Interval-valued Fuzzy Ideals of Regular and Intraregular Semigroups

D.Singaram\(^1\) and PR. Kandasamy\(^2\)

\(^1\)Department of Mathematics, Karpagam University
Coimbatore - 641 021, Tamilnadu, India
\(^1\)Department of Mathematics, PSG college of Technology
Peelamedu, Coimbatore - 641 004, Tamilnadu, India
Email: dsingaram@yahoo.co.in

\(^2\)Department of Computer Applications, Hindustan Institute of Technology
Coimbatore - 641 032, Tamilnadu, India
Email: pr_kandasamy@yahoo.com

Received 12 December 2013; accepted 24 December 2013

Abstract. Let \(S\) be a semigroup. A mapping \(\tilde{A} : S \rightarrow D[0, 1]\) is called an interval-valued fuzzy subset of \(S\) where \(D[0, 1]\) denotes the family of all closed sub intervals of \([0, 1]\). A semigroup \(S\) is called an intraregular semigroup if for each element \(a \in S\) there exist \(x, y \in S\) such that \(a = xa^2y\). In this paper, intraregular semigroups are characterized by means of interval-valued fuzzy left ideals (resp. right ideals, bi-ideals, interior ideals).

Keywords: Semigroup, Interval-valued fuzzy subsemigroup, Interval-valued fuzzy ideal, Interval-valued fuzzy bi-ideal, Interval-valued fuzzy interior ideal

AMS Mathematics Subject Classification (2010): 20N25, 20M12, 03E72, 08A72

1. Introduction

Interval-valued Fuzzy Ideals of Regular and Intraregular Semigroups

2. Preliminaries
Let \( S \) be a semigroup.
A non-empty subset \( A \) of \( S \) is called a subsemigroup of \( S \) if \( AA \subseteq A \) and is called a left (resp. right) ideal of \( S \) if \( SA \subseteq A \) (resp. \( AS \subseteq A \)).

By two sided ideal or simply ideal, we mean a non-empty subset of \( S \) which is both a left and a right ideal of \( S \).
A subsemigroup \( A \) of \( S \) is called a bi-ideal of \( S \) if \( ASA \subseteq A \).
A non-empty subset \( A \) of \( S \) is called an interiorideal of \( S \) if \( SAS \subseteq A \).

**Definition 2.1.** A semigroup \( S \) is called regular if for each element \( a \in S \) there exists \( x \in S \) such that \( a = axa \). In other words \( S \) is regular if \( a \in Sa \) \( \forall a \in S \).

**Definition 2.2.** A semigroup \( S \) is called intraregular if for each element \( a \in S \) there exist \( x, y \in S \) such that \( a = xa^2y \). In other words \( S \) is intraregular if \( a \in Sa^2S \) \( \forall a \in S \).

We now review some fuzzy concepts.

A fuzzy subset \( A \) of a non-empty set \( X \) is a mapping from \( X \) to \([0, 1]\).

Let \( S \) be a semigroup. A fuzzy subset \( A \) of \( S \) is called a fuzzy subsemigroup of \( S \) if \( A(xy) \geq \min\{A(x), A(y)\} \) \( \forall x, y \in S \). A fuzzy subset \( A \) of \( S \) is called a fuzzy left (resp. right) ideal of \( S \) if \( A(xy) \geq A(x)(\text{resp. } A(xy) \geq A(y)) \) \( \forall x, y \in S \).

A fuzzy subset \( A \) of \( S \) is called a fuzzy two-sided ideal or simply fuzzy ideal of \( S \) if it is both a fuzzy left ideal and a fuzzy right ideal of \( S \).

A fuzzy subsemigroup \( A \) of \( S \) is called a fuzzy bi-ideal of \( S \) if \( A(xyz) \geq \min\{A(x), A(z)\} \) \( \forall x, y, z \in S \).

An interval number on \([0, 1]\), say \( \bar{a} \), is a closed subinterval of \([0, 1]\), that is \( \bar{a} = [a^-, a^+] \) where \( 0 \leq a^- \leq a^+ \leq 1 \). Let \( D[0, 1] \) denote the family of all closed subintervals of \([0, 1]\), \( \bar{0} = [0, 0] \) and \( \bar{1} = [1, 1] \).

For any two elements \( \bar{a} = [a^-, a^+] \) and \( \bar{b} = [b^-, b^+] \) in \( D[0, 1] \), we define

(i) \( \bar{a} \leq \bar{b} \) if and only if \( a^- \leq b^- \) and \( a^+ \leq b^+ \),

(ii) \( \bar{a} = \bar{b} \) if and only if \( a^- = b^- \) and \( a^+ = b^+ \),

(iii) \( \text{Min} \{\bar{a}, \bar{b}\} = \{\min \{a^-, b^-\}, \min \{a^+, b^+\}\} \),

(iv) \( \text{Max} \{\bar{a}, \bar{b}\} = \{\max \{a^-, b^-\}, \max \{a^+, b^+\}\} \).

Similarly we can define \( \text{Inf} \) and \( \text{Sup} \) in case of family of elements in \( D[0, 1] \).

A mapping \( \tilde{A} : X \rightarrow D[0, 1] \) is called an interval-valued fuzzy subset (briefly, an i-v fuzzy subset) of \( X \), where \( \tilde{A}(x) = [A^-(x), A^+(x)] \) \( \forall x \in X \). \( A^- \) and \( A^+ \) are fuzzy subsets in \( X \) such that \( A^-(x) \leq A^+(x) \) \( \forall x \in X \).

**Definition 2.3.** Let \( \tilde{A}, \tilde{B} \) be i-v fuzzy subsets of \( X \). Then we have the following:

(i) \( \tilde{A} \leq \tilde{B} \) if and only if \( \tilde{A}(x) \leq \tilde{B}(x) \) \( \forall x \).

(ii) \( \tilde{A} = \tilde{B} \) if and only if \( \tilde{A}(x) = \tilde{B}(x) \) \( \forall x \).

(iii) \( \tilde{A} \cup \tilde{B} \)(\( x \)) = \text{Max} \{\tilde{A}(x), \tilde{B}(x)\}

(iv) \( \tilde{A} \cap \tilde{B} \)(\( x \)) = \text{Min} \{\tilde{A}(x), \tilde{B}(x)\}.
D. Singaram and PR. Kandasamy

**Definition 2.4.** Let \( \cdot \) be a binary composition in a set \( S \). The product \( \tilde{A} \circ \tilde{B} \) of any two i-v fuzzy subsets \( \tilde{A}, \tilde{B} \) of \( S \) is defined by

\[
(\tilde{A} \circ \tilde{B})(x) = \begin{cases} 
\sup x = a \cdot b \{ \min x = a \cdot b \{ \tilde{A}(a), \tilde{B}(b) \} \} & \text{if } x \text{ is expressible as } x = a \cdot b \\
0 & \text{otherwise}
\end{cases}
\]

Since semigroup \( S \) is associative, the operation \( \circ \) is associative. We denote \( xy \) instead of \( x \cdot y \) and \( \tilde{A} \circ \tilde{B} \) for \( \tilde{A} \circ \tilde{B} \).

Let \( \tilde{B} \) be a subset of a set \( X \). Define a function \( \overline{\tilde{B}}: X \to D[0,1] \) by

\[
\overline{\tilde{B}}(x) = \begin{cases} 
1 & \text{if } x \in \tilde{B} \\
0 & \text{otherwise}
\end{cases} \forall x \in X.
\]

Clearly \( \overline{\tilde{B}} \) is an i-v fuzzy subset of \( X \). Throughout this paper \( \overline{\tilde{A}} \) is denoted by \( \tilde{S} \) and \( S \) will denote a semigroup unless otherwise mentioned.

An i-v fuzzy subset \( \tilde{A} \) of \( S \) is called an interval–valued fuzzy subsemigroup (briefly, an i-v fuzzy subsemigroup) of \( S \) if \( \tilde{A}(ab) \geq \min \{ \tilde{A}(a), \tilde{A}(b) \} \forall a, b \in S \).

An i-v fuzzy subset \( \tilde{A} \) of \( S \) is called an interval-valued fuzzy left (resp. right) ideal (briefly, an i-v fuzzy left (resp. right) ideal) of \( S \) if \( \tilde{A}(ab) \geq \tilde{A}(b)(\text{resp. } \tilde{A}(ab) \geq \tilde{A}(a)) \) for all \( a, b \in S \).

Every i-v fuzzy right (left, two sided) ideal of \( S \) is an i-v fuzzy subsemigroup of \( S \). However the converse is not true.

An i-v fuzzy subset \( \overline{\tilde{A}} \) of \( S \) is called an interval-valued fuzzy two-sided ideal or simply i-v fuzzy ideal of \( S \) if it is both an i-v fuzzy left ideal and an i-v fuzzy right ideal of \( S \).

An i-v fuzzy subsemigroup \( \tilde{A} \) of \( S \) is called an i-v fuzzy bi-ideal of \( S \) if \( \tilde{A}(xyz) \geq \min \{ \tilde{A}(x), \tilde{A}(y) \} \forall x, y, z \in S \).

An i-v fuzzy subsemigroup \( \tilde{A} \) of \( S \) is called an i-v fuzzy interior ideal of \( S \) if \( \tilde{A}(xay) \geq \tilde{A}(a) \).

**3. Results**

In this section, we obtained the structure of i-v fuzzy interior ideal of an intraregular semigroup and obtained equivalent conditions for a semigroup to be intraregular and showed that in an intraregular semigroup the concept of an i-v fuzzy ideal and an i-v fuzzy interior ideal are identical.

**Theorem 3.1.** Let \( S \) be an intraregular semigroup. Then \( \tilde{A} = \tilde{S}\tilde{A}\tilde{S} \) for every i-v fuzzy interior ideal \( \tilde{A} \) of \( S \).

**Proof:** Let \( \tilde{A} \) be an i-v fuzzy interior ideal of an intraregular semigroup \( S \).

\[
\tilde{S}\tilde{A}\tilde{S}(a) = \sup_{a = xy} \left\{ \min_{x = a \cdot b} \{ \tilde{S}\tilde{A}(x), \tilde{S}(y) \} \right\} \\
= \sup_{a = xy} \{ \tilde{S}\tilde{A}(x) \} \\
= \sup_{a = xy} \left\{ \sup_{x = au} \{ \min_{x = au} \{ \tilde{S}(u), \tilde{A}(v) \} \} \right\} \\
= \sup_{a = xy} \left\{ \sup_{x = au} \{ \tilde{A}(v) \} \right\}
\]

52
Conversely, assume that

By Lemma 3.2, we have

Thus

Let

Therefore

Then

By our assumption,

Proof:

Now, let

Therefore

Therefore,

\[ \bar{SA}S(a) = \sup_{a = pq} \left\{ \min_i \left( \bar{S}(p), (\bar{A}S)(q) \right) \right\} \]

\[ \geq \min_i \left( \bar{S}(xa), (\bar{A}S)(ay) \right) \]

\[ = (\bar{A}S)(ay) \]

\[ = \sup_{ay = uv} \left\{ \min_i \left( \bar{A}(u), S(v) \right) \right\} \]

\[ \geq \min_i \left( \bar{A}(xaa), S(y^2) \right) \]

\[ = \bar{A}(xa) \]

\[ = \bar{A}(a) \], since \( \bar{A} \) is an interior ideal.

Thus \( \bar{A} = \bar{SA}S \).

Lemma 3.2. [5] For a semigroup \( S \), the following conditions are equivalent:

(i) \( S \) is intraregular

(ii) \( A \cap B \subseteq AB \) holds for every left ideal \( A \) and every right ideal \( B \) of \( S \).

Theorem 3.3. For a semigroup \( S \), the following conditions are equivalent

(i) \( S \) is intraregular

(ii) \( A \cap B \subseteq A \ast B \) holds for every i-v fuzzy left ideal \( A \) and every i-v fuzzy right ideal \( B \) of \( S \).

Proof: Assume that \( S \) is intraregular. Therefore \( \forall a \in S, \exists x, y \in S \) such that \( a = xa^2y \).

Then we have \( (A \ast B)(a) = \sup_{a = pq} \left\{ \min_i \left( \bar{A}(u), \bar{A}(v) \right) \right\} \)

\[ \geq \min_i \left( \bar{A}(xa), \bar{B}(ay) \right) \]

\[ \geq \min_i \left( \bar{A}(a), \bar{B}(a) \right) \]

\[ = (\bar{A} \cap \bar{B})(a) \quad \forall a \in S \]

Hence by Definition 2.3(i) \( A \ast B \geq A \ast B \).

Conversely, assume that \( A \cap B \subseteq A \ast B \) for every left ideal \( A \) and every right ideal \( B \) of \( S \).

Let \( R \) be a right ideal and \( L \) be a left ideal of \( S \) respectively.

Then \( \bar{SLR} \) is an i-v fuzzy right ideal of \( S \) and \( \bar{SLR} \) is an i-v fuzzy left ideal of \( S \).

By our assumption, \( \bar{SL} \cap \bar{SR} \subseteq \bar{SL} \ast \bar{SR} \).

Since \( \bar{SL} \cap \bar{SR} = \bar{SLR} \) (lemma 2.3.12[2]) we have \( \bar{SLR} \subseteq \bar{SL} \ast \bar{SR} \).

Now, let \( a \in L \cap R \).

Therefore \( \bar{SLR}(a) = \bar{1} \) and by our assumption \( \bar{SL} \ast \bar{SR}(a) = \bar{1} \) that is

\[ \sup_{a = uv} \left\{ \min_i \left( \bar{SL}(u), \bar{SR}(v) \right) \right\} = \bar{1} \].

Therefore \( \exists x \in L \) and \( \exists y \in R \) such that \( a = xy \) which implies that \( a \in LR \).

Therefore \( L \cap R \subseteq LR \) for every left ideal \( L \) and every right ideal \( R \) of \( S \).

By Lemma 3.2, we have \( S \) is intraregular.
D. Singaram and PR. Kandasamy

Theorem 3.4. A semigroup $S$ is intraregular if and only if $(\forall a \in S)$ $\bar{A}(a) = \bar{A}(a^2)$ for every i-v fuzzy ideal $\bar{A}$ of $S$.

Proof: $\Rightarrow$ Let $\bar{A}$ be an i-v fuzzy ideal of $S$. And $a \in S$.

Then by hypothesis, as $S$ is intraregular $\exists x, y \in S$ such that $a = xa^2y$.

And $\bar{A}(a) = \bar{A}(xa^2y) \geq \bar{A}(a^2y) \geq \bar{A}(a^2) \geq \bar{A}(aa) \geq \bar{A}(a)$.

This implies that $\bar{A}(a) = \bar{A}(a^2)$.

$\Leftarrow$ $l(a^2)$ is an ideal of $S$ generated by $a^2$.

And $l(a^2) = \{a^2\} \cup \{Sa^2\} \cup \{a^2S\}$.

$\bar{x}(a^2)$ is an i-v fuzzy ideal of $S$.

By our assumption $\bar{x}(a^2)(a^2) = \bar{x}(a^2)(a)$.

We have $\bar{x}(a^2)(a) = 1$.

Hence $a \in l(a^2) = \{a^2\} \cup \{Sa^2\} \cup \{a^2S\}$.

Suppose $a \in \{a^2\}$

Then $a = a^2 = aa = a^2a^2 = aa^2a$.

Therefore $a \in Sa^2S$.

If $a \in \{Sa^2\}$, then $\exists x \in S$ such that $a = xa^2 = xaa = x(xa^2)a = x^2a^2a$.

Therefore $a \in Sa^2S$.

If $a \in \{a^2S\}$, then $\exists x \in S$ such that $a = a^2x = aax = a(a^2x)x = aa^2x^2$.

Therefore $a \in Sa^2S$.

Thus in all the cases, by Definition 2.2, $S$ is intraregular.

This completes the proof.

Theorem 3.5. Let $S$ be an intraregular semigroup. Then for any i-v fuzzy ideal $\bar{A}$ of $S$, we have, $\bar{A}(ab) = \bar{A}(ba)$, $\forall a, b \in S$

Proof: Let $\bar{A}$ be any i-v fuzzy ideal of $S$ and $a, b \in S$.

Then by theorem 3.4 and hypothesis, we have

$\bar{A}(ab) = \bar{A}((ab)^2) = \bar{A}(a(ba)b) \geq \bar{A}(ba) = \bar{A}((ba)^2) = \bar{A}(b(ab)a) \geq \bar{A}(ab)$

Thus we have $\bar{A}(ab) = \bar{A}(ba)$.

Lemma 3.6. A semigroup $S$ is regular and intraregular if and only if every bi-ideal of $S$ is idempotent, that is $B = B^2$ for every bi-ideal $B$ of $S$.

Proof: Let $S$ be both regular and intraregular semigroup and $B$ be a bi-ideal of $S$.

Since $B$ is a bi-ideal, we have $BSB \subseteq B$ and since $S$ is both regular and intraregular, we have $B \subseteq BSB$ and $B \subseteq SB^2S$.

Thus $B \subseteq BSB$

$\subseteq BS( SB^2S )SB$

$\subseteq BS^2B^2S^2B$

$\subseteq BSB^2SB$

$\subseteq BSBBSB$

$\subseteq BSB$

$= B^2$

That is $B \subseteq B^2$.

On the other hand, since $B$ is a bi-ideal of $S$ we have $B^2 \subseteq B$.

Hence we have $B = B^2$.

54
Interval-valued Fuzzy Ideals of Regular and Intraregular Semigroups

Conversely, let \( B = B^2 \) for every bi-ideal of \( S \) and let \( a \in S \).
But \( B(a) = \{ a \cup a^2 \cup aSa \} \) is a biideal.
Since \( a \in B(a) \), by our assumption \( B(a) = B^2(a) \), \( a \in B^2(a) = \{ a^2 \cup a^2Sa \cup aSa^2 \} \).
Therefore either \( a = a^2 \) or \( a \in a^2Sa \) or \( a \in aSa^2 \).
In all the cases it can easily seen that \( a \in aS\alpha \) and \( a \in S\alpha \).
This is true for any \( a \in S \).
Therefore \( S \) is both regular and intraregular.

**Theorem 3.7.** Let \( S \) be an intraregular semigroup. Then the following are equivalent.
(i) \( A \) is an i-v fuzzy ideal of \( S \).
(ii) \( A \) is an i-v fuzzy interior ideal of \( S \).
**Proof:** (i) \( \Rightarrow \) (ii). By Lemma 2.4.9 [2] every i-v fuzzy ideal of a semigroup \( S \) is an i-v fuzzy interior ideal of \( S \).
(ii) \( \Rightarrow \) (i). Assume that \( A \) is an i-v fuzzy interior ideal of a intraregular semigroup \( S \).
Let \( a, b \in S \). Then since \( S \) is intraregular \( \exists x, y, x', y' \in S \) such that \( a = xa^2y \) and \( b = xb^2y' \).
Thus \( A(ab) = A((xa)a(yb)) \geq A(a) \) and \( A(ab) = A(ax'b^2y') \geq A(b) \).
This implies that \( A \) is an i-v fuzzy ideal of \( S \).

**Definition 3.8.** An i-v fuzzy subset \( A \) of a semigroup \( S \) is called idempotent if \( (A\bar{A})(x) = A(x) \ \forall x \in S \).

**Theorem 3.9.** Let \( S \) be a semigroup. Then the following are equivalent.
(i) \( S \) is both regular and intraregular semigroup
(ii) Every i-v fuzzy bi-ideal \( A \) is idempotent.
**Proof:** Let \( A \) be an i-v fuzzy bi-ideal of an regular and intraregular semigroup \( S \).
Since \( A \) is an i-v fuzzy bi-ideal \( A(xy) \geq \text{Min}^1\{A(x), A(y)\} \) and \( A(xyz) \geq \text{Min}^1\{A(x), A(y)\} \) ……..(1)
First we will prove that \( A \circ A \subseteq A \bar{A} \).
Let \( a \in S \). Since \( S \) is regular \( \exists x \in S \) such that \( a = axa \)
\[ (A \circ A)(a) = \sup_{a = uv} \{ \text{Min}^1\{A(u), A(v)\} \} \]
\[ \leq \sup_{a = uv} \{ A(uv) \} \]
\[ = \text{Min}^1\{A(a)\} \]
\[ = A(a) \]
That is \( A \circ A \subseteq A \).
Now we will prove that \( A \subseteq A \circ A \). Since \( S \) is regular \( \forall a \in S, \exists x \in S \) such that \( a = axa \).
And since \( S \) is intraregular \( \forall \alpha, \exists x', y' \in S \) such that \( a = x'a^2y' \).
Therefore \( a = axa \)
\[ = (ax)(x'a^2y')(xa) \]
\[ = (axxa)(ayxa). \]
D. Singaram and PR. Kandasamy

Now \((\bar{A} \circ \bar{A})(a) = \Sup_{a = uv} \{\Min_{i}(\bar{A}(u), \bar{A}(v))\}\)
\[\geq \Min_{i}\{\bar{A}(axx')a, \bar{A}(ay'xa)\}\]
\[\geq \Min_{i}\{\Min_{i}(\bar{A}(a), \bar{A}(a)), \Min_{i}(\bar{A}(a), \bar{A}(a))\}\quad \text{by (1)}\]
\[= \Min_{i}\{\bar{A}(a), \bar{A}(a)\}\]
\[= \bar{A}(a)\]
\[\bar{A} \circ \bar{A} \supseteq \bar{A}\]

Therefore \(\bar{A}\) is idempotent.

Conversely assume that any i-v fuzzy bi-ideal \(\bar{A}\) of \(S\) is idempotent. That is \(\bar{A} \circ \bar{A} = \bar{A}\).

Now let \(B\) be any bi-ideal of \(S\). Therefore \(B^2 \subseteq \bar{B}\) and \(\bar{B}\) is an i-v fuzzy bi-ideal of \(S\).

By our assumption \(\bar{B} \circ \bar{B} = \bar{B}\).

Let \(a \in B\). Therefore \(\bar{B}(a) = 1\) which implies \((\bar{B} \circ \bar{B})(a) = 1\) and therefore
\[\Sup_{a = uv} \{\Min_{i}(\bar{B}(u), \bar{B}(v))\} = 1\]

Thus there exist \(b, c \in B\) such that \(a = bc\). Therefore \(a \in B^2\)

Hence \(B \subseteq B^2\) and hence \(B = B^2\).

Then by lemma 3.6 \(S\) is both regular and intraregular.

**Lemma 3.10.** (Theorem 2.7.2 [2]) A semigroup is regular if and only if for every i-v fuzzy right ideal \(\bar{A}\) and every i-v fuzzy left ideal \(\bar{B}\) of \(S\), we have \(\bar{A} \circ \bar{B} = \bar{A} \cap \bar{B}\).

**Theorem 3.11.** Let \(S\) be an ordered semigroup. Then the following are equivalent.

(i) \(S\) is regular and intraregular

(ii) \(\bar{A} \cap \bar{B} \subseteq (\bar{A} \circ \bar{B}) \cap (\bar{B} \circ \bar{A})\) for any i-v fuzzy bi-ideals \(\bar{A}\) and \(\bar{B}\) of \(S\)

(iii) \(\bar{A} \cap \bar{B} \subseteq (\bar{A} \circ \bar{B}) \cap (\bar{B} \circ \bar{A})\) for every i-v fuzzy bi-ideal \(\bar{A}\) and every i-v fuzzy left ideal \(\bar{B}\) of \(S\).

(iv) \(\bar{A} \cap \bar{B} \subseteq (\bar{A} \circ \bar{B}) \cap (\bar{B} \circ \bar{A})\) for every i-v fuzzy right ideal \(\bar{A}\) and every i-v fuzzy bi-ideal \(\bar{B}\) of \(S\).

(v) \(\bar{A} \cap \bar{B} \subseteq (\bar{A} \circ \bar{B}) \cap (\bar{B} \circ \bar{A})\) for every i-v fuzzy right ideal \(\bar{A}\) and every i-v fuzzy left ideal \(\bar{B}\) of \(S\).

**Proof:** (i) \(\Rightarrow\) (ii). Let \(\bar{A}\) and \(\bar{B}\) be i-v fuzzy bi-ideals of \(S\). And \(a \in S\).

Then since \(S\) is both regular and intraregular, there exists \(x \in S\) such that \(a = axa = axaxa\)

And there exist \(y, z \in S\) such that \(a = ya^2z\).

Thus \(a = axa = axaxa = ax(ya^2z)xa = (axya)(axxa)\).

Since \(\bar{A}\) and \(\bar{B}\) are i-v fuzzy bi-ideals of \(S\), we have
\[\bar{A}(axya) \geq \Min_{i}\{\bar{A}(a), \bar{A}(a)\} = \bar{A}(a)\]
\[\text{and}\]
\[\bar{B}(axxa) \geq \Min_{i}\{\bar{B}(a), \bar{B}(a)\} = \bar{B}(a)\.

Then \((\bar{A} \circ \bar{B})(a) = \Sup_{a = uv} \{\Min_{i}(\bar{A}(u), \bar{B}(v))\}\)
\[\geq \Min_{i}\{\bar{A}(axya), \bar{B}(axxa)\}\]
\[\geq \Min_{i}\{\bar{A}(a), \bar{B}(a)\}\]
\[= (\bar{A} \cap \bar{B})(a)\]

which means that \(\bar{A} \cap \bar{B} \subseteq \bar{A} \circ \bar{B}\).
Interval-valued Fuzzy Ideals of Regular and Intraregular Semigroups

In the same way we can show that \( \bar{A} \cap \bar{B} \subseteq \bar{B} \circ \bar{A} \).

Hence \( \bar{A} \cap \bar{B} \subseteq (\bar{A} \circ \bar{B}) \cap (\bar{B} \circ \bar{A}) \).

Since every i-v fuzzy left (right) ideal of \( S \) is a i-v fuzzy bi-ideal of \( S \), we have

\[(ii) \Rightarrow (iii), (ii) \Rightarrow (iv), (ii) \Rightarrow (v), (iii) \Rightarrow (v) \text{ and } (iv) \Rightarrow (v) \text{ are clear.}

(v) \Rightarrow (i). \]

Let \( \bar{A} \) and \( \bar{B} \) be i-v fuzzy right ideal and a i-v fuzzy left ideal of \( S \) respectively.

By hypothesis, \( \bar{A} \cap \bar{B} \subseteq (\bar{A} \circ \bar{B}) \cap (\bar{B} \circ \bar{A}) \subseteq \bar{B} \circ \bar{A} \)

By Theorem 3.3 \( S \) is intraregular.

On the other hand, \( \bar{A} \cap \bar{B} \subseteq (\bar{A} \circ \bar{B}) \cap (\bar{B} \circ \bar{A}) \subseteq \bar{A} \circ \bar{B} \)

But \( \bar{A} \circ \bar{B} \subseteq \bar{A} \circ \bar{S} \subseteq \bar{A} \) and \( \bar{A} \circ \bar{B} \subseteq \bar{S} \circ \bar{B} \subseteq \bar{B} \) implies \( \bar{A} \circ \bar{B} \subseteq \bar{A} \cap \bar{B} \)

Thus \( \bar{A} \circ \bar{B} = \bar{A} \cap \bar{B} \)

By Lemma 3.10 \( S \) is regular.

Thus \( S \) is both regular and intraregular.

This completes the proof.

REFERENCES

2. V. Chinnadurai, Contributions to the study of some fuzzy algebraic structures Doctoral Thesis, Annamalai University, 2010