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A Multi-Objective Production Planning Problem Based on NeutrosophicLinear Programming Approach

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Abstract. Neutrosophic set is a powerful general formal framework that has been proposed in 1995 by Smarandache. The paper aims to give a computational algorithm to solve a multi-objective linear programming problem (MOLPP) using Neutrosophic optimization method. The developed algorithm has been illustrated by a production planning problem. We made a comparative study of optimal solution between intuitionistic fuzzy optimization and Neutrosophic optimization technique.

Keywords: Neutrosophic set, single valued neutrosophic set, neutrosophic optimization method.

***AMS Mathematics Subject Classification (2010)*:** 90C05, 90C29

1. Introduction

The concept of fuzzy sets was introduced by Zadeh in 1965 [1]. The fuzzy sets and fuzzy logic have been applied in many real applications to handle uncertainty. The traditional fuzzy sets uses one real value $\mu_A(x) \in [0,1]$ to represents the truth membership function of fuzzy set A defined on universe X. Sometimes $\mu_A(x)$ it self is uncertain and hard to be defined by a crisp value. So the concept of interval valued fuzzy sets was proposed [2] to capture the uncertainty of truth membership. In some applications we should consider not only the truth membership supported by the evident but also the falsity membership against by the evident. That is beyond the scope of fuzzy sets and interval valued fuzzy sets. In 1986, Atanassov [3,5] devolved the idea of intuitionistic fuzzy set A characterized by the membership degree $\mu_A(x) \in [0,1]$ as well as non-membership degree $\nu_A(x) \in [0,1]$ with some restriction $0 \leq \mu_A(x)+\nu_A(x) \leq 1$. Therefore certain amount of indeterminacy $1 -(\mu_A(x)+\nu_A(x))$ remains by default. However one may also consider the possibility

$\mu_A(x)$ + $\nu_A(x)$> 1, so that inconsistent beliefs are also allowed.In neutrosophic sets indeterminacy is quantified explicitly and truth membership, indeterminacy membership and falsity membership are independent. Neutrosophic set (NS) was introduced by Smarandache in 1995 [4] which is actually generalization of different types of FSs and IFSs. In 1978 a paper Fuzzy linear programming with several objective functions has been published by Zimmermann **[11]**. In 2007, Jana and Roy **[9]** have studied multi-objective intuitionistic fuzzy linear programming problem and its application in Transportation model. The motivation of the present study is to give computational algorithm for solving multi-objective linear programming problem by single valued neutrosophic optimization approach. We also aim to study the impact of truth membership, indeterminacy membership and falsity membership functions in such optimization process and thus have made comparative study in intuitionistic fuzzy and neutrosophic optimization technique.

2. Some preliminaries

Definition 1. (Fuzzy set) [1]

Let X be a fixed set. A fuzzy set A of X is an object having the form $\tilde{A}$ = {(x,μ_A (x)), x∈ X} where the function $\mu_A(x)$: X → [0, 1] define the truth membership of the element x ∈ X to the set A.

Definition 2. (Intuitionistic fuzzy set) [3]

Let a set X be fixed. An intuitionistic fuzzy set or IFS $\tilde{A}^i$ in X is an object of the form $\tilde{A}^i$ = $\{< X, \mu_A(x), \nu_A(x) > / x \in X\}$ where $\mu_A(x)$: X→ [0, 1] and $\nu_A(x)$: X→ [0, 1] define the truth-membership and falsity-membership respectively, for every element of x ∈ X , $0 \leq \mu_A(x) + \nu_A(x) \leq 1$

Definition 3. (Neutrosophic set) [4]

Let X be a space of points (objects) and $x \in X$. A neutrosophic set $\tilde{A}^n$ in X is defined by a Truth-membership function$\mu_A(x)$, an indeterminacy-membership function $\sigma_A(x)$ and a falsity-membership function $\nu_A(x)$ and having the form $\tilde{A}^n$ ={$< X, \mu_A(x), \sigma_A(x), \nu_A(x) > / x \in X$}. $\mu_A(x), \sigma_A(x)\ and\ \nu_A(x)$ are real standard or non-standard subsets of

$]\ 0^-, 1^+[$. that is
$\mu_A(x)$: X→ $]\ 0^-, 1^+[$
$\sigma_A(x)$: X→ $]\ 0^-, 1^+[$
$\nu_A(x)$: X→ $]\ 0^-, 1^+[$

There is no restriction on the sum of$\mu_A(x), \sigma_A(x)\ and\ \nu_A(x)$, so

$$0^- \leq \sup \mu_A(x) + \sup \sigma_A(x) + \sup \nu_A(x) \leq 3^+$$

Definition 4. (Single valued Neutrosophic sets) [6]

Let X be a universe of discourse. A single valued neutrosophic set $\tilde{A}^n$ over X is an object having the form $\tilde{A}^n = \{< X, \mu_A(x), \sigma_A(x), \nu_A(x) > / x \in X\}$ where $\mu_A(x) : X \rightarrow [0, 1]$, $\sigma_A(x) : X \rightarrow [0, 1]$ and $\nu_A(x) : X \rightarrow [0, 1]$ with $0 \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 3$ for all $x \in X$.

Example 1. Assume that $X = [x_1, x_2, x_3]$. X_1 is capability, x_2 is trustworthiness and x_3 is price. The values of x_1, $x_2$and x_3 are in [0, 1]. They are obtained from the questionnaire of some domain experts, their option could be a degree of "good service", a degree of indeterminacy and a degree of "poor service". $\tilde{A}^n$is a single valued neutrosophic set of X defined by

$\tilde{A}^n = \langle 0.3,0.4,0.5\rangle/x_1 + \langle 0.5,0.2,0.3\rangle/x_2 + \langle 0.7,0.2,0.2\rangle/x_3$.$\tilde{B}^n$ is a single valued neutrosophic set of X defined by $\tilde{B}^n = \langle 0.6,0.1,0.2\rangle/x_1 + \langle 0.3,0.2,0.6\rangle/x_2 + \langle 0.4,0.1,0.5\rangle/x_3$

Definition 5. (Complement) [6] The complement of a single valued neutrosophic set $\tilde{A}^n$is denoted by $c(\tilde{A}^n)$ and is defined by
$\mu_{c(\tilde{A}^n)}(x) = \nu_{\tilde{A}^n}(x)$
$\sigma_{c(\tilde{A}^n)}(x) = 1 - \sigma_{\tilde{A}^n}(x)$
$\nu_{c(\tilde{A}^n)}(x) = \mu_{\tilde{A}^n}(x)$ for all x in X.

Example 2. let $\tilde{A}^n$ be a single valued neutrosophic set defined in Example 1. Then, $c(\tilde{A}^n) = \langle 0.5,0.6,0.3\rangle/x_1 + \langle 0.3,0.8,0.5\rangle/x_2 + \langle 0.2,0.8,0.7\rangle/x_3$.

Definition 6. (Union) [6] The union of two single valued neutrosophic sets $\tilde{A}^n$ and $\tilde{B}^n$is a single valued neutrosophic set $\tilde{C}^n$, written as $\tilde{C}^n = \tilde{A}^n \cup \tilde{B}^n$, whose truth-membership, indeterminacy-membership and falsity-membership functions are given by
$\mu_{c(A)}(x) = \max(\mu_A(x), \mu_B(x))$,
$\sigma_{c(A)}(x) = \max(\sigma_A(x), \sigma_B(x))$,
$\nu_{c(A)}(x) = \min(\nu_A(x), \nu_B(x))$ for all x in X

Example 3. Let A and B be two single valued neutrosophic sets defined in Example 1. Then, $A \cup B = \langle 0.6,0.4,0.2\rangle/x_1 + \langle 0.5,0.2,0.3\rangle/x_2 + \langle 0.7,0.2,0.2\rangle/x_3$.

Definition 7 (Intersection) [6] The Intersection of two single valued neutrosophic sets A and B is a single valued neutrosophic set C, written as $C = A \cap B$, whose truth-membership, indeterminacy-membership and falsity-membership functions are are given by

$\mu_{c(A)}(x) = \min(\mu_A(x), \mu_B(x))$
$\sigma_{c(A)}(x) = \min(\sigma_A(x), \sigma_B(x))\nu_{c(A)}(x) = \max(\nu_A(x), \nu_B(x))$ for all x in X

Example 4. Let A and B be two single valued neutrosophic sets defined in Example 1. Then, $A \cap B = \langle 0.3,0.1,0.5\rangle/x_1 + \langle 0.3,0.2,0.6\rangle/x_2 + \langle 0.4,0.1,0.5\rangle/x_3$.

Here, we notice that by the definition of complement, union and intersection of single valued neutrosophic sets, single valued neutrosophic sets satisfy the most properties of classic set, fuzzy set and intuitionistic fuzzy set. Same as fuzzy set and intuitionistic fuzzy set, it does not satisfy the principle of middle exclude.

3. Multi-objective linear programming problem (MOLPP)

A general multi-objective linear programming problem with p objectives, q constraints and n decision variables may be taken in the following form

Maximize $f_1(x)$ = c_1 X
Maximize $f_2(x)$ = c_2 X
…….
……....
Maximize $f_p(x)$ = c_p X
Subject to A X$\leq$ b and X$\geq$ 0
where C= $(c_{i1}, c_{i2}, \dots\dots\dots, c_{in})$ for i=1,2,…….p.
A= $[a_{j\,i}]_{q.n}$, X = $(x_1, x_2, \dots\dots\dots, x_n)^T$, b=$\left(b_1, b_2, \dots\dots\dots, b_q\right)^T$.
for j=1,2,……q; i=1,2,…….n.

4. Neutrosophic decision making

Decision making is a process of solving the problem involving pursuing the goals under constraints. The outcome is a decision which should in an action. Decision making plays an important role in an economic and business, management sciences, engineering and manufacturing, social and political science, biology and medicine, military, computer science etc. It is difficult process due to factors like incomplete and imprecise information which tend to be presented in real life situations. In the decision making process, our main target is to find the value from the selected set with the highest degree of membership in the decision set and these values support the goals under constraints only. But there may be situations arise where some values from selected set cannot support i.e. such values strongly against the goals under constraints which are non-admissible. In this case we find such values from the selected set with last degree of non-membership in the decision sets. Intuitionistic fuzzy sets can only handle incomplete information not the indeterminate information and inconsistent information which exists commonly in belief system. In neutrosophic set, indeterminacy is quantified explicitly and truth-membership, indeterminacy-membership and falsity-membership are independent. So it is natural to adopt for that purpose the value from the selected set with highest degree of truth-membership, indeterminacy-membership and least degree of falsity-membership on the decision set. These factors indicate that a decision making process takes place in neutrosophic environment.

Consider the multi-objective linear programming problem as

$$\text{Maximize } \{f_1(x), f_2(x), \dots\dots\dots f_p(x)\} \quad (1)$$

Subject to AX≤ b

where A = $(a_{ij})_{q\,.n}$, X= $(x_1, x_2, \dots\dots\dots, x_n)^T$, b = $\left(b_1, b_2, \dots\dots\dots, b_q\right)^T$.

Now the decision set $\widetilde{D}^n$, a conjunction of Neutrosophic objectives and constraints is defined as

$\widetilde{D}^n = (\bigcap_{k=1}^{p} \tilde{G}_k{}^n) \cap (\bigcap_{j=1}^{q} \tilde{C}_j{}^n) = \{(x, \mu_{\widetilde{D}^n}(x),\ \sigma_{\widetilde{D}^n}(x), \nu_{\widetilde{D}^n}(x))\}$

Here $\mu_{\widetilde{D}^n}(x) = \min(\mu_{\tilde{G}_1{}^n}(x), \mu_{\tilde{G}_2{}^n}(x), \dots\dots\dots, \mu_{\tilde{G}_p{}^n}(x);\ \mu_{\tilde{C}_1{}^n}(x), \mu_{\tilde{C}_2{}^n}(x), \dots\dots. \mu_{\tilde{C}_q{}^n}(x))$ for $all\ x \in X$

$\sigma_{\widetilde{D}^n}(x) = \min(\sigma_{\tilde{G}_1{}^n}(x), \sigma_{\tilde{G}_2{}^n}(x), \dots\dots\dots. \sigma_{\tilde{G}_p{}^n}(x);\ \sigma_{\tilde{C}_1{}^n}(x), \sigma_{\tilde{C}_2{}^n}(x), \dots\dots. \sigma_{\tilde{C}_q{}^n}(x))$ for all $x \in X$

$\nu_{\widetilde{D}^n}(x) = \max(\nu_{\tilde{G}_1{}^n}(x), \nu_{\tilde{G}_2{}^n}(x), \dots\dots\dots. \nu_{\tilde{G}_p{}^n}(x);\ \nu_{\tilde{C}_1{}^n}(x), \nu_{\tilde{C}_2{}^n}(x), \dots\dots. \nu_{\tilde{C}_q{}^n}(x))$ for $all\ x \in X$

where $\mu_{\widetilde{D}^n}(x)$, $\sigma_{\widetilde{D}^n}(x)$, $\nu_{\widetilde{D}^n}(x)$ are truth membership function, Indeterminacy membership function, falsity membership function of Neutrosophic decision set respectively. Now using the neutrosophic optimization the problem (1) is transformed to the linear programming problem as

Max α (2)
Min β
Max γ
Such that
$\mu_{\tilde{G}_k{}^n}(x) \geq \alpha$
$\mu_{\tilde{C}_k{}^n}(x) \geq \alpha$
$\sigma_{\tilde{G}_k{}^n}(x) \geq \gamma$
$\sigma_{\tilde{C}_k{}^n}(x) \geq \gamma$
$\nu_{\tilde{G}_k{}^n}(x) \leq \beta$
$\nu_{\tilde{C}_k{}^n}(x) \leq \beta$ for k=1,2,……..p.
α +β +γ ≤ 3
α ≥ β
α ≥ γ
α ,β, γ ∈ [0, 1]

Now this linear programming problem (2) can be easily solved by simplex method to give solution of multi-objective linear programming problem (1) by neutrosophic optimization approach.

5. Computational algorithm

A. Algorithm 1 (Linear membership functions)

Step 1. Pick the first objective function and solve it as a single objective subject to the constraints. Continue the process k-times for k different objective functions. Find value of objective functions and decision variables.

Step 2. To build membership functions, goals and tolerances should be determined at first. Using the ideal solutions, obtained in step-1 we find the values of all the objective functions at each ideal solution and construct pay-off matrix as follows:

$$\begin{bmatrix} f_1^*(x^1) f_2(x^1) \dots \dots \dots \dots f_p(x^1) \\ f_1(x^2) f_2^*(x^2) \dots \dots \dots \dots f_p(x^2) \\ \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \\ f_1(x^p) f_2(x^p) \dots \dots \dots \dots f_p^*(x^p) \end{bmatrix}$$

Step 3. From step-2 we find the upper and lower bounds of each objective functions.

$U_k{}^{\mu}$ = max $\{f_k(x_r{}^*)\}$ and $L_k{}^{\mu}$ = min $\{f_k(x_r{}^*)\}$ where 1≤ r ≤ k

For truth membership of objectives.

Step 4. We represents upper and lower bounds for indeterminacy and falsity membership of objectives as follows:

$L_k{}^{\nu} = L_k{}^{\mu}$ and $U_k{}^{\nu} = U_k{}^{\mu} - \lambda\,(U_k{}^{\mu} - L_k{}^{\mu})$

$U_k{}^{\sigma} = U_k{}^{\mu}$ and $L_k{}^{\sigma} = U_k{}^{\mu} - t\,(U_k{}^{\mu} - U_k{}^{\mu})$

Here λ and t are to predetermined real number in (0, 1).

Step 5. Define truth membership, Indeterminacy membership, falsity membership functions as follows:

$$\mu_k(f_k(x)) = \begin{cases} 0 & if\ f_k(x) \le L_k{}^{\mu} \\ \frac{f_k(x) - L_k{}^{\mu}}{U_k{}^{\mu} - L_k{}^{\mu}} & if\ L_k{}^{\mu} \le f_k(x) \le U_k{}^{\mu} \\ 1 & if\ f_k(x) \ge U_k{}^{\mu} \end{cases}$$

$$\sigma_k(f_k(x)) = \begin{cases} 0 & if\ f_k(x) \le L_k{}^{\sigma} \\ \frac{f_k(x) - L_k{}^{\sigma}}{U_k{}^{\sigma} - L_k{}^{\sigma}} & if\ L_k{}^{\sigma} \le f_k(x) \le U_k{}^{\sigma} \\ 1 & if\ f_k(x) \ge U_k{}^{\sigma} \end{cases}$$

$$\nu_k(f_k(x)) = \begin{cases} 1 & if\ f_k(x) \le L_k{}^{\nu} \\ \frac{U_k{}^{\nu} - f_k(x)}{U_k{}^{\nu} - L_k{}^{\nu}} & if\ L_k{}^{\nu} \le f_k(x) \le U_k{}^{\nu} \\ 0 & if\ f_k(x) \ge U_k{}^{\nu} \end{cases}$$

Step 6. Now neutrosophic optimization method for MOLP problem gives an equivalent linear programming problem as:

Max $\alpha + \beta + \gamma$ (3)

Such that

$\mu_k(f_k(x)) \geq \alpha$, $\sigma_k(f_k(x)) \geq \gamma$, $\nu_k(f_k(x)) \leq \beta$ for k=1,2, 3, ...,… p

$\alpha + \beta + \gamma \leq 3$,

$\alpha \geq \beta$, $\alpha \geq \gamma$

$\alpha, \beta, \gamma \in [0, 1]$

$AX \leq b$, $x \geq 0$,

where $A = (a_{ij})_{q.n}$, $X = (x_1, x_2, \dots\dots\dots, x_n)^T$, $b = (b_1, b_2, \dots\dots\dots, b_q)^T$.

k=1,2,…….p; j=1,2,……..q

which is reduced to equivalent linear-programming problem as

Max $\alpha + \beta + \gamma$ (4)

Such that

$f_k(x) - (U_k{}^{\mu} - L_k{}^{\mu}).\alpha \geq L_k{}^{\mu}$

$f_k(x) - (U_k{}^{\sigma} - L_k{}^{\sigma}).\gamma \geq L_k{}^{\sigma}$

$f_k(x) + (U_k{}^{\nu} - L_k{}^{\nu}).\beta \geq U_k{}^{\nu}$

$\alpha + \beta + \gamma \leq 3$

$\alpha \geq \beta$

$\alpha \geq \gamma$

$\alpha, \beta, \gamma \in [0, 1]$

$AX \leq b$, $x \geq 0$,

where $A = (a_{ij})_{q.n}$, $X = (x_1, x_2, \dots\dots\dots, x_n)^T$, $b = (b_1, b_2, \dots\dots\dots, b_q)^T$.

k=1,2,…….p; j=1,2,……..q.

B. **Algorithm 2 (Non-linear membership function)**

Repeat steps 1 to 4 and construct pay off matrix.

Step 5. Assume that solutions so far computed by algorithm follow exponential function for Truth membership, hyperbolic function for Falsity membership and exponential function for Indeterminacy membership function given as

$$\mu_k(f_k(x)) = \begin{cases} 0 & if\ f_k(x) \leq L_k{}^{\mu} \\ 1 - Exp\left\{-\Psi \frac{f_k(x) - L_k{}^{\mu}}{U_k{}^{\mu} - L_k{}^{\mu}}\right\} & if\ L_k{}^{\mu} \leq f_k(x) \leq U_k{}^{\mu} \\ 1 & if\ f_k(x) \geq U_k{}^{\mu} \end{cases}$$

$$\sigma_k(f_k(x)) = \begin{cases} 0 & if\ f_k(x) \leq L_k{}^{\sigma} \\ Exp\left(\frac{f_k(x) - L_k{}^{\sigma}}{U_k{}^{\sigma} - L_k{}^{\sigma}}\right) & if\ L_k{}^{\sigma} \leq f_k(x) \leq U_k{}^{\sigma} \\ 1 & if\ f_k(x) \geq U_k{}^{\sigma} \end{cases}$$

$$\nu_k(f_k(x)) = \begin{cases} 1 & if\ f_k(x) \le {L_k}^{\nu} \\ \frac{1}{2}+\frac{1}{2}tanh\left(\delta_k \frac{{U_k}^{\nu}+{L_k}^{\nu}}{2} - f_k(x)\right) & if\ {L_k}^{\nu} \le f_k(x) \le {U_k}^{\nu} \\ 0 & if\ f_k(x) \ge {U_k}^{\nu} \end{cases}$$

where Ψ, δ_k are nonzero parameters prescribed by the decision maker.

Step 6. Now neutrosophic optimization method for MOLP problem with the exponential Truth membership, hyperbolic Falsity membership and exponential indeterminacy membership functions gives the equivalent linear programming problem:

Max $\alpha + \beta + \gamma$ (5)

Such that

$\mu_k(f_k(x)) \ge \alpha$, $\sigma_k(f_k(x)) \ge \gamma$,

$\nu_k(f_k(x)) \le \beta$

$\alpha + \beta + \gamma \le 3$

$\alpha \ge \beta$

$\alpha \ge \gamma$

$\alpha, \beta, \gamma \in [0, 1]$

$AX \le b$

where $A = (a_{ij})_{q.n}$, $X = (x_1, x_2, \dots\dots\dots, x_n)^T$, $b = \left(b_1, b_2, \dots\dots\dots, b_q\right)^T$.

For solution convenience the above problem is transformed to

Maximize $\theta + \xi - \eta$ (6)

Subject to $f_k(x) - \frac{\theta({U_k}^{\mu}-{L_k}^{\mu})}{4} \ge {L_k}^{\mu}$

$f_k(x) - \frac{\eta}{\delta_k} \ge \frac{{U_k}^{\nu}+{L_k}^{\nu}}{2}$

$f_k(x) - \xi\,({U_k}^{\sigma} - {L_k}^{\sigma}) \ge {L_k}^{\sigma}$

where $\theta = -\log(1-\alpha)$, $\xi = \log\gamma$, $\eta = -\,tanh^{-1}(2\beta - 1)$, $\Psi = 4$, $\delta_k = \frac{6}{{U_k}^{\nu}-{L_k}^{\nu}}$

$\theta + \xi + \eta \le 3$

$\theta \ge \xi$

$\theta \ge \eta$

$\theta, \xi, \eta \in [0, 1]$

$AX \le b$

where $A = (a_{ij})_{q.n}$, $X = (x_1, x_2, \dots\dots\dots, x_n)^T$, $b = \left(b_1, b_2, \dots\dots\dots, b_q\right)^T$.

5. Illustrated example

Production planning problem

Consider a park of six machine types whose capacities are to be devoted to production of three products. A current capacity portfolio is available, measured in machine hours per week for each machine capacity unit priced according to machine type. Necessary data is summarized below in table 1.

Table 1: Physical parameter values

Machine type ($100 per hour)	Machine hours	Unit price	Products		
			x_1	x_2	x_3
Milling	1400	0.75	12	17	0
machine	1000	0.60	3	9	8
Lathe	1750	0.35	10	13	15
Grinder	1325	0.50	6	0	16
Jig saw	900	1.15	0	9.5	7
Drill press	1075	0.65	12	9.5	4
Band saw					
Total capacity cost	$4658.75				

Let x_1, x_2, x_3 denote three products, then the complete mathematical formulation of the above mentioned problem as a Multi-objective Linear Programming problem is given as:

Max $f_1(x) = 50x_1 + 100x_2 + 17.5x_3$(profit)

Max $f_2(x)$ $92x_1 + 75x_2 + 50x_3$(quality)

Max $f_3(x)$ $25x_1 + 100x_2 + 75x_3$(worker satisfaction)

Subject to the constraints

$12x_1 + 17x_2 \leq 1400$

$3x_1 + 9x_2 + 8x_3 \leq 1400$

$10x_1 + 13x_2 + 15x_3 \leq 1750$

$6x_1 + 16x_3 \leq 1325$

$x_1, x_2, x_3 \geq 0$.

Table 2: Positive ideal solution

	$f_1 f_2 f_3$		
Max f_1	8041.14	10020.33	9319.25
Max f_2	5452.63	10950.59	5903.00
Max f_3	7983.60	10056.99	9355.90

Table 3: Comparison of optimal solutions by IFO and NSO technique.

Optimization techniques	Optimal Decision Variables x_1^*, x_2^*, x_3^*	Optimal Objective Functions f_1^*, f_2^*, f_3^*	Aspiration levels of truth, falsity and indeterminacy membership functions	Sum of optimal objective values
Intuitionistic fuzzy optimization(IFO)	62.82, 38.005,	7673.2 10721.81	$\alpha^* = 0.75$ $\beta^* = 0.056$	26903.51

	41.84	8508.5		
Proposed neutrosophic optimization(NSO) Algorithm -1	64.01, 38.16, 41.77	7747.47 10839.42 8549.00	$\alpha^* = 0.73$ $\beta^* = 0.077$ $\gamma^* = 0.34$	27135.89
Proposed neutrosophic optimization(NSO) Algorithm-2	48.93038 45.41035 44.69078	7769.64 10341.91 9116.10	$\alpha^* = 0.4071$ $\beta^* = 0.50$ $\gamma^* = 0.999$	27227.653

Table 3 shows that Neutrosophic optimization gives better result than Intuitionistic fuzzy optimization.

5. Conclusions

In this paper, we presents simple Neutrosophic optimization approach to solve Multi-objective linear programming problem.it can be considered as an extension of fuzzy and intuitionistic fuzzy optimization .Also lower and upper bounds for the indeterminacy membership functions are defined. The empirical tests show that optimal solutions of Neutrosophic optimization approach can satisfy the objective function with higher degree than the solutions of fuzzy and intuitionistic fuzzy programming approach. The results thus obtained also reveal that neutrosophic optimization by proposed algorithm- 2 using non-linear Truth, Indeterminacy, Falsity membership functions give a better result than neutrosophic optimization by proposed algorithm- 1 using linear Truth, Indeterminacy, Falsity membership functions.

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