**g Closed Sets in Topological Spaces

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Abstract. The aim of this paper is to introduce a new class of sets called **g closed sets and investigate some of the basic properties of this class of sets.

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1. Introduction

Levine [5] introduced generalized closed sets and semi open sets. Abd Monsef, Deeb and Mahmoud introduced β sets and Njastad introduced α sets and Mashour, Abd El-Monsef and Deeb introduced pre-open sets. Andregvic called β sets as semi pre-open sets. Veerakumar [12] introduced g* closed sets. The aim of this paper is to introduce a new type of closed sets namely **g closed sets and investigate some of the basic properties of this class of sets.

2. Preliminaries

Definition 2.1. A subset A of topological space (X, τ ) is called

(1) a generalized closed [5] (briefly g-closed) set, if cl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ); the complement of a g-closed set is called a g-open set.

(2) a generalized semi-closed (briefly gs-closed) set, if scl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ); the complement of a gs-closed set is called a gs-open set.

(3) a semi-generalized closed (briefly sg-closed) set, if scl(A) ⊆ U and U is semi-open in (X, τ); the complement of a sg-closed set is called a sg-open set.

(4) a Ψ-closed set [9], if scl(A) ⊆ U whenever A ⊆ U and U is sg-open in (X, τ); the complement of a Ψ-closed set is called a Ψ-open set.

(5) a α-generalized closed [5] (briefly α g-closed) set, if α cl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ); the complement of an α g-closed set is called an α g-open set.
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(6) a **generalized α - closed** [4] (briefly g α - closed) set, if α cl(A) ⊆ U whenever A ⊆ U and U is α - open in (X, τ); the complement of a g α -closed set is called a g α -open set.

(7) a **generalized pre-closed** (briefly gp-closed) set, if pcl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ); the complement of a gp-closed set is called a gp-open set.

(8) a **generalized semi-pre closed** [2] (briefly gsp- closed) set, if Spcl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ); the complement of a gsp-closed set is called a gsp-open set.

(9) a **generalized*-closed** (briefly g*-closed) set, if cl(A) ⊆ U whenever A ⊆ U and U is g open in (X, τ); the complement of a g*-closed set is called a g*-open set.

(10) a **generalized pre*-closed** (briefly gp*-closed) set, if cl (A) ⊆ U whenever A ⊆ U and U is gp-open in (X, τ); the complement of a gp*-closed set is called a gp*-open set.

(11) a **generalized * closed** [7] (briefly g# closed) set, if cl(A) ⊆ U whenever A ⊆ U and U is α g open in (X, τ); the complement of a g# closed set is called a g# open set.

(12) a **regular generalized –star closed** (briefly rg* closed) set, if rcl(A) ⊆ U whenever A ⊆ U and U is regular open in (X, τ); the complement of a rg* closed set is called a rg* open set.

(13) a **generalized closed** (briefly *g closed) set, if cl(int(A)) ⊆ U whenever A ⊆ U and U is g – open in (X, τ); the complement of a *g-closed set is called a *g-open set.

(14) a **strongly g* closed** (briefly sg* closed) set, if cl(int(A)) ⊆ U whenever A ⊆ U and U is g open in (X, τ); the complement of a.

(15) a **regular weakly generalized closed** (briefly rwg closed) set, if cl(int(A)) ⊆ U whenever A ⊆ U and U is regular open in (X, τ); the complement of a rwg closed set is called rwg open set.

3. **g closed sets

**Definition 3.1.** If a subset A of a topological space (X, τ) is called **g closed set, if cl(A) ⊆ U whenever A ⊆ U and U is g* open in (X, τ); the complement of a **g closed set is called **g open set.

**Example 3.1.** X={a,b,c}

τ = {X, φ, {a}, {b}, {a,b}}

(Closed sets)={X, φ, {c}, {b,c}, {a,c}}

g closed sets={X, φ, {c}, {b,c}, {a,c}}

g* closed sets={X, φ, {c}, {b,c}, {a,c}}

g* open sets={X, φ, {a}, {b}, {a,b}}

**g closed sets={X, φ, {c}, {b,c}, {a,c}}

**g open sets={X, φ, {a,b}, {a}, {b}}.

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Theorem 3.1. Every closed set is \(\ast\ast\)g closed set.

Proof: Let \(A\) be a closed set.

TPT: \(A\) is \(\ast\ast\)g closed set, \(\text{i.e.}\) TPT: if \(\text{cl}(A) \subseteq U\), whenever \(A \subseteq U\) and \(U\) is \(g^*\) open.

Since \(A\) is closed, \(A = \text{cl}(A)\).

Let \(A \subseteq U\), \(U\) is \(g^*\) open.

Now \(\text{cl}(A) = A \subseteq U\)
\(\Rightarrow\) \(A\) is \(\ast\ast\)g closed.

Example 3.2. The Converse of the above theorem is need not be true as seen from the following example \(X = \{a, b, c\}\).

\[\tau = \{X, \emptyset, \{a\}, \{a, c\}\}\]
\([\text{Closed sets}] = \{X, \emptyset, \{b\}, \{b, c\}\}\]
\(\ast\ast\)g closed sets = \(\{X, \emptyset, \{b\}, \{a, b\}\{b, c\}\}\)

Here \(A = \{a, b\}\) be a \(\ast\ast\)g closed set but not a closed set.

Theorem 3.2. Every \(g^*\) closed set is \(\ast\ast\)g closed set.

Proof: Let \(A\) be \(g^*\) closed set

TPT: \(A\) is \(\ast\ast\)g closed, \(\text{i.e.}\) TPT if \(\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(g^*\) open.

By the definition of \(g^*\) closed set, if \(\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(g\) open.

Let \(A \subseteq U\) where \(U\) is \(g^*\) open.

Since \(U\) is \(g^*\) open \(\Rightarrow\) \(U\) is \(g\) open.

Thus \(\text{cl}(A) \subseteq U\), \(U\) is \(g^*\) open.

\(\Rightarrow\) \(A\) is \(\ast\ast\)g closed.

Example 3.3. The converse of the above theorem is need not be true as seen from the following example \(X = \{a, b, c\}\).

\[\tau = \{X, \emptyset, \{a\}, \{a, c\}\}\]
\([\text{Closed sets}] = \{X, \emptyset, \{b\}, \{b, c\}\}\]
\(g^*\) closed sets = \(\{X, \emptyset, \{c\}, \{a, c\}\{b, c\}\}\)
\(\ast\ast\)g closed sets = \(\{X, \emptyset, \{b\}, \{a, b\}\{b, c\}\}\)

Here \(A = \{a, b\}\) is a \(\ast\ast\)g closed set, but not a \(g^*\) closed set.

Remark 3.1. \(\ast\ast\)g closedness is independent of \(g^*\)\(\psi\) closedness. It can be seen from the following examples.

Example 3.4. \(X = \{a, b, c, d\}\).

\[\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}\]
\([\text{Closed sets}] = \{X, \emptyset, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}\]
\(\ast\ast\)g closed sets = \(\{X, \emptyset, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, \{a, d\}, \{b, d\}, \{a, b, d\}\}\)

Here \(A = \{b\}\) be a \(g^*\)\(\psi\) closed set, but not \(\ast\ast\)g closed set.

Example 3.5. \(X = \{a, b, c\}\).

\[\tau = \{X, \emptyset, \{a\}, \{b\}\}\]
\([\text{Closed sets}] = \{X, \emptyset, \{a\}, \{b, c\}\}\]
\(\ast\ast\)g closed sets = \(\{X, \emptyset, \{a\}, \{b\}\,\{c\}, \{a, b\}, \{b, c\}\}\)

Here \(A = \{a, b\}\) is a \(\ast\ast\)g closed set, but not \(g^*\)\(\psi\) set.

Remark 3.2. Similarly we can prove \(\ast\ast\)g closedness is independent of \(g^#\) closedness.
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**Theorem 3.3.** If A is strongly g* closed and A is open then A is **g closed set.

**Proof:** Let A ⊆ U and U is g* open.
Since A is strongly g* closed set.
cl(int(A)) ⊆ U whenever A ⊆ U and U is g open in X.
Since A is open, int(A) = A, also g* open => g open.
Thus cl(A) ⊆ U, U is g* open.
⇒ A is **g closed.

3.1. Properties of **g closed sets

**Theorem 3.1.1.** If A and B are **g closed sets then A ∪ B is **g closed set.

**Proof:** Let U be a g* open in X such that A ⊆ U.
Since A and B are **g closed sets.
Then, cl(A) ⊆ U, A ⊆ U and U is g*open.
Similarly cl(B) ⊆ U, B ⊆ U and U is g*open.
⇒ cl(A ∪ B) = cl(A) ∪ cl(B) ⊆ U
⇒ A ∪ B is **g closed.

**Example 3.1.1.** X = {a, b, c}
τ = {X, ϕ, {a}, {b}, {a, b}}
**g closed sets = {X, ϕ, {c}, {b, c}, {a, c}}
Let A = {c} and B = {b, c}.
Then A ∪ B = {b, c} which is **g closed.

**Remark 3.1.1.** The intersection of two **g closed sets need not be **g closed set.

**Theorem 3.1.2.** If a subset A of X is **g closed set if cl(A) – A does not contain any non-empty g*closed set.

**Proof:** Let A be a **g closed set.
Suppose cl(A) – A contains a non-empty g*closed set namely F, (ie) F ⊂ cl(A) – A
F ⊂ cl(A) & F ⊂ A ⊆ cl(A)
⇒ cl(A) ⊆ F
F ⊂ cl(A) ∩ cl(A)
⇒ F = ϕ
This is a contradiction to our assumption.
Hence the theorem.
The converse is not true.

**Theorem 3.1.3.** If A is a **g closed set of X such that A ⊆ B ⊆ cl(A) then B is also a **g closed set.

**Proof:** Let U be a g*open set of X such that B ⊆ U.
Now B ⊆ cl(A)
Theorem 3.1.4. If A is g* open and **g closed set then A is closed set.

Proof: Let A be both g*open and **g closed.

TPT: A is closed

Since A is **g closed, cl(A) \( \subseteq U \), U is g*open
Since A is g*open , we can take A=U

\[ \Rightarrow \text{Cl}(A) \subseteq A \text{ but } A \subseteq \text{cl}(A) \]
\[ \Rightarrow \text{A}=\text{cl}(A) \]
\[ \Rightarrow A \text{ is closed.} \]

Theorem 3.1.5. If a subset A of a topological space \((X, \tau)\) is both open and **g closed then A is closed set.

Proof: Let A be both open and **g closed

TPT: A is closed

cl(A) \( \subseteq U \) where U is g*open
We know that, open \( \Rightarrow \text{g*open} \)

\[ \Rightarrow \text{A is g*open} \]
Now take U=A
cl(A) \( \subseteq A \)
Also A \( \subseteq \text{cl}(A) \)

\[ \Rightarrow \text{cl}(A)=A \]
\[ \Rightarrow A \text{ is closed.} \]

Theorem 3.1.6. If a subset A of a topological space \((X, \tau)\) is both open and regular closed then A is **g closed set.

Proof: Let A be both open and regular closed.

TPT: A be a **g closed

A=cl(int(A))
Let A \( \subseteq U \) whenever A \( \subseteq U \) and U is g*open.
Since A is open, int(A)=A.
Since A is regular closed, cl(int(A))=A.

\[ \Rightarrow \text{cl(int(A))}=A \]
\[ \Rightarrow \text{cl(A)} \subseteq U \text{ [ since int(A)=A]} \]
\[ \Rightarrow A \text{ is **g closed.} \]

Theorem 3.1.7. If a subset A of a topological space \((X, \tau)\) is both open and **g closed then A is both regular open and regular closed set.

Proof: Let A be both open and **g closed.

Then by the above theorem (4.7)
A is closed, (ie) cl(A)=A
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Also as A is open \(\text{int}(A) = A\) \(\rightarrow\) (*).

Sub \(A = \text{cl}(A)\) in eq (*).
\[\Rightarrow \text{int}(\text{cl}(A)) = A\]
\[\Rightarrow A \text{ is regular open}\]

Since A is closed, \(\text{cl}(A) = A\)
\[\Rightarrow \text{cl}(\text{int}(A)) = A\] [since \(\text{int}(A) = A\)]
\[\Rightarrow A \text{ is regular closed.}\]

**Theorem 3.1.8.** If a subset A of a topological space \((X, \tau)\) is both open and **g closed then A is rg closed set.

**Proof:** If A is both open and **g closed then by the above theorem (3.1.5).
A is closed, (ie) \(\text{cl}(A) = A\)
Also by the above theorem (3.1.7)
A is both regular open and regular closed
Let \(A \subseteq U\) where \(U\) is regular open
\[\text{cl}(A) \subseteq U = A\]
\[\Rightarrow A \text{ is rg closed.}\]

**Theorem 3.1.9.** Let A be a **g closed and suppose that F is closed then \(A \cap F\) is **g closed.

**Proof:** Let A be a **g closed.

TPT : \(A \cap F\) is **g closed.
Let F be such that \(A \cap F \subseteq U\) where \(U\) is g*open.
Since F is closed \(A \cap F\) is closed in A.
\[\Rightarrow \text{cl}(A \cap F) = A \cap F \subseteq U\]
\[\Rightarrow \text{cl}(A \cap F) \subseteq U\]
\[\Rightarrow A \cap F\] is **g closed.

**REFERENCES**