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An Inventory Model with Three Rates of Production and Time Dependent Deterioration Rate for Quadratic Demand Rate

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Abstract. An inventory model with three different rates of production and quadratic demand rate is considered. The shortages are allowed and deterioration rate is time dependent. The objective is to determine the optimal total cost and the optimal time schedule of the plan for the proposed model. To illustrate the results of this model, numerical example is presented.

Keywords: Inventory, quadratic demand, time dependent deterioration

AMS Mathematics Subject Classification (2010): 90B05

1. Introduction

The inventory system is taking an important part of cost controlling in business. For the last fifteen years, researchers in this area have extended investigation into various models with considerations of item shortage, item deterioration, demand patterns, item order cycles and their combinations. The controlling and regulating of deteriorating items is a measure problem in any inventory system. Certain products like vegetables, fruits, electronic components, chemicals deteriorate during their normal storage period. Hence when developing an optimal inventory policy for such products, the loss of inventory due to deterioration cannot be ignored. The researchers have continuously modified the deteriorating inventory models so as to more practicable and realistic. The analysis of deteriorating inventory model is initiated by Ghare and Scheader [2] with a constant rate of decay. Several researchers have extended their idea to different situations in deterioration on inventory model. In all those models, the demand rate and the deterioration rate were constants, the replenishment rate was finite and no shortage in inventory was allowed. Researchers started to develop inventory systems allowing time variability in one or more than one parameter. Liquan Ji [5] developed EOQ inventory model with shortages in starting and without shortages in ending. This was followed by another model by Misra [7] with Weibull deteriorating items, permissible delay in payments under inflation. Maragatham and Lakshmidevi [6] developed an inventory model for non-instantaneous deteriorating items under conditions of permissible delay in payments for n-cycles. Yu [8] discussed a ordering policy for two – phase deteriorating items with changing deterioration rate. A production inventory model with two rates of production, backorders and variable production cycle is analyzed by Bhowmick and Samauta. The optimal values of inventory levels are derived using Hessian matrix. In this paper, a continuous production control inventory model with three rates of production and time dependent deterioration rate for quadratic demand rate is developed. The production is started at one rate and after some time it may be switched over to another rate is possible to real life situations. Such a situation is desirable in the sense that starting at low rate of production, a large quantum stock of manufactured items at the initial stage is avoided, leading to reduction in the holding cost, the new production rate is used. For demand, the quadratic function in time and time dependent deterioration rate is considered.

2. Notations and assumption

Notations

I_1 - The inventory level at time t_1
I_2 - The inventory level at time t_2
I_3 - The inventory level at time t_4
$[t_{3,}t_4]$ – The shortage period
T – The time length of the plan
$I(t)$ – The inventory level at time t
p_1 - The production rate in $[0,t_1]$
p_2 - The production rate in $[t_1,t_2]$
p_3 - The production rate in $[t_4,T]$
h – The holding cost per unit per unit time
d – The deteriorating cost per unit per unit time
s – The shortage cost per unit per unit time
p – The production cost per unit per unit time
TC- The total cost per unit time

Assumptions

1. The demand rate $f(t) = a + bt + ct^2, c > 0, b > a > 0$
2. The deterioration rate $\theta(t) = \alpha t, \alpha > 0$
3. The production rates $p_3 > p_2 > p_1$
4. The shortages are allowed.
5. There is no replacement or repair of deteriorated items during the cycle under consideration.
6. The deterioration is instantaneous.
7. The time for allowing shortages is same as back order time

3. Model formulation

The production of the item is started initially at $t = 0$ ata ratep_1.When $t = t_1$, the rate of production is switches over to $p_2(> p_1)$ and the production is stopped at time t_2 and the inventory depleted at a rate $f(t)$. The inventory level reaches zero at t_3. It is decided to backlog demand upto $t = t_4$ which occur during stock – out line. The production is started at a faster rate $p_3(> p_2 > p_1)$ so as to clear the backlog and when the inventory level reaches zero (ie), the backlog cleared, the next production cycle starts.At the time duration$[0, t_1]$, the production is at the rate p_1 and the consumption by demand. At the time duration $[t_1, t_2]$, the production is at the rate p_2 and the consumption by demand. At the time duration $[t_2, t_3]$, there is no production, but only consumption by demand. At the time duration $[t_3, t_4]$, the shortages is allowed. $[t_4 T]$isthe duration of time to backlog at production rate p_3. The cycle then repeats itself after time T. The deterioration is instantaneous with the rate αt in the duration $[0, t_3]$. The model is represented by figure 1.

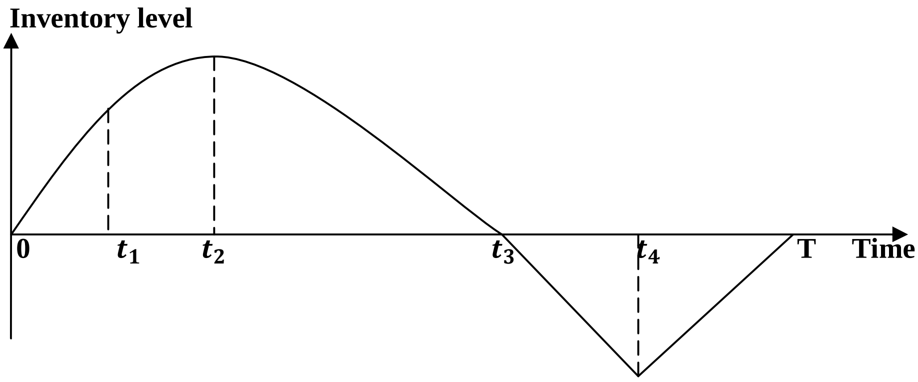

Figure 1: A production inventory model

The change of inventory level can be described as follows

$$\frac{dI(t)}{dt} + \theta(t)I(t) = p_1 - f(t), \qquad 0 < t < t_1 \quad (1)$$

$$\frac{dI(t)}{dt} + \theta(t)I(t) = p_2 - f(t), \qquad t_1 < t < t_2 \quad (2)$$

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -f(t), \qquad t_2 < t < t_3 \quad (3)$$

$$\frac{dI(t)}{dt} = -f(t), \qquad t_3 < t < t_4 \quad (4)$$

$$\frac{dI(t)}{dt} = p_3 - f(t), \qquad t_4 < t < T \quad (5)$$

With boundary conditions$I(0) = 0, I(t_1) = I_1, I(t_2) = I_2, I(t_3) = 0, I(t_4) = I_3, I(T) = 0$ (6)

The solution of the above equations are given by

$$I(t) = \begin{cases} e^{-\frac{\alpha t^2}{2}}\left[p_1 t - at - \frac{bt^2}{2} - \frac{ct^3}{3} + \frac{p_1 \alpha t^3}{6} - \frac{a\alpha t^3}{6} - \frac{b\alpha t^4}{8} - \frac{c\alpha t^5}{10}\right], 0 < t < t_1 \\ e^{-\frac{\alpha t^2}{2}}\left[\begin{array}{c} p_2(t - t_1) - a(t - t_1) - \frac{b}{2}(t^2 - t_1^2) - \frac{c}{3}(t^3 - t_1^3) + p_2\alpha(t^3 - t_1^3) - \frac{a\alpha}{6}(t^3 - t_1^3) \\ - \frac{b\alpha}{8}(t^4 - t_1^4) - \frac{c\alpha}{10}(t^5 - t_1^5) + I_1 e^{\frac{\alpha t_1^2}{2}} \end{array}\right], t_1 < t < t_2 \\ e^{-\frac{\alpha t^2}{2}}\left[-a(t - t_2) - \frac{b}{2}(t^2 - t_2^2) - \frac{c}{3}(t^3 - t_2^3) - \frac{a\alpha}{6}(t^3 - t_2^3) - \frac{b\alpha}{8}(t^4 - t_2^4) - \frac{c\alpha}{10}(t^5 - t_2^5) + I_2 e^{\frac{\alpha t_2^2}{2}}\right], t_2 < t < t_3 \\ -a(t - t_3) - \frac{b}{2}(t^2 - t_3^2) - \frac{c}{3}(t^3 - t_3^3), t_3 < t < t_4 \\ p_3(t - t_4) - a(t - t_4) - \frac{b}{2}(t^2 - t_4^2) - \frac{c}{3}(t^3 - t_4^3) + I_3, t_4 < t < T \end{cases} \quad (7)$$

By considering the continuity at t_4,$I_3 = -a(t_4 - t_3) - \frac{b}{2}(t_4^2 - t_3^2) - \frac{c}{3}(t_4^3 - t_3^3)$ (8)

By considering the continuity at t_2,

$$I_2 = e^{-\frac{\alpha t_2^2}{2}}\left[a(t_3 - t_2) + \frac{b}{2}(t_3^2 - t_2^2) + \frac{c}{3}(t_3^3 - t_2^3) + \frac{a\alpha}{6}(t_3^3 - t_2^3) + \frac{b\alpha}{8}(t_3^4 - t_2^4) + \frac{c\alpha}{10}(t_3^5 - t_2^5)\right]$$
(9)

By considering the continuity at t_1,

$$\mathrm{I}_1 = \mathrm{e}^{-\frac{\alpha t_1^2}{2}}\left[\mathrm{p}_1\mathrm{t}_1 - \mathrm{at}_1 - \frac{\mathrm{bt}_1^2}{2} - \frac{\mathrm{ct}_1^3}{3} + \frac{\mathrm{p}_1\alpha\mathrm{t}_1^3}{6} - \frac{\mathrm{a}\alpha\mathrm{t}_1^3}{6} - \frac{\mathrm{b}\alpha\mathrm{t}_1^4}{8} - \frac{\mathrm{c}\alpha\mathrm{t}_1^5}{10}\right] \quad (10)$$

Therefore, after considering the continuity points, the solution becomes

$$I(t) = \begin{cases} e^{-\frac{\alpha t^2}{2}}\left[p_1 t - at - \frac{bt^2}{2} - \frac{ct^3}{3} + \frac{p_1\alpha t^3}{6} - \frac{a\alpha t^3}{6} - \frac{b\alpha t^4}{8} - \frac{c\alpha t^5}{10}\right], 0 < t < t_1 \\ e^{-\frac{\alpha t^2}{2}}\left[\begin{array}{c} p_1 t_1 + p_2(t - t_1) - at - \frac{bt^2}{2} - \frac{ct^3}{3} + \frac{p_1\alpha t^3}{6} + p_2\alpha(t^3 - t_1^3) - \frac{a\alpha t^3}{6} \\ - \frac{b\alpha t^4}{8} - \frac{c\alpha t^5}{10} \end{array}\right], t_1 < t < t_2 \\ e^{-\frac{\alpha t^2}{2}}\left[-a(t - t_3) - \frac{b}{2}(t^2 - t_3^2) - \frac{c}{3}(t^3 - t_3^3) - \frac{a\alpha}{6}(t^3 - t_3^3) - \frac{b\alpha}{8}(t^4 - t_3^4) - \frac{c\alpha}{10}(t^5 - t_3^5)\right], t_2 < t < t_3 \\ -a(t - t_3) - \frac{b}{2}(t^2 - t_3^2) - \frac{c}{3}(t^3 - t_3^3), t_3 < t < t_4 \\ p_3(t - t_4) - a(t - t_3) - \frac{b}{2}(t^2 - t_3^2) - \frac{c}{3}(t^3 - t_3^3) + I_3, t_4 < t < T \end{cases} \quad (11)$$

Total inventory carried over the period $[0, T] = \int_0^{t_1} I(t)dt + \int_{t_1}^{t_2} I(t)dt + \int_{t_2}^{t_3} I(t)dt$ (12)

Total number of deteriorating items in $[0, t_3] = p_1 t_1 + p_2(t_2 - t_1) - \int_0^{t_3} f(t)dt$

$= p_1 t_1 + p_2(t_2 - t_1) - at_3 - \frac{1}{2}bt_3^2 - \frac{1}{3}ct_3^3$ (13)

Total shortage occurred in $[t_3, T] = -\int_{t_3}^{t_4} I(t)dt - \int_{t_4}^{T} I(t)dt$ (14)

Total number of units produced in $[0, T] = p_1 t_1 + p_2(t_2 - t_1) + p_3(T - t_4)$ (15)

Total cost $TC = HC + DC - SC + PC$

$$= h\left[\int_0^{t_1} I(t)dt + \int_{t_1}^{t_2} I(t)dt + \int_{t_2}^{t_3} I(t)dt\right] + d\left[p_1 t_1 + p_2(t_2 - t_1) - \int_0^{t_3} f(t)dt\right]$$

$$-s\left[-\int_{t_3}^{t_4} I(t)dt - \int_{t_4}^{T} I(t)dt\right] + p[p_1 t_1 + p_2(t_2 - t_1) + p_3(T - t_4)] \quad (16)$$

The time for allowing shortages is same as back order time (ie), $t_4 - t_3 = T - t_4$

Therefore $t_3 = 2 * t_4 - T$ (17)

The necessary conditions for $TC(t_1, t_2, t_4, T)$ to be minimum are

$$\frac{\partial TC}{\partial t_1} = 0, \frac{\partial TC}{\partial t_2} = 0, \frac{\partial TC}{\partial t_4} = 0, \frac{\partial TC}{\partial T} = 0 \quad (18)$$

Solving these equations , we get the optimal values t_1^*, t_2^*, t_4^*, T^* which minimize total cost provided they satisfy the following sufficient condition

$H \quad = TheHessianMatrixofTC$

$$= \begin{pmatrix} \frac{\partial^2 TC}{\partial {t_1}^2} & \frac{\partial^2 TC}{\partial t_1 \partial t_2} & \frac{\partial^2 TC}{\partial t_1 \partial t_4} & \frac{\partial^2 TC}{\partial t_1 \partial T} \\ \frac{\partial^2 TC}{\partial t_2 \partial t_1} & \frac{\partial^2 TC}{\partial {t_2}^2} & \frac{\partial^2 TC}{\partial t_2 \partial t_4} & \frac{\partial^2 TC}{\partial t_2 \partial T} \\ \frac{\partial^2 TC}{\partial t_4 \partial t_1} & \frac{\partial^2 TC}{\partial t_4 \partial t_2} & \frac{\partial^2 TC}{\partial {t_4}^2} & \frac{\partial^2 TC}{\partial t_4 \partial T} \\ \frac{\partial^2 TC}{\partial T \partial t_1} & \frac{\partial^2 TC}{\partial T \partial t_2} & \frac{\partial^2 TC}{\partial T \partial t_4} & \frac{\partial^2 TC}{\partial T^2} \end{pmatrix} \text{is positive definite.} \qquad (19)$$

If the solutions obtained from equations (18) do not satisfy the sufficient condition (19), then no feasible solution will be optimal for the set of parameter values taken to solve equations (18) is considered. Such a situation will imply that the parameter values are inconsistent and there is some error in their estimation.

3.1. Numercial example

Let $a = 0.1, b = 0.2, c = 0.003, \alpha = 0.02, p_1 = 2, p_2 = 4, p_3 = 6, hc = Rs.2,$ $dc = Rs3, sc = Rs6, pc = Rs10$ Using MATLAB program the following are calculated, Then optimal values of $t_1 = 1.3496, t_2 = 3.0748, t_3 = 3.1419, t_4 = 3.5117, T = 3.8815$.

The total cost $TC = Rs167.7252$.

The production quantity = 11.8188.

4. Conclusion

A continuous production inventory model for time dependent deteriorating items with shortages in which three different rates of production and quadratic demand rate is considered. The case of change of production is very useful in practical situations. By starting at a low rate of production, a large quantum stock of manufactured item, at the initial stage is avoided, leading to reduction in the holding cost. The variation in production rate provides a way resulting consumer satisfaction and earning potential profit.The total cost of the system and the optimal values for t_1, t_2, t_4, T is derived for quadratic demand rate and time dependent deterioration rate.

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