatrix} 1 & 9 \\ 4 & 2 \end{pmatrix}, N = \begin{pmatrix} 5 & 1 \\ 2 & 6 \end{pmatrix}$$

$$b = \begin{pmatrix} 23 \\ 46 \end{pmatrix}, g = \begin{pmatrix} 33 \\ 24 \end{pmatrix}, h = \begin{pmatrix} 62 \\ 70 \end{pmatrix}, k = \begin{pmatrix} 45 \\ 66 \end{pmatrix}$$

$A = U_1 D_1 V_1^T$.

$\begin{pmatrix}1 & 8\\6 & 2\end{pmatrix} = \begin{pmatrix}-0.8643 & -0.5030\\-0.5030 & 0.8643\end{pmatrix}\begin{pmatrix}8.8206 & 0\\0 & 5.2151\end{pmatrix}\begin{pmatrix}-0.4401 & -0.8979\\0.8979 & -0.4401\end{pmatrix}$

$x = V_1 D_1^{-1} U_1^T b$

$x = \begin{pmatrix}-0.4401 & 0.8979\\-0.8979 & -0.4401\end{pmatrix}\begin{pmatrix}0.1134 & 0\\0 & 0.1918\end{pmatrix}\begin{pmatrix}-0.8643 & -0.5030\\-0.5030 & 0.8643\end{pmatrix}\begin{pmatrix}23\\46\end{pmatrix} = \begin{pmatrix}7\\2\end{pmatrix}$

$B = U_2 D_2 V_2^T$

$\begin{pmatrix}1 & 5\\4 & 2\end{pmatrix} = \begin{pmatrix}-0.7777 & -0.6287\\-0.6287 & 0.7777\end{pmatrix}\begin{pmatrix}6.1088 & 0\\0 & 2.9466\end{pmatrix}\begin{pmatrix}-0.5390 & -0.8423\\0.8423 & -0.5390\end{pmatrix}$

$y = V_2 D_2^{-1} U_2^T g$

$y = \begin{pmatrix}-0.5390 & 0.8423\\-0.8423 & -0.5390\end{pmatrix}\begin{pmatrix}0.1637 & 0\\0 & 0.3394\end{pmatrix}\begin{pmatrix}-0.7777 & -0.6287\\-0.6287 & 0.7777\end{pmatrix}\begin{pmatrix}33\\24\end{pmatrix} = \begin{pmatrix}3\\6\end{pmatrix}$

z = $V_1 D_1^{-1} U_1^T (h - Mx) = \begin{pmatrix}5\\4\end{pmatrix}$

w = $V_2 D_2^{-1} U_2^T (k - Ny) = \begin{pmatrix}4\\4\end{pmatrix}$

Hence the solution of the fully fuzzy linear system is $\tilde{x} = \begin{bmatrix}(7,3,5,4)\\(2,6,4,4)\end{bmatrix}$.

Example4.2. Consider the symmetric positive definite FFLS

$\begin{bmatrix}(8,5,6,2) & (3,1,2,1)\\(3,1,2,1) & (8,5,6,2)\end{bmatrix}\begin{bmatrix}(x_1, y_1, z_1, w_1)\\(x_2, y_2, z_2, w_2)\end{bmatrix} = \begin{bmatrix}(30,30,60,50)\\(25,30,46,46)\end{bmatrix}$

$A = \begin{pmatrix}8 & 3\\3 & 8\end{pmatrix}, B = \begin{pmatrix}5 & 1\\1 & 5\end{pmatrix}, M = \begin{pmatrix}6 & 2\\2 & 6\end{pmatrix}, N = \begin{pmatrix}2 & 1\\1 & 2\end{pmatrix}$

$b = \begin{pmatrix}30\\25\end{pmatrix}, g = \begin{pmatrix}30\\30\end{pmatrix}, h = \begin{pmatrix}60\\46\end{pmatrix}, k = \begin{pmatrix}50\\46\end{pmatrix}$

Since A is symmetric and positive definite $U_1 = V_1$.

$\begin{pmatrix}8 & 3\\3 & 8\end{pmatrix} = \begin{pmatrix}-1/\sqrt{2} & -1/\sqrt{2}\\-1/\sqrt{2} & 1/\sqrt{2}\end{pmatrix}\begin{pmatrix}11 & 0\\0 & 5\end{pmatrix}\begin{pmatrix}-1/\sqrt{2} & -1/\sqrt{2}\\-1/\sqrt{2} & 1/\sqrt{2}\end{pmatrix}$

$$x = V_1 D_1^{-1} U_1^T b$$

x = $\begin{pmatrix}-1/\sqrt{2} & -1/\sqrt{2}\\-1/\sqrt{2} & 1/\sqrt{2}\end{pmatrix}\begin{pmatrix}1/11 & 0\\0 & 1/5\end{pmatrix}\begin{pmatrix}-1/\sqrt{2} & -1/\sqrt{2}\\-1/\sqrt{2} & 1/\sqrt{2}\end{pmatrix}\begin{pmatrix}30\\20\end{pmatrix} = \begin{pmatrix}3\\2\end{pmatrix}$

Since A is symmetric and positive definite $U_2 = V_2$

$B = U_2 D_2 V_2^T$

$\begin{pmatrix}5 & 1\\1 & 5\end{pmatrix} = \begin{pmatrix}1/\sqrt{2} & 1/\sqrt{2}\\1/\sqrt{2} & -1/\sqrt{2}\end{pmatrix}\begin{pmatrix}6 & 0\\0 & 4\end{pmatrix}\begin{pmatrix}1/\sqrt{2} & 1/\sqrt{2}\\1/\sqrt{2} & -1/\sqrt{2}\end{pmatrix}$

$y = V_2 D_2^{-1} U_2^T g$

y = $\begin{pmatrix}1/\sqrt{2} & 1/\sqrt{2}\\1/\sqrt{2} & -1/\sqrt{2}\end{pmatrix}\begin{pmatrix}1/6 & 0\\0 & 1/4\end{pmatrix}\begin{pmatrix}1/\sqrt{2} & 1/\sqrt{2}\\1/\sqrt{2} & -1/\sqrt{2}\end{pmatrix}\begin{pmatrix}30\\30\end{pmatrix} = \begin{pmatrix}5\\5\end{pmatrix}$

z = $V_1 D_1^{-1} U_1^T (h - Mx) = \begin{pmatrix}4\\2\end{pmatrix}$

w = $V_2 D_2^{-1} U_2^T (k - Ny) = \begin{pmatrix}6\\5\end{pmatrix}$

Hence the solution of the fully fuzzy linear system is $\tilde{x} = \begin{bmatrix}(3,5,4,6)\\(2,5,2,5)\end{bmatrix}$

Example 4.3. Consider the FFLS

$$\begin{bmatrix}(1,1,1,1) & (2,1,1,1)\\(2,2,2,2) & (4,2,2,2)\end{bmatrix}\begin{bmatrix}(x_1, y_1, z_1, w_1)\\(x_2, y_2, z_2, w_2)\end{bmatrix}=\begin{bmatrix}(10,10,16,20)\\(20,20,32,40)\end{bmatrix}$$

$A = \begin{pmatrix}1 & 2\\2 & 4\end{pmatrix}$, $B = \begin{pmatrix}1 & 1\\2 & 2\end{pmatrix}$, $M = \begin{pmatrix}1 & 1\\2 & 2\end{pmatrix}$, $N = \begin{pmatrix}1 & 1\\2 & 2\end{pmatrix}$

$b = \begin{pmatrix}10\\20\end{pmatrix}$, $g = \begin{pmatrix}10\\20\end{pmatrix}$, $h = \begin{pmatrix}16\\32\end{pmatrix}$, $k = \begin{pmatrix}20\\40\end{pmatrix}$.

Since *A* is singular matrix then $A^+ = V_1 D_1^+ U_1^T$ where A^+ is the Moore–Penrose generalized inverse of *A*.

$$A = U_1 D_1 V_1^T$$

$$\begin{pmatrix}1 & 2\\2 & 4\end{pmatrix} = \begin{pmatrix}-0.4472 & -0.8944\\-0.8944 & 0.4472\end{pmatrix}\begin{pmatrix}5 & 0\\0 & 0\end{pmatrix}\begin{pmatrix}-0.4472 & -0.8944\\-0.8944 & 0.4472\end{pmatrix}$$

$$x = V_1 D_1^+ U_1^T b$$

$$x = \begin{pmatrix}-0.4472 & -0.8944\\-0.8944 & 0.4472\end{pmatrix}\begin{pmatrix}0.2 & 0\\0 & 0\end{pmatrix}\begin{pmatrix}-0.8643 & -0.5030\\-0.5030 & 0.8643\end{pmatrix}\begin{pmatrix}10\\20\end{pmatrix}=\begin{pmatrix}2\\4\end{pmatrix}$$

Since *B* is singular matrix then $B^+ = V_2 D_2^+ U_2^T$ where B^+ is the Moore–Penrose generalized inverse of *B*.

$$B = U_2 D_2 V_2^T$$

$$\begin{pmatrix}1 & 1\\2 & 2\end{pmatrix} = \begin{pmatrix}-0.4472 & -0.8944\\-0.8944 & 0.4472\end{pmatrix}\begin{pmatrix}3.1622 & 0\\0 & 0\end{pmatrix}\begin{pmatrix}-0.7071 & -0.7071\\-0.7071 & 0.7071\end{pmatrix}$$

$$y = V_2 D_2^+ U_2^T g$$

$$y = \begin{pmatrix}-0.7071 & -0.7071\\-0.7071 & 0.7071\end{pmatrix}\begin{pmatrix}0.3163 & 0\\0 & 0\end{pmatrix}\begin{pmatrix}-0.4472 & -0.8944\\-0.8944 & 0.4472\end{pmatrix}\begin{pmatrix}10\\20\end{pmatrix}=\begin{pmatrix}5\\5\end{pmatrix}$$

$$z = V_1 D_1^+ U_1^T (h - Mx) = \begin{pmatrix}2\\4\end{pmatrix}$$

$$w = V_2 D_2^+ U_2^T (k - Ny) = \begin{pmatrix}5\\5\end{pmatrix}$$

Hence the solution of the fully fuzzy linear system is $\tilde{x} = \begin{bmatrix}(2,5,2,5)\\(4,5,4,5)\end{bmatrix}$

5. Conclusion

In this paper solution of FFLS $\tilde{A} \otimes \tilde{x} = \tilde{b}$ is obtained by SVD by a new methodology in the form of trapezoidal fuzzy number matrices. By using orthogonal matrices throughout SVD reduces the risk of numerical errors. The Moore – Penrose generalized inverse can be easily obtained from SVD. Hence SVD is very efficient for the system $\tilde{A} \otimes \tilde{x} = \tilde{b}$ when $\tilde{A}$ is singular.

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