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A Fuzzy Inventory Model for Deteriorating Items with Price Dependent Demand

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Abstract. A fuzzy inventory model with price dependent demand is considered. The shortages are allowed . The objective is to determine the optimal total cost and the optimal time length of the plan for the proposed model. The trapezoidal fuzzy numbers are introduced to achieve this objective. The signed distance method is used for defuzzification process. To illustrate the results of this model, sensitivity analysis is presented for both crisp and fuzzy model.

Keywords: Inventory, deterioration, trapezoidal fuzzy number, signed distance method

AMS Mathematics Subject Classification (2010): 90B05

1. Introduction

The inventory system is taking an important part of cost controlling in business. For the last fifteen years, researchers in this area have extended investigation into various models with considerations of item shortage, item deterioration, demand patterns, item order cycles and their combinations.

The controlling and regulating of deteriorating items is a measure problem in any inventory system. Certain products like vegetables, fruits, electronic components, chemicals deteriorate during their normal storage period. Hence when developing an optimal inventory policy for such products, the loss of inventory due to deterioration cannot be ignored. The researchers have continuously modified the deteriorating inventory models so as to more practicable and realistic. The analysis of deteriorating inventory model is initiated by Ghare and Scheader [3] with a constant rate of decay. Several researchers have extended their idea to different situations in deterioration on inventory model. The models for these type products have been developed by Bhowmick. Samanta [4], LiqunJi [6], Misra, Sahu, Bhaula and Raju [7] .

In conventional inventory models, various types of uncertainties are classically modeled using the approaches from the probability theory. However, there are uncertainties that cannot be appropriately treated by usual probabilistic models. To define inventory optimization tasks in such environment and to interpret optimal solutions, fuzzy set theory in inventory modeling renders an authenticity to the model formulated since fuzziness is the closest possible approach to reality.

In [2], Dutta and Pavan Kumar, discussed a fuzzy inventory model without shortage using Tapezoidal fuzzy number with sensitivity analysis, by using fuzzy trapezoidal numbers for holding cost and ordering cost. Syed [8] developed a fuzzy inventory model without shortages using signed distance method. He also used fuzzy triangular numbers for both ordering cost and holding cost. Umap [9] formed a fuzzy EOQ model for deteriorating items with two warehouses. He considered fuzzy numbers for the parameters such as holding cost and deteriorating cost for two ware houses. He used signed distance method and function principle method for defuzzification of total inventory costs as well as optimum order quantity. Economic order quantity model for deteriorating items with imperfect quality are discussed in [1]. Kasthuri et al. [5] developed a fuzzy inventory model for unit price related demand function. They used Kuhn-Tucker conditions for defuzzification.

In real situations, the demand is always depends on purchasing price . Therefore in this paper, a fuzzy inventory model with price dependent demand is considered. The price and hence demand are represented by trapezoidal fuzzy numbers. For defuzzification signed distance method is used. The optimal total cost and the optimal time length of the plan are derived. The model is illustrated by sensitivity analysis for both crisp and fuzzy model.

2. Prelimiaries

Definition 2.1. A fuzzy set $\tilde{a}$ on $R = (-\infty, \infty)$ is called a fuzzy point if its membership function is

$$\mu_{\tilde{a}}(x) = \begin{cases} 1, x = a \\ 0, x \neq a \end{cases}$$

Where the point 'a' is called the support of fuzzy set $\tilde{a}$.

Definition 2.2. A trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is represented with membership function $\mu_{\tilde{A}}$ is defined as $\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a}, when\ a \leq x \leq b \\ 1, when\ b \leq x \leq c \\ \frac{d-x}{d-c}, when\ c \leq x \leq d \\ 0, otherwise \end{cases}$

Definition 2.3. A fuzzy set $\tilde{A}$ is defined on R. Then the signed distance of $\tilde{A}$ is defined as $d(\tilde{A}, 0) = \frac{1}{2}\int_0^1 [A_L(\alpha) + A_R(\alpha)]d\alpha$, where $A_\alpha = [A_L(\alpha), A_R(\alpha)]$ $= [a + (b-a)\alpha, d - (d-c)\alpha], \alpha \in [0,1]$is the α-cut of fuzzy set $\tilde{A}$.

Definition 2.4. Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers, then arithmetical operations are defined as

Addition: $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$
Subtraction: $\tilde{A} - \tilde{B} = (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4)$
Multiplication: $\tilde{A} \times \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)$

Division: $\frac{\tilde{A}}{\tilde{B}} = \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}\right)$

Scalar Multiplication:

For any real constant k, $k\tilde{A} = \begin{cases} (ka_1, ka_2, ka_3, ka_4), k \geq 0 \\ (ka_4, ka_3, ka_2, ka_1), k < 0 \end{cases}$

3. Notations and assumptions

3.1. Notations

$q(t)$= The inventory level at time t
D = The demand rate (function of purchase cost)
α_1= The deterioration rate in $[0, t_1]$
M = The shortage level of inventory
R = The inventory level at $t = 0$
Q = The lot size
t_1 = The time at which the inventory attains zero
t_2 = The time duration at which the shortages are allowed
T = The time length of the plan
h = The holding cost per unit item
s = Then shortage cost per unit item
p = The purchase cost per unit item
O = The ordering cost per order
TC = The total cost for the period [0,T]
$\tilde{p}$ = The fuzzy purchase cost per unit item
$\widetilde{D}$ = The fuzzy demand rare
$\widetilde{TC}$ = The fuzzy total cost for the period [0,T]
$\widetilde{TC}_{dG}$ = The defuzzified value of $\widetilde{TC}$ by signed distance method
$\widetilde{t_1}$ = The fuzzy time at which inventory attains zero
$\widetilde{t_2}$ = The fuzzy time duration at which shortages are allowed
$\widetilde{t_1}_{dG}$ = The defuzzified value of $\widetilde{t_1}$ by signed distance method
$\widetilde{t_2}_{dG}$ = The defuzzified value of $\widetilde{t_2}$ by signed distance method

3.2. Assumptions

1. The demand is related to the unit price as $D = Ap^{\beta}$, where A(>0), 0<β<1.
2. The shortages are allowed.
3. There is no replacement or repair of deteriorated items during the cycle under consideration.
4. The purchase cost and hence demand are fuzzy in nature.
5. The signed distance method is used for defuzzification.
6. The lead time is zero.
7. The deterioration is instantaneous.

4. Mathematical model

The behavior of inventory system goes as follows

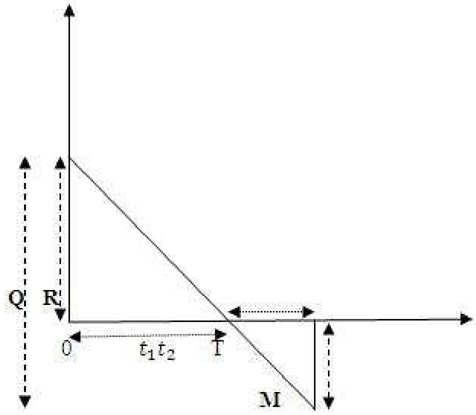

Figure 1:

This system starts with inventory level R at $t = 0$. The inventory level R decreases owing to the demand as well as deterioration with deterioration rate α_1 and reaches zero at time t_1. Then shortages are allowed accumulate up to time T. The replenishment is done at time T with quantity M. This quantity is decreases to make up the shortages that accumulated in the time interval $[t_1$,T]. The entire process is repeated.

4.1. Crisp model

The change of inventory level can be described by the following equation

$$\frac{dq(t)}{dt} = -\alpha_1 q(t) - D, \qquad 0 < t < t_1$$

$$\frac{dq(t)}{dt} = -D, \qquad t_1 < t < T \tag{1}$$

with the boundary conditions $q(0) = R, q(t_1) = 0, q(T) = M$

The solution of (1) are given by

$$q(t) = \begin{cases} -\frac{D}{\alpha_1} + \left(R + \frac{D}{\alpha_1}\right) e^{-\alpha_1 t}, 0 < t < t_1 \\ -D(t - t_1), t_1 < t < T \end{cases} \tag{2}$$

The order quantity $Q = DT + \alpha_1 t_1 = Dt_1 + Dt_2 + \alpha_1 t_1 = R + M$ (3)

The holding cost $HC = h \int_0^{t_1} q(t)dt$

$$= h\left[-\frac{D}{\alpha_1} t_1 + \frac{1}{\alpha_1}\left(R + \frac{D}{\alpha_1}\right)(1 - e^{-\alpha_1 t_1})\right]$$

$$= h\left[\frac{1}{\alpha_1}\left(R + \frac{D}{\alpha_1}\right)\left(1 - e^{-\frac{\alpha_1 R}{(D+\alpha_1)}}\right) - \frac{DR}{\alpha_1(D+\alpha_1)}\right] \tag{4}$$

The shortage cost $SC = s \int_{t_1}^{T} q(t)dt$

$$= sD\left[t_1 T - \frac{{t_1}^2}{2} - \frac{T^2}{2}\right] = \frac{-sM^2}{2D} \tag{5}$$

The deterioration cost $DC = d[R - Dt_1] = dR\left[1 - \frac{D}{D+\alpha_1}\right]$ (6)

The total cost

$$TC = Purchase\ cost + Ordering\ cost + Holding\ \ cost + Deterioration\ cost - \ Shortage\ cost$$

$$= pQ + O + h\left[\frac{1}{\alpha_1}\left(Q - M + \frac{D}{\alpha_1}\right)\left(1 - e^{-\frac{\alpha_1(Q-M)}{(D+\alpha_1)}}\right) - \frac{D(Q-M)}{\alpha_1(D+\alpha_1)}\right] + \frac{sM^2}{2D} + d(Q-M)\left[1 - \frac{D}{D+\alpha_1}\right] \quad (7)$$

For a minimum of total cost TC(M), $\frac{dTC(M)}{dM} = 0$ and $\frac{d^2TC(M)}{dM^2} > 0$

$$\frac{dTC}{dM} = h\left[-\frac{1}{D+\alpha_1}\left(Q - M + \frac{D}{\alpha_1}\right)e^{-\frac{\alpha_1(Q-M)}{(D+\alpha_1)}} - \frac{\left(1 - e^{-\frac{\alpha_1(Q-M)}{(D+\alpha_1)}}\right)}{\alpha_1} + \frac{D}{\alpha_1(D+\alpha_1)}\right] + \frac{sM}{D} - d\left(1 - \frac{D}{D+\alpha_1}\right) \quad (8)$$

Here $t_1 = \frac{R}{D+\alpha_1} = \frac{Q-M}{D+\alpha_1}$, $t_2 = \frac{M}{D}$ and $t_1 + t_2 = T$.

4.2. Fuzzy model

The inventory model in fuzzy environment is considered, since purchase cost and hence demand are trapezoidal fuzzy numbers.

Let $\tilde{p} = The\ fuzzy\ purchasing\ cost\ per\ unit\ item\ per\ unit\ time.$

$\tilde{D} = A\tilde{p}^{\beta} = The\ fuzzy\ demand\ over\ the\ planning\ time\ period\ [0, T]$

$$\widetilde{TC} = \tilde{p}Q + O + h\left[\frac{1}{\alpha_1}\left(Q - M + \frac{\tilde{D}}{\alpha_1}\right)\left(1 - e^{-\frac{\alpha_1(Q-M)}{(\tilde{D}+\alpha_1)}}\right) - \frac{\tilde{D}(Q-M)}{\alpha_1(\tilde{D}+\alpha_1)}\right] + \frac{sM^2}{2\tilde{D}} + d(Q-M)\left(1 - \frac{\tilde{D}}{\tilde{D}+\alpha_1}\right) \quad (9)$$

$$= (p_1, p_2, p_3, p_4)Q + O + h\left[\frac{1}{\alpha_1}\left(Q - M + \frac{Ap_1{}^{\beta}}{\alpha_1}, Q - M + \frac{Ap_2{}^{\beta}}{\alpha_1}, Q - M + \frac{Ap_3{}^{\beta}}{\alpha_1}, Q - M + \frac{Ap_4{}^{\beta}}{\alpha_1}\right)\right] - h\left[\frac{1}{\alpha_1}\left(\left(Q - M + \frac{Ap_1{}^{\beta}}{\alpha_1}\right)e^{\frac{-\alpha_1(Q-M)}{Ap_1{}^{\beta}+\alpha_1}}, \left(Q - M + \frac{Ap_2{}^{\beta}}{\alpha_1}\right)e^{\frac{-\alpha_1(Q-M)}{Ap_2{}^{\beta}+\alpha_1}}, \left(Q - M + \frac{Ap_3{}^{\beta}}{\alpha_1}\right)e^{\frac{-\alpha_1(Q-M)}{Ap_3{}^{\beta}+\alpha_1}}, \left(Q - M + \frac{Ap_4{}^{\beta}}{\alpha_1}\right)e^{\frac{-\alpha_1(Q-M)}{Ap_4{}^{\beta}+\alpha_1}}\right) + \left(\frac{Ap_1{}^{\beta}(Q-M)}{\alpha_1(Ap_4{}^{\beta}+\alpha_1)}, \frac{Ap_2{}^{\beta}(Q-M)}{\alpha_1(Ap_3{}^{\beta}+\alpha_1)}, \frac{Ap_3{}^{\beta}(Q-M)}{\alpha_1(Ap_2{}^{\beta}+\alpha_1)}, \frac{Ap_4{}^{\beta}(Q-M)}{\alpha_1(Ap_1{}^{\beta}+\alpha_1)}\right)\right] + sM^2\left[\frac{1}{2Ap_4{}^{\beta}}, \frac{1}{2Ap_3{}^{\beta}}, \frac{1}{2Ap_2{}^{\beta}}, \frac{1}{2Ap_1{}^{\beta}}\right] - d(Q-M)\left[\frac{Ap_1{}^{\beta}}{(Ap_4{}^{\beta}+\alpha_1)}, \frac{Ap_2{}^{\beta}}{(Ap_3{}^{\beta}+\alpha_1)}, \frac{Ap_3{}^{\beta}}{(Ap_2{}^{\beta}+\alpha_1)}, \frac{Ap_4{}^{\beta}}{(Ap_1{}^{\beta}+\alpha_1)}\right] + d(Q-M) \quad (10)$$

$$= (a, b, c, d)$$

$$a = p_1Q + O + h\left[\frac{1}{\alpha_1}\left(Q - M + \frac{Ap_1{}^{\beta}}{\alpha_1}\right)\right] - h\left[\frac{1}{\alpha_1}\left(Q - M + \frac{Ap_4{}^{\beta}}{\alpha_1}\right)e^{\frac{-\alpha_1(Q-M)}{Ap_4{}^{\beta}+\alpha_1}} + \frac{Ap_4{}^{\beta}(Q-M)}{\alpha_1(Ap_1{}^{\beta}+\alpha_1)}\right] + s\left(\frac{M^2}{2Ap_4{}^{\beta}}\right) - d\left[\frac{(Q-M)Ap_4{}^{\beta}}{(Ap_1{}^{\beta}+\alpha_1)}\right] + d(Q-M) \quad (11)$$

$$b = p_2Q + O + h\left[\frac{1}{\alpha_1}\left(Q - M + \frac{Ap_2{}^{\beta}}{\alpha_1}\right)\right] - h\left[\frac{1}{\alpha_1}\left(Q - M + \frac{Ap_3{}^{\beta}}{\alpha_1}\right)e^{\frac{-\alpha_1(Q-M)}{Ap_3{}^{\beta}+\alpha_1}} +\right.$$

$$\left.\frac{Ap_3{}^{\beta}(Q-M)}{\alpha_1(Ap_2{}^{\beta}+\alpha_1)}\right]+s\left(\frac{M^2}{2Ap_3{}^{\beta}}\right)-\quad d\left[\frac{(Q-M)Ap_3{}^{\beta}}{(Ap_2{}^{\beta}+\alpha_1)}\right]+d(Q-M) \tag{12}$$

$$c=p_3Q+O+h\left[\frac{1}{\alpha_1}\left(Q-M+\frac{Ap_3{}^{\beta}}{\alpha_1}\right)\right]-h\left[\frac{1}{\alpha_1}\left(Q-M+\frac{Ap_2{}^{\beta}}{\alpha_1}\right)e^{\frac{-\alpha_1(Q-M)}{Ap_2{}^{\beta}+\alpha_1}}+\right.$$

$$\left.\frac{Ap_2{}^{\beta}(Q-M)}{\alpha_1(Ap_3{}^{\beta}+\alpha_1)}\right]+s\left(\frac{M^2}{2Ap_2{}^{\beta}}\right)-\quad d\left[\frac{(Q-M)Ap_2{}^{\beta}}{(Ap_3{}^{\beta}+\alpha_1)}\right]+d(Q-M) \tag{13}$$

$$d=p_4Q+O+h\left[\frac{1}{\alpha_1}\left(Q-M+\frac{Ap_4{}^{\beta}}{\alpha_1}\right)\right]-h\left[\frac{1}{\alpha_1}\left(Q-M+\frac{Ap_1{}^{\beta}}{\alpha_1}\right)e^{\frac{-\alpha_1(Q-M)}{Ap_1{}^{\beta}+\alpha_1}}+\right.$$

$$\left.\frac{Ap_1{}^{\beta}(Q-M)}{\alpha_1(Ap_4{}^{\beta}+\alpha_1)}\right]+s\left(\frac{M^2}{2Ap_1{}^{\beta}}\right)-\qquad d\left[\frac{(Q-M)Ap_1{}^{\beta}}{(Ap_4{}^{\beta}+\alpha_1)}\right]+d(Q-M) \tag{14}$$

Defuzzification of fuzzy total cost by signed distance method is as follows

$$d(\widetilde{TC},0)=\frac{1}{2}\int_0^1[A_L(\alpha)+A_R(\alpha)]\,d\alpha$$

$$=\frac{1}{2}\int_0^1[a+(b-a)\alpha+d-(d-c)\alpha]\,d\alpha$$

Therefore

$$d(\widetilde{TC},0)=\frac{1}{2}\Bigg[(p_1+p_4)Q+2O+h\left[\frac{1}{\alpha_1}\left\{\left(2Q-2M+\frac{A(p_1{}^{\beta}+p_4{}^{\beta})}{\alpha_1}\right)\right\}\right]-$$

$$h\left[\frac{1}{\alpha_1}\left\{\left(Q-M+\frac{Ap_4{}^{\beta}}{\alpha_1}\right)e^{\frac{-\alpha_1(Q-M)}{Ap_4{}^{\beta}+\alpha_1}}+\left(Q-M+\frac{Ap_1{}^{\beta}}{\alpha_1}\right)e^{\frac{-\alpha_1(Q-M)}{Ap_1{}^{\beta}+\alpha_1}}\right\}+\right.$$

$$\left.\frac{A(Q-M)}{\alpha_1}\left\{\frac{p_4{}^{\beta}}{Ap_1{}^{\beta}+\alpha_1}+\frac{p_1{}^{\beta}}{Ap_4{}^{\beta}+\alpha_1}\right\}\right]+\frac{sM^2}{2A}\left[\frac{1}{p_4{}^{\beta}}+\frac{1}{p_1{}^{\beta}}\right]-d(Q-M)A\left[\frac{p_4{}^{\beta}}{Ap_1{}^{\beta}+\alpha_1}+\right.$$

$$\left.\frac{p_1{}^{\beta}}{Ap_4{}^{\beta}+\alpha_1}\right]+2d(Q-M)\Bigg]+\frac{1}{4}\Bigg[(p_2-p_1)Q+h\left[\frac{1}{\alpha_1}\left\{\frac{A}{\alpha_1}\left(p_2{}^{\beta}-p_1{}^{\beta}\right)\right\}\right]-$$

$$h\left[\frac{1}{\alpha_1}\left\{\left(Q-M+\frac{Ap_3{}^{\beta}}{\alpha_1}\right)e^{\frac{-\alpha_1(Q-M)}{Ap_3{}^{\beta}+\alpha_1}}-\left(Q-M+\frac{Ap_4{}^{\beta}}{\alpha_1}\right)e^{\frac{-\alpha_1(Q-M)}{Ap_4{}^{\beta}+\alpha_1}}\right\}+\right.$$

$$\left.\frac{A(Q-M)}{\alpha_1}\left\{\frac{p_3{}^{\beta}}{Ap_2{}^{\beta}+\alpha_1}-\frac{p_4{}^{\beta}}{Ap_1{}^{\beta}+\alpha_1}\right\}\right]+\frac{sM^2}{2A}\left[\frac{1}{p_3{}^{\beta}}-\frac{1}{p_4{}^{\beta}}\right]-d(Q-M)A\left[\frac{p_3{}^{\beta}}{Ap_2{}^{\beta}+\alpha_1}-\right.$$

$$\left.\left.\frac{p_4{}^{\beta}}{Ap_1{}^{\beta}+\alpha_1}\right]\right]-\frac{1}{4}\Bigg[(p_4-p_3)Q+h\left[\frac{1}{\alpha_1}\left\{\frac{A}{\alpha_1}\left(p_4{}^{\beta}-p_3{}^{\beta}\right)\right\}\right]-h\left[\frac{1}{\alpha_1}\left\{\left(Q-M+\right.\right.\right.$$

$$\left.\left.\frac{Ap_1{}^{\beta}}{\alpha_1}\right)e^{\frac{-\alpha_1(Q-M)}{Ap_1{}^{\beta}+\alpha_1}}-\left(Q-M+\frac{Ap_2{}^{\beta}}{\alpha_1}\right)e^{\frac{-\alpha_1(Q-M)}{Ap_2{}^{\beta}+\alpha_1}}\right\}+\frac{A(Q-M)}{\alpha_1}\left\{\frac{p_1{}^{\beta}}{Ap_4{}^{\beta}+\alpha_1}-\frac{p_2{}^{\beta}}{Ap_3{}^{\beta}+\alpha_1}\right\}\right]+$$

$$\frac{sM^2}{2A}\left[\frac{1}{p_1{}^{\beta}}-\frac{1}{p_2{}^{\beta}}\right]-d(Q-M)A\left[\frac{p_1{}^{\beta}}{Ap_4{}^{\beta}+\alpha_1}-\frac{p_2{}^{\beta}}{Ap_3{}^{\beta}+\alpha_1}\right]\Bigg]=F(M) \tag{15}$$

$$\frac{dF(M)}{dM}=\frac{1}{2}\Bigg[-h\left[\frac{2}{\alpha_1}+\frac{1}{\alpha_1}\left\{\left(Q-M+\frac{Ap_4{}^{\beta}}{\alpha_1}\right)\left(\frac{\alpha_1}{Ap_4{}^{\beta}+\alpha_1}\right)e^{-\frac{-\alpha_1(Q-M)}{Ap_4{}^{\beta}+\alpha_1}}-e^{-\frac{-\alpha_1(Q-M)}{Ap_4{}^{\beta}+\alpha_1}}+\right.\right.$$

$$\left.\left.\left(Q-M+\frac{Ap_1{}^{\beta}}{\alpha_1}\right)\left(\frac{\alpha_1}{Ap_1{}^{\beta}+\alpha_1}\right)e^{\frac{-\alpha_1(Q-M)}{Ap_1{}^{\beta}+\alpha_1}}-e^{\frac{-\alpha_1(Q-M)}{Ap_1{}^{\beta}+\alpha_1}}\right\}-\frac{A}{\alpha_1}\left\{\frac{p_4{}^{\beta}}{Ap_1{}^{\beta}+\alpha_1}+\frac{p_1{}^{\beta}}{Ap_4{}^{\beta}+\alpha_1}\right\}\right]+$$

$$\frac{sM}{A}\left[\frac{1}{p_4{}^{\beta}}+\frac{1}{p_1{}^{\beta}}\right]+dA\left\{\frac{p_4{}^{\beta}}{Ap_1{}^{\beta}+\alpha_1}+\frac{p_1{}^{\beta}}{Ap_4{}^{\beta}+\alpha_1}\right\}-2d\Bigg]+$$

$$\frac{1}{4}\left[-h\left(\frac{1}{\alpha_1}\left\{\left(Q-M+\frac{Ap_3{}^{\beta}}{\alpha_1}\right)\left(\frac{\alpha_1}{Ap_3{}^{\beta}+\alpha_1}\right)e^{\frac{-\alpha_1(Q-M)}{Ap_3{}^{\beta}+\alpha_1}}-e^{\frac{-\alpha_1(Q-M)}{Ap_3{}^{\beta}+\alpha_1}}-\left(Q-M+\frac{Ap_4{}^{\beta}}{\alpha_1}\right)\alpha_1\frac{e^{-\frac{-\alpha_1(Q-M)}{Ap_4{}^{\beta}+\alpha_1}}}{Ap_4{}^{\beta}+\alpha_1}+e^{\frac{-\alpha_1(Q-M)}{Ap_4{}^{\beta}+\alpha_1}}\right\}-\frac{A}{\alpha_1}\left\{\frac{p_3{}^{\beta}}{Ap_2{}^{\beta}+\alpha_1}-\frac{p_4{}^{\beta}}{Ap_1{}^{\beta}+\alpha_1}\right\}\right)+\frac{sM}{A}\left[\frac{1}{p_3{}^{\beta}}-\frac{1}{p_4{}^{\beta}}\right]+dA\left\{\frac{p_3{}^{\beta}}{Ap_2{}^{\beta}+\alpha_1}-\frac{p_4{}^{\beta}}{Ap_1{}^{\beta}+\alpha_1}\right\}\right]-\frac{1}{4}\left[-h\left\{\frac{1}{\alpha_1}\left(\left(Q-M+\frac{Ap_1{}^{\beta}}{\alpha_1}\right)\alpha_1\left(\frac{e^{\frac{-\alpha_1(Q-M)}{Ap_1{}^{\beta}+\alpha_1}}}{Ap_1{}^{\beta}+\alpha_1}\right)-\left(e^{\frac{-\alpha_1(Q-M)}{Ap_1{}^{\beta}+\alpha_1}}\right)-\left(Q-M+\frac{Ap_2{}^{\beta}}{\alpha_1}\right)\alpha_1\left(\frac{e^{\frac{-\alpha_1(Q-M)}{Ap_2{}^{\beta}+\alpha_1}}}{Ap_2{}^{\beta}+\alpha_1}\right)+\left(e^{\frac{-\alpha_1(Q-M)}{Ap_2{}^{\beta}+\alpha_1}}\right)\right)-\frac{A}{\alpha_1}\left\{\frac{p_1{}^{\beta}}{Ap_4{}^{\beta}+\alpha_1}-\frac{p_2{}^{\beta}}{Ap_3{}^{\beta}+\alpha_1}\right\}\right\}+\frac{sM}{A}\left[\frac{1}{p_1{}^{\beta}}-\frac{1}{p_2{}^{\beta}}\right]+dA\left\{\frac{p_1{}^{\beta}}{Ap_4{}^{\beta}+\alpha_1}-\frac{p_2{}^{\beta}}{Ap_3{}^{\beta}+\alpha_1}\right\}\right] \quad (16)$$

$$\widetilde{t_1}=\frac{R}{\widetilde{D}+\alpha_1}=\left(\frac{R}{D_4+\alpha_1},\frac{R}{D_3+\alpha_1},\frac{R}{D_2+\alpha_1},\frac{R}{D_1+\alpha_1}\right)$$

The defuzzified value of

$$t_1=d(\widetilde{t_1},0)=\frac{R}{2}\left[\frac{1}{D_4+\alpha_1}+\frac{1}{D_1+\alpha_1}\right]+\frac{R}{4}\left[\frac{1}{D_3+\alpha_1}-\frac{1}{D_4+\alpha_1}\right]-\frac{R}{4}\left[\frac{1}{D_1+\alpha_1}-\frac{1}{D_2+\alpha_1}\right] \quad (17)$$

$$\widetilde{t_2}=\frac{M}{\widetilde{D}+\alpha_1}=\left(\frac{M}{D_4},\frac{M}{D_3},\frac{M}{D_2},\frac{M}{D_1}\right)$$

The defuzzified value of $t_2=d(\widetilde{t_2},0)$

$$=\frac{M}{2}\left[\frac{1}{D_4}+\frac{1}{D_1}\right]+\frac{M}{4}\left[\frac{1}{D_3}-\frac{1}{D_4}\right]-\frac{M}{4}\left[\frac{1}{D_1}-\frac{1}{D_2}\right] \quad (18)$$

5. Numerical example

Let $\alpha_1 = 0.2, O = Rs1000$ per order,$Q = 200$,$d = Rs.12\ per\ unit, A = 100, \beta = 0.01$,$p = Rs.50\ per\ unit, D = 103.9896$.

Table 5.1: Sensitivity Analysis for Crisp Model with increase of both holding and shortage costs

S.No	**h**	**s**	**M**	**R**	t_1	t_2	**T**	**TC**
1	8	10	77.6509	122.3491	1.1743	0.7467	1.9210	11785.9
2	10	12	79.7945	120.2355	1.1540	0.7670	1.9210	11966.8
3	12	14	81.2361	118.7639	1.1399	0.7812	1.9211	12147.1
4	14	16	82.3193	117.6803	1.1295	0.7916	1.9211	12327.2
5	16	18	83.15	116.85	1.1215	0.7996	1.9211	12507.0

Table 5.2: Sensitivity Analysis for Crisp Model with the increase of holding cost

S.N	**h**	**s**	**M**	**R**	t_1	t_2	**T**	**TC**

o								
1	8	10	77.6509	122.3491	1.1743	0.7467	1.9210	11785.9
2	10	10	89.5856	110.4144	1.0597	0.8615	1.9212	11898.1
3	12	10	99.6114	100.3886	0.9635	0.9579	1.9214	11991.4
4	14	10	108.1063	91.8937	0.8820	1.0396	1.9216	12069.9
5	16	10	115.3671	84.6329	0.8123	0.1094	1.9217	12136.8

Table 5.3: Sensitivity Analysis for Crisp Model with the increase of shortage cost

S.No	**h**	**s**	**M**	**R**	t_1	t_2	**T**	**TC**
1	8	10	77.6509	122.3491	1.1743	0.7467	1.9210	11785.9
2	8	12	68.3757	131.6243	1.2633	0.6575	1.9208	11837.0
3	8	14	61.0121	138.9879	1.3340	0.5867	1.9207	11877.1
4	8	16	55.0404	144.9596	1.3913	0.5293	1.9206	11909.4
5	8	18	50.1096	149.8904	1.4386	0.4819	1.9205	11935.9

Table 5.4: Sensitivity Analysis for Fuzzy Model with increase of both holding and shortage costs

S.No	**h**	**s**	$\tilde{p}$	**M**	**R**	t_1	t_2	**T**	**TC**
1	8	10	(49,50,51,52)	77.6655	122.3345	1.1740	0.7468	1.9208	11884.6
2	10	12	(49,50,51,52)	79.7801	120.2199	1.1537	0.7671	1.9208	12065.5
3	12	14	(49,50,51,52)	81.2521	118.7479	1.1396	0.7813	1.9209	12245.8
4	14	16	(49,50,51,52)	82.3353	117.6647	1.1292	0.7917	1.9209	12425.9
5	16	18	(49,50,51,52)	83.1661	116.8339	1.1213	0.7997	1.921	12605.5

Table 5.5: Sensitivity Analysis for Fuzzy Model with increase of holding cost

S.No	**h**	**s**	$\tilde{p}$	**M**	**R**	t_1	t_2	**T**	**TC**
1	8	10	(49,50,51,52)	77.6655	122.3345	1.1740	0.7468	1.9208	11884.6
2	10	10	(49,50,51,52)	89.6028	110.3972	1.0595	0.8616	1.9211	11996.8
3	12	10	(49,50,51,52)	99.6297	100.3703	0.9632	0.9580	1.9212	12090.2
4	14	10	(49,50,51,52)	108.1257	91.8744	0.8817	1.0397	1.9214	12168.9
5	16	10	(49,50,51,52)	115.3874	84.6126	0.8120	1.1095	1.9215	12235.8

Table 5.6: Sensitivity Analysis for Fuzzy Model with increase of shortage cost

S.No	**h**	**s**	$\tilde{p}$	**M**	**R**	t_1	t_2	**T**	**TC**
1	8	10	(49,50,51,52)	77.6655	122.3345	1.1740	0.7468	1.9208	11884.6
2	8	12	(49,50,51,52)	68.3891	131.6109	1.2631	0.6576	1.9207	11935.5
3	8	14	(49,50,51,52)	61.0243	138.9757	1.3337	0.5868	1.9205	11975.6
4	8	16	(49,50,51,52)	55.0514	144.9486	1.3911	0.5293	1.9204	12007.9
5	8	18	(49,50,51,52)	50.1201	149.8799	1.4384	0.4819	1.9203	12034.3

6. Conclusion

A fuzzy inventory model for deteriorating items with price dependent demand is developed. A proper EOQ deterioration inventory model is built to prove that there exists the unique optimal solution to minimize total cost and the analytic solution of the optimal order cycle is derived. Instead of having on hand inventory, allowing shortages is the best method to minimize the total cost. The price dependent demand is suitable to many real life situations. For fuzzy model, the unit price and hence the demand rate are represented by trapezoidal fuzzy numbers. Signed distance method is used for defuzzification. This model can be developed with many limitations, such as their inventory level, warehouse space and budget limitations, etc.

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